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Study of Thermal Stability of the System and Oscillatory Modes in Newtonian Fluids

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ABSTRACT

In this present paper, we studied about thermal stability in a Newtonian fluid. The theory has been applied to the study of some simple lubrication problems. According to the theory, couple-stresses are found to appear in noticeable magnitudes in fluids with very large molecules. Since the long chain hyaluronic acid molecules are found as additives in synovial fluid, Walicki and Walicka [80] modeled synovial fluid as a couple-stress fluid in human joints. A human joint is a dynamically loaded bearing which has particular cartilage as the bearing and synovial fluid as the lubricant. Normal synovial fluid is clear or yellowish and is a non-Newtonian, viscous fluid.

Keywords : Thermal Stability, Newtonian Fluid, Instability, Rotation, Magnetic Field.

I. INTRODUCTION

G. K. [1] showed that the bearing with a couple of stresses in fluid as the lubricant improves the squeeze film characteristic and results in a longer bearing life. All diseases of joints are caused by or connected with a malfunction of the lubrication. One of the applications of couple-stresses in fluid is its use in the study of the mechanism of lubrication of synovial joints, which has become the objective of scientific research. The problem of couple-stress fluid heated from below in a porous medium is considered by Roberts [5] and Lundquist [2]. Lehnert [3], Cowling et al [6], Bateman et al [7], and Sutton et al [4] considered the effect of compressibility, suspended particles, and rotation on thermal convection in an elastic-viscous fluid in hydromagnetic.

II. MATERIALS AND METHODS

Consider an infinite, horizontal, electrically conducting, incompressible, couple-stress fluid layer to thickness d, bounded by the planes $z=0$ and $z=d$.

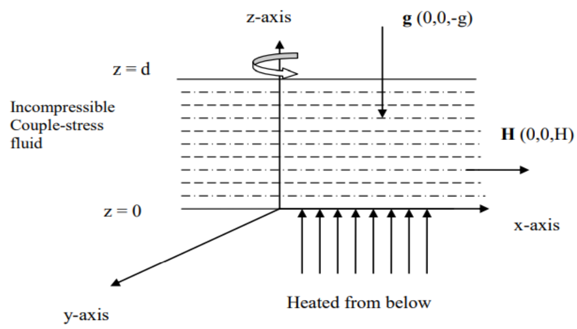

Figure 1: Geometrical configuration

This fluid layer is heated from below so that a uniform temperature gradient $\beta\left(\left|\frac{dT}{dz}\right|\right)$ is maintained and the layer is acted upon by the gravity field g(0,0,-g), a uniform vertical magnetic field H(0,0 *H*) and rotation $\Omega(0,0,\Omega)$.

Let $p, \rho, T, a, v, \mu', k, and\ q(u, v, w)$ denote respectively pressure, density, temperature, thermal coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity and velocity of the fluid. The equation of motion, continuity and heat conduction of couple-stress fluid are

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) + \left(v - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q + 2(q \times \Omega) + \frac{\mu_e}{4\pi\rho_0}[\nabla \times H \times H] \qquad (1)$$

$$\nabla.q = 0 \qquad (2)$$

$$\frac{\partial T}{\partial t} + (q.\nabla)T = k\nabla^2 T \qquad (3)$$

$$\frac{\partial H}{\partial t} = (H.\nabla)q + \eta\nabla^2 H \qquad (4)$$

$$\nabla.H = 0 \qquad (5)$$

The equation of state is

$$\rho = \rho_0[1 - \alpha)T - T_0)] \qquad (6)$$

where the suffix zero refers to the values at the reference level z=0.

The basic motionless solution is

$q = (0,0,0), p = p(z), T = T_o - \beta z, \rho = \rho_o(1 + \alpha\beta z), N = N_o$, a constant. (7)

Assume small perturbations around the basic solution and let $\delta p, \delta\rho, \theta, N, H\ (h_x, h_y, H + h_z)$ and q(u,v,w,) denote respectively the perturbations in pressure, density, temperature, number density, magnetic field and couple-stress fluid velocity (0,0,0). The change in density $\delta\rho$ caused mainly by the perturbbation θ in temperature is given by

$$\delta\rho = -\alpha\,\rho_o\theta \qquad (8)$$

Then the linearized perturbation equations of the couple-stress fluid become

$$\frac{\partial q}{\partial t} = -\frac{1}{\rho_o}\nabla\delta p - ga\theta + \left(v - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q + 2(q \times \Omega) + \frac{\mu_e}{4\pi\rho_0}[\nabla \times H \times H] \qquad (9)$$

