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Studies on Growth of Oscillation in a Class of Nonlinear Oscillator

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ABSTRACT
Growth of oscillation in Rayleigh-Duffing type oscillator has been studied using modern method of nonlinear dynamics. Primary aim is to predict the parameter range of the system where system becomes oscillatory. Analytically predicted range of system parameters are found to be in close agreement with the corresponding estimated values obtained by numerically solving system equations. This study is very helpful to explain the behaviour of a microwave Gunn oscillator and any other types of nonlinear oscillators in the free running as well as periodically driven condition.

Keywords : Duffing Oscillator, Rayleigh Duffing Oscillator, Nonlinear Dynamics

Introduction:

Duffing oscillator (DO) is well studied and leads to interesting conclusions among different oscillator model found in literatures [1-2]. It was proposed to explore the conservative motion in a double well potential observed in a magneto-elastic mechanical system [3-4]. A forced Duffing oscillator model is also applicable to describe the self-oscillations in Josephson junctions, plasma oscillations, optical bi-stability etc. [5-6]. Besides the restoring force terms written as a power series of displacement, DO equations contain linear as well as nonlinear damping terms dependent on the velocity. Now to describe the nonlinear dynamics of some systems some higher order velocity damping term is used, for example to explain the dynamics of Brusselator, Selkov, rolling response, certain micro-electromechanical systems (MEMS) a cubic order velocity dissipative term is introduced in the system equation of Duffing oscillator [7-10]. This cubic order of velocity damping term is known as Rayleigh dissipation and with this dissipation term, the Duffing oscillator is known as Rayleigh-Duffing oscillator (RDO). Different damping terms present in a system, sometimes may change the sign depending on velocity or displacement values, and provide the excitation energy to the system [7]. Chaotic responses in a periodically driven Duffing oscillator have been reported in the literature [11-12]. Similarly some articles on the dynamics of RDO have also been reported [13-15]. In most of the work the mechanisms by which the strange attractors arise and are modified with the variation of system parameters have been studied. In this paper, we have theoretically derived the condition of growth of oscillation in RDO by considering unequal values of linear and cubic damping coefficients.

The rest of the paper is organized in the following way. The dynamics of a free running RDO has been studied in section-2, The values of damping coefficients required for stable oscillations have been determined analytically in section-3. Finally the paper ends in section-4, with some concluding remarks.

Dynamics of a Free Running R-D Oscillator:

The system equation of RDO is a second order differential equation in terms of a state variable *q* as given below:

$$\frac{d^2y}{d\tau^2} = ay - by^3 + c(\frac{dy}{d\tau}) - d(\frac{dy}{d\tau})^3 \qquad (1)$$

Here, τ is the normalized time and *a, b, c* and *d* are parameters used to express the relative strengths of respective restoring and damping forces. The state variable *q* is equivalent to the displacement and its time derivative, denoted by $p = (dq/d\tau)$, is equivalent to the velocity of the mechanical system. For an electrical oscillator, state variables q and p are equivalent to normalized charge and current respectively. The parameters *a* and *b* are respectively linear and nonlinear restoring force coefficients and in an electrical oscillator, they are dependent on the magnitudes of storage elements. Similarly, *c* and *d* represent linear and nonlinear damping force coefficients. The equation of a conventional Duffing oscillator is similar to (1), with the value of *c* negative and the parameter $d=0$. Without an external forcing signal, sustained autonomous oscillations are not possible in a Duffing oscillator. The presence of cubic order restoring force in (1) is the reason of naming it as a Duffing oscillator and the cubic order damping term introduces the qualifier Rayleigh in its name.

The conditions of an autonomous limit cycle oscillation of the RDO, described by equation (1), could be obtained by examining the stable points of the system as reported in [15-16]. For this purpose, we decompose equation (1) into two first order differential equations describing time evolution of two state variables as follows:

$$\frac{dq}{d\tau} = p \qquad (2a)$$

$$\frac{dp}{d\tau} = \alpha q - bq^3 + cp - dp^3 \qquad (2b)$$

These two equations would be used to explore the dynamics analytically as well as numerically.

