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Perfectly g*b-Continuous Functions

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ABSTRACT

The purpose of this paper is to introduce new classes of functions of g*b-irresolute function, strongly g*b-continuous functions and perfectly g*b-continuous function in topological spaces. Some of their properties and several characterizations of these types of functions are discussed. Also we investigate the relationship between these classes of functions.

Keywords: b closed sets, strongly continuous functions, g-continuous functions, b irresolute functions.

I. INTRODUCTION

Several authors working in the field of general topology have shown more interest in studying the properties of generalizations of continuous and closed functions. Some of them are Arokiarani et al [1,3,4], Devi et al [5,6], Mashour et al [7]. Noiri [9] introduced strong forms of g-irresolute functions and studied their properties.. Iyappan and Nagaveni [10] introduced sgb-continuous functions and semi generalized b-continuous maps. Recently R S Wali et al [11] introduced and studied the properties of αrω-continuous and αrω-irresolute functions in topological spaces. The concept of regular continuous and Completely–continuous functions was first introduced by Arya. S. P. and Gupta.R [2]. In 1981, Munshy and Bassan [8] introduced the notion of generalized continuous (briefly g-continuous) functions which are called in as g-irresolute functions.

II. METHODS AND MATERIAL

2. Preliminaries

We recall the following definitions which are useful in the sequel

Definition 2.1 A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called

(i). Strongly continuous if $f^{-1}(V)$ is both open and closed in (X,τ) for each subset V of (Y,σ).

(ii). strongly g-continuous if the inverse image of every g-open set in (Y,σ) is open in (X,τ).

(iii). perfectly g-continuous if the inverse image of every g-open set in (Y,σ) is both open and closed in (X,τ).

(iv). glc-continuous if $f^{-1}(V)$ is glc-closed in (X,τ) for every $V\in(Y,\sigma)$

(v). irresolute if $f^{-1}(V)$ is semi-open in (X,τ) for each semi-open set V of (Y,σ),

(vi). gc- irresolute if $f^{-1}(V)$ is g-closed in (X,τ) for each g-closed set V of (Y,σ),

(vii). αg- irresolute if $f^{-1}(V)$ is αg-closed in (X,τ) for each g-closed set V of (Y,σ),

(viii). gs-irrresolute if $f^{-1}(V)$ is gs-closed in (X,τ) for each gs-closed set V of (Y,σ),

(ix). gα-irresolute in (X,τ) if and only if g-irresolute in (X,τ^{α}) .

3. g*b IRRESOLUTE FUNCTIONS

Definition 3.2 A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called g* b-irresolute (g*b-irresolute) if the inverse image of every g*b-closed set V of (Y,σ) is g*b-closed in (X,τ).

Remark 3.3 The following examples shows that the notion of gc-irresolute functions and g*b closed irresolute functions are independent.

Example 3.4 **(i)**Let $X=Y=\{a,b,c\}$, $\tau=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$ and $\sigma=\{X,\varphi,\{a\},\{b,c\}\}$ Then the identity function on X is g*b-irresolute but not gc-irresolute.

Example 3.5 Let $X=Y=\{a,b,c\}$, $\tau=\{X,\varphi,\{a\},\{b,c\}\}$ and $\sigma=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$.

Then the function $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by $f(a)=b$, $f(b)=a$, $f(c)=c$ is gc-irresolute but not g*b-irresolute.

Remark 3.6 The irresolute functions and g*b-irresolute functions are independent and can be seen from the following example.

Example 3.7 Let $X=Y=\{a,b,c\}$, $\tau=\{X,\varphi,\{b\},\{a,b\}\}$ and $\sigma=\{X,\varphi,\{a\},\{a,b\}\}$. Then the function $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by $f(a)=b$, $f(b)=a$, $f(c)=c$ is irresolute but not g*b-irresolute.

Example 3.8 $X=Y=\{a,b,c\}$, $\tau=\{X,\varphi,\{a\},\{a,b\}\}$ and $\sigma=\{X,\varphi,\{a\}\}$ .Then the function $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by $f(a)=b$, $f(b)=a$, $f(c)=c$ is g*b-irresolute but not irresolute.

Theorem 3.9 A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is g*b-irresolute if and only if the inverse image of every g*b-open set in (Y,σ) is g*b-open in (X,τ).

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$) be g*b-irresolute and U be a g*b-open set in (Y,σ). Then U^c is g*b-closed in (Y,σ) and since f is g*b-irresolute, $f^{-1}(U^c)$ is g*b-closed in (X,τ). But $f^{-1}(U^c)$ = $(f^{-1}(U))^c$ and so $f^{-1}(U)$ is g*b-open in (X,τ).