Analytical study:

In the two dimensional (*q, p*) phase plane, there are three fixed points of the system described by equation (2). It is obtained by equating time derivatives of *q* and *p* to zero. The (*q, p*) co-ordinates of the fixed points are (0, 0), $(+\sqrt{a/b}$, 0) and $(-\sqrt{a/b}$, 0). One can proceed to find the stability of these fixed points by finding the linearized transformation Jacobian at these fixed points and evaluating the nature of the roots of the characteristic equations of the Jacobian matrix[15]. The fixed point (0, 0) is a saddle point, since both the roots of the characteristic equation at this point are real, one of them is positive and the other is negative; this is true for a positive value of *a* and any value of *c*. The roots of the characteristic equation at other two fixed points are $\frac{1}{2}\left(c+\sqrt{c^2-8a}\right)$ and $\frac{1}{2}\left(c-\sqrt{c^2-8a}\right)$. Here we get a pair of complex roots when $c^2 < 8a$ for both positive and negative values of *c*. If *c* be negative, the real parts of the complex roots are negative and thus the system dynamics converges to spiral nodes and so steady oscillations could not be obtained at that condition. For a positive value of *c,* the complex roots have positive real parts and thus the system dynamics diverges in a spiral path. However, the presence of the cubic type nonlinear damping term in the equation, proportional to *d*, would limit the growing amplitude of oscillation. Thus an amplitude stabilizing effect would be obtained and we get sustained oscillations. For *c* value at zero one can observe a bifurcation in the dynamics because then the roots are purely imaginary with zero real parts. This is a case of Hopf bifurcation and the frequency of limit cycle oscillation is dependent on the parameter *a* of the RDO equation. Mathematically we can study the variation of the system dynamics with the variation of *c*, for fixed values of other parameters. We summarize the conditions of autonomous oscillation of equation (1) as follows:

(i) the parameters *a* and *b* should be positive. This makes the potential function of the system a double well-type in nature, the stable points are symmetrically placed about q=0;

(ii) The magnitude of *c* should be greater than or equal to zero for building up of self-oscillations; i.e., with a negative value of *c*, no sustained oscillations are possible in this oscillator;

(iii) The presence of the parameter *d* of positive sign is necessary to stabilize the sustained oscillation of finite amplitude.

The analytical prediction has been shown by drawing phase plane plot of the oscillation after numerical simulation of the system equation. The phase plane plots for positive and negative values of "c" have been shown in Fig.1.

We do further analyze the effect of parameter variation in the dynamics of system described by (2) by converting the Cartesian (*q-p*) phase space into a polar form. We substitute $q = r\cos(\theta)$ and $p = r\sin(\theta)$ in the two sides of (2b), where r and θ are the time dependent radius vector and argument of the instantaneous phase point [22]. For a better modeling of the nonlinear damping, we include the effect of a fifth order damping term ep^5 in the right hand side of (2b) where *e* is another system parameter. Further we take the contributions of only the fundamental components, like sin(θ) and cos(θ), in the expansions of higher degrees of these trigonometric functions because of the tuned condition of the oscillator circuit. After simplification, we get the following equations involving $\dot{r}$ and $\dot{\theta}$:

$$\dot{r}\sin(\theta) + r\dot{\theta}\cos(\theta) = \left(ar - \frac{3}{4}br^3 \right)\cos(\theta) + \left(cr - \frac{3}{4}dr^3 + \frac{3}{8}er^5 \right)\sin(\theta) \qquad (3)$$

From (3) we get time evolution equations of r and θ as follows:

$$\dot{r} = cr - \frac{3}{4}dr^3 + \frac{3}{8}er^5 \qquad (4a)$$

$$\dot{\theta} = a - \frac{3}{4}br^2 \qquad (4b)$$

The implication of (4b) is that the frequency of the oscillator has a dependence on the amplitude of oscillation as is evident by the presence of the term $\frac{3}{4}br^2$ in the right hand side. The steady state amplitude of oscillation depends on the system parameters in a complicated way which is derived by putting $\dot{r}$ equal to zero in (4a). We have one of the possible steady states as $r=0$, i.e. no oscillation what so ever. Moreover, when $e=0$, $r^2 = \frac{4c}{3d}$; this means, for a positive d, the value of c cannot be negative to have oscillation in the system. For the case $e>0$, an interesting situation is observed with the variation of *c* for fixed values of *d* and *e*. We search for real positive roots for r^2, so that real value of the radius vector could be obtained. The obtained values of r are plotted for different values of c in the Fig 2.