Conversely, assume that $f^{-1}(U)$ is g*b-open in (X,τ) for each open set U in (Y,σ). Let f be a g*b-closed set in (Y,σ). Then U^c is g*b-open in (Y,σ) and by assumption $f^{-1}(U)$ is g*b-open in (X,τ). Since $f^{-1}(U^c)$ = $(f^{-1}(U))^c$, we have $f^{-1}(F)$ is g*b-closed in (X,τ) and so f is g*b-irresolute.

Theorem 3.10 If a function $f:(X,\tau)\rightarrow(Y,\sigma)$is g*b-irresolute then it is g*b-continuous but not conversely

Example 3.11 Let $X=Y=\{a,b,c,d\}$, $\tau=\{X,\varphi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}=\sigma$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be defined by $f(a)=b$, $f(b)=c$, $f(c)=d$, $f(d)=a$. Then f is g*b-continuous but not g*b-irresolute since $f^{-1}(b)=\{a\}$ is not g*b-closed in X.

Theorem 3.12 Let (X,τ) be any topological space, (Y,σ) be a T_{g*b}-space and $f:(X,\tau)\rightarrow(Y,\sigma)$be a function**.** Then the following are equivalent

(i) f is g*b-irresolute

(ii) f is g*b-continuous.

Proof: (i) $\Rightarrow$ (ii): obvious

(ii) $\Rightarrow$ (i) Let F be an g*b-closed set in (Y,σ). Since (Y,σ) is a Tg*b-space, F is a closed set in (Y,σ) and by hypothesis, $f^{-1}(F)$ is g*b-closed in (X,τ). Therefore f is g*b-irresolute.

Theorem 3.13 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is bijective, b-open and g*b-continuous then f is g*b-irresolute.

Proof: Let V be any g*b-closed set in Y and let $f^{-1}(V)\subset U$ where U is b-open in X. Then $V\subset f(U)$. Since $f(U)$ is b-open in Y and V is g*b-closed in Y , then cl(V) $\subset$ $f(U)$ implies that $f^{-1}cl(V)\subset U$. Since f is g*b-continuous, $f^{-1}cl(V)$ is g*b-closed in X. Hence $cl(f^{-1}cl(V))\subset U$ .Therefore $cl(f^{-1}(V))\subset cl(f^{-1}cl(V))\subset U$. That is $cl(f^{-1}(V))\subset U$. This shows that f is g*b-irresolute.

Theorem 3.14 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is bijective, closed and irresolute then the inverse function $f^{-1}:(Y,\sigma)\rightarrow(X,\tau)$ is g*b-irresolute.

Proof: Let A be g*b-closed in (X,τ). Let $(f^{-1})^{-1}(A)=f(A)\subseteq U$, where U is g-open in (Y,σ). Then $A\subseteq f^{-1}(U)$ holds. Since $f^{-1}(U)$ is g-open in (X,τ) and A is g*b-closed in (X,τ), $bcl(A)\subseteq f^{-1}(U)$ and hence $f(bcl(A))\subseteq U$. Since f is b-closed and bcl(A) is closed in (X,τ), $f(bcl(A))$ is b-closed in (Y,σ). Therefore $bcl(f(bcl(A)))\subseteq U$ and hence $bcl(f(A))\subseteq(U)$. Thus $f(A)$ is g*b-closed in (Y,σ) and so $f^{-1}$is g*b-irresolute.

Definition 3.15 A function $f:(X,\tau)\rightarrow(Y,\sigma)$is called strongly g*b-continuous if the inverse image of every g*b-open set in (Y,σ) is open in (X, τ).

Theorem 3.16 If $f:(X,\tau)\to(Y,\sigma)$ is strongly g*b-continuous, then it is strongly g-continuous but not conversely.

Example 3.17 Let $X=\{a,b,c\}$, $\tau=\{X,\varphi,\{a\},\{b\},\{a,b\}\}=\sigma$. Let $f:(X,\tau)\to(Y,\sigma)$ be defined by $f(a)=b$, $f(b)=a$, $f(c)=c$. Then f is strongly g-continuous but not strongly g*b-continuous.

Theorem 3.18 Let (X,τ)be any topological space and (Y,σ) be a T_{g*b}-space and $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is strongly g*b-continuous

(ii) f is continuous.

Theorem 3.19 If $f:(X,\tau)\to(Y,\sigma)$ is strongly continuous then it is strongly g*b-continuous but not conversely.

Theorem 3.20 Let $f:(X,\tau)\to(Y,\sigma)$ be onto, g*b-irresolute and b-closed. If (X,τ) is a T_b -space, then (Y,σ) is also a T_b -space**.**

Proof: Let A be a g*b-closed subset of(Y,σ)**.** Since f is g*b-irresolute, then $f^{-1}(A)$ is g*b-closed in (X,τ).Since (X,τ) is a T_b -space, then $f^{-1}(A)$ is a b-closed in (X,τ).Thus A is b-closed in (Y,σ) because f is surjective. Hence (Y,σ) is a T_b -space.