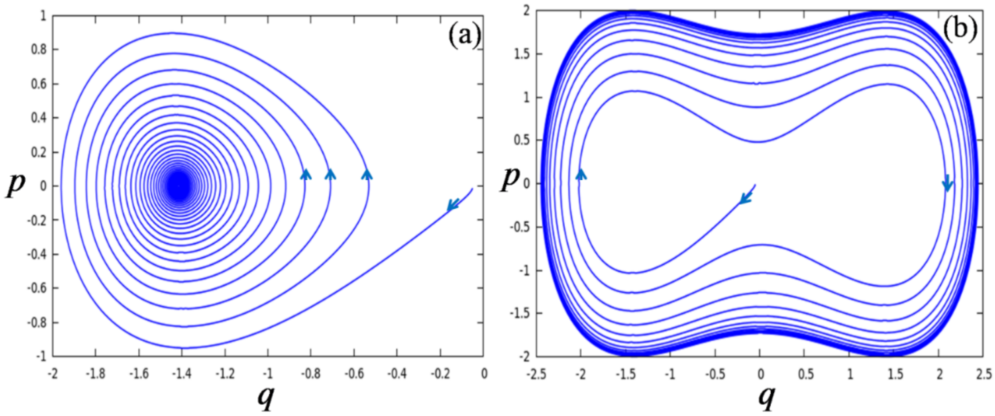

Fig. 1: Numerically computed phase plane plot (q-p) of the oscillator for (a) $c < 0$ (non-oscillatory condition), (b) $c > 0$ (Normal oscillating condition) $[a = 1, b = 0.5, d = 0.015]$.

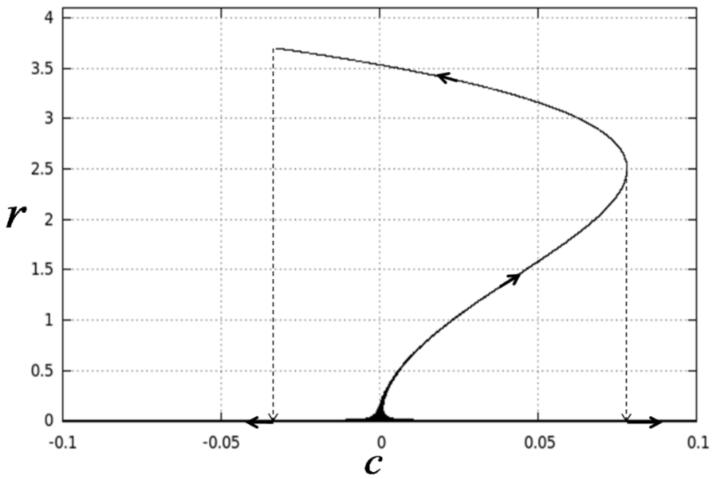

Fig.2. Numerically computed variation of r with the parameter c [Equation 4(a)] $[a = 1, b = 0.5, d = 0.015]$.

Following conclusions could be drawn from these results: (i) there is a specific range of c where r is a positive real number; (ii) r increases with c up to a limit and then decreases following a different path; (iii) there is a clear evidence of hysteresis phenomenon in the onset and quenching of oscillation in the free running Rayleigh- Duffing oscillator obtained by the parameter variation exercise.

Conclusion

Oscillatory behavior of RDO has been analytically explained with the help of modern theory of nonlinear dynamics. Damping parameter of RDO plays the important role to control oscillatory states. Growth of oscillation for different values of damping parameters has also been analyzed and numerically verified. Numerical results prove the analytical predictions. This study is very fruitful to describe DO and RDO type different electronic oscillator systems different biological systems to predict different behaviors.

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