Theorem 3.21 Let $f:(X,\tau)\to(Y,\sigma)$ is g*b-irresolute and closed. If (X,τ) is almost weakly Hausdroff, then (Y,σ) is almost weakly Hausdroff space.

Proof: Let V be any g*b-closed set in Y. Since f is g*b-irresolute, $f^{-1}(V)$ is g*b-closed in X. Since (X,τ) is is almost weakly Hausdroff, $f^{-1}(V)$ is closed in X, then V is closed because f is closed and onto. Hence Y is almost weakly Hausdorff space.

Theorem 3.22 Let (X,τ) be a discrete topological space, (Y,σ) be a g*b-space and $f:(X,\tau)\to(Y,\sigma)$ be a function. Then the following are equivalent.

(i) f is strongly continuous

(ii) f is strongly g*b-continuous.

Proof: (i) $\Rightarrow$ (ii) Follows from the Theorem 3.19.

(ii) $\Rightarrow$ (i) Let U be any g*b-set in (Y,σ). Since (Y,σ) is a g*b-space, U is a g*b-open subset of (Y,σ) and by hypothesis, $f^{-1}(U)$ is open in (X,τ). But (X,τ) is a discrete space and so $f^{-1}(U)$ is also closed in (X,τ). That is $f^{-1}(U)$ is both open and closed in (X,τ) and so f is strongly continuous.

Theorem 3.23 Let $f:(X,\tau)\to(Y,\sigma)$ be a function and both (X,τ) and (Y,σ) be T_{g*b} -spaces. Then the following are equivalent.

(i) f is strongly g*b-continuous,

(ii) f is continuous,

(iii) f is g*b-continuous.

Theorem 3.24 A function $f:(X,\tau)\to(Y,\sigma)$ is strongly g*b-continuous if and only if the inverse image of every g*b-closed set in (Y,σ) is closed in (X,τ)

Theorem 3.25 If $f:(X,\tau)\to(Y,\sigma)$ and g: (Y,σ) →(Z,η) are strongly g*b-continuous, then their composition $g\circ f:(X,\tau)\to(Z,\eta)$ is also strongly continuous.

Proof: Let U be a g*b-open set in (Z,η). Since g is strongly g*b-continuous, $g^{-1}(U)$ is open in (Y,σ). Since $g^{-1}(U)$ is open, it is g*b-open in (Y,σ). As f is also strongly g*b-continuous, $f^{-1}(g^{-1}(U))=(g\circ f)^{-1}(U)$ is open in (X,τ) and so gof is strongly continuous.

Theorem 3.26 If $f:(X,\tau)\to(Y,\sigma)$ and g: (Y,σ) →(Z,η) be any two functions then their composition $g\circ f:(X,\tau)\to(Z,\eta)$ is

(i) Strongly g*b-continuous if g is strongly g*b-continuous and f is strongly g*b-continuous.

(ii) g*b-irresolute if g is strongly g*b-continuous and f is g*b-continuous (or f is g*b-irresolute).

(iii) Strongly g*b-continuous if g is strongly continuous and f is irresolute.

(iv) Continuous if g is g*b-continuous and f is strongly g*b-continuous.

We next introduce perfectly g*b-continuous functions in topological spaces.

Definition 3.27 A function $f:(X,\tau)\to(Y,\sigma)$ is called perfectly g*b-continuous if the inverse image of every g*b-open set in (Y,σ) is both open and closed in (X,τ).

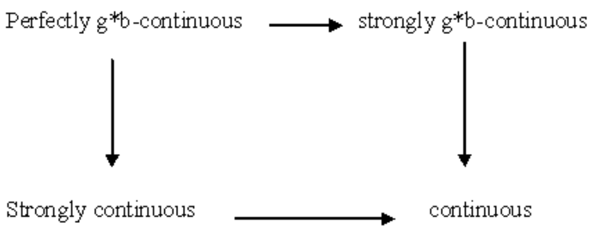
Theorem 3.28 If $f:(X,\tau)\to(Y,\sigma)$ is perfectly g*b-continuous then it is strongly g*b- continuous.

Proof: Since $f:(X,\tau)\to(Y,\sigma)$ is perfectly g*b-continuous, $f^{-1}(U)$ is both open and closed in X, for every g*b-open set Y in X. Therefore f is strongly g*b-continuous.

Theorem 3.29 If $f:(X,\tau)\to(Y,\sigma)$ is strongly continuous then it is perfectly g*b-continuous.

Proof: Since $f:(X,\tau)\to(Y,\sigma)$ is strongly continuous, $f^{-1}(U)$ is both open and closed in (X,τ), for every g*b-open set U in (Y,σ). Therefore f is perfectly g*b-continuous.

We have the following implications for the above results.



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