$$\nabla.q = 0 \qquad (10)$$

$$\frac{\partial\theta}{\partial t} = \beta w + k\nabla^2\theta \qquad (11)$$

$$\frac{\partial q}{\partial t} = (H.\nabla)q + \eta\nabla^2 h \qquad (12)$$

$$\nabla.h = 0 \qquad (13)$$

where $k = \frac{q}{\rho_0 c_v}$

Dispersion Relation

Analyze the perturbations in to normal modes by seeking solutions in the form $[w, \theta, h, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)]\exp(ik_x x + ik_y y + nt)$. where k_x, k_y are the wave numbers along x and y directions respectively, and $k = \left(k_x^2 + k_y^2\right)^{1/2}$ is the resultant wave number of the disturbance and n is the growth rate which is, in general, a complex constant and $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\ and\ \xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively.

The non-dimensional system of equations eliminating the physical quantities is

$$[\sigma - (1 - F(D^2 - a^2))(D^2 - a^2)](D^2 - a^2)W = -\frac{g\alpha a^2 d^2}{\upsilon}\Theta - \frac{2\Omega d^3}{\upsilon}DZ + \frac{\mu_e H d}{4\pi\rho_0 \nu}(D^2 - a^2)DK$$

(14)

$$[\sigma - (1 - F(D^2 - a^2))(D^2 - a^2)]Z = \left(\frac{2\Omega d}{\upsilon}\right)DW + \frac{\mu_e H d}{4\pi\rho_0 \nu}DK \quad (15)$$

$$(D^2 - a^2 - \sigma p_1)\Theta = -\left(\frac{\beta d^2}{k}\right)W \quad (16)$$

$$(D^2 - a^2 - p_2\sigma)\mathrm{K} = -\frac{Hd}{\eta}DW \quad (17)$$

$$(D^2 - a^2 - p_2\sigma)\mathrm{X} = -\frac{Hd}{\eta}DZ \quad (18)$$

Eliminating Z, X, Θ and K between equations (14) – (18), we obtain

$$[\sigma + F(D^2 - a^2)^2 - (D^2 - a^2)][D^2 - a^2 - \sigma p_1][D^2 - a^2 - \sigma p_2][D^2 - a^2]W$$

$$+R\lambda a^2[D^2 - a^2 - \sigma p_2]W + Q[D^2 - a^2 - \sigma p_1][D^2 - a^2]D^2W \quad (19) \quad +T_A \frac{[D^2-a^2-\sigma p_2]^2[D^2-a^2-\sigma p_1]D^2W}{[\{\sigma+F(D^2-a^2)-(D^2-a^2)\}(D^2-a^2-\sigma p_2)+QD]} = 0$$

where $R = \frac{g\alpha\beta d^4}{\nu k}$ is the thermal Rayleigh number, $T_A = \left(\frac{2\Omega d^2}{y}\right)^2$ is the Taylor's number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_o \nu\eta}$ is the Chandrasekhar number.

Consider the case in which both the boundaries are free, the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The boundary conditions, appropriate for the problem, are

$$W = 0 = Z = \Theta \ and \ D^2W = 0 \quad at \quad z = 0 \ and \ z = 1 \quad (20)$$

$$W = W_0 \sin\pi z \quad (21)$$

where W_0 is constant. Substituting the proper solution (21) to equation (19), we obtain the dispersion relation

$$R_1 = \frac{(1+x)}{\lambda x}[i\sigma + F_1 + (1+x)][1 + x + i\sigma p_1] + \frac{Q_1(1+x)[1+x+i\sigma_1 p_1]}{\lambda x[1+x+i\sigma_1 p_1]}$$

$$+ \frac{T_{A_1}[1+x+i\sigma_1 p_2][1+x+i\sigma_1 p_1]}{\lambda x[\{i\sigma+F_1(1+x)^2+(1+x)\}(1+x+i\sigma_1 p_2)+Q_1]} \quad (22)$$

where, $R_1 = \frac{R}{\pi^4}, T_{A_1} = \frac{T_A}{\pi^4}, i\sigma_1 = \frac{\sigma}{\pi^2}, Q_1 = \frac{Q}{\pi^2} \ and \ F_1 = \pi^2 F$

The above relation expresses the modified Rayleigh number R_1 as a function of couple stress parameter F_1, rotation parameter T_{A_1}, magnetic field parameter Q_1 and dimension less wave number x.

Stationary Convection

For stationary convection, the marginal state will be characterized by $\sigma = 0$. Thus equation (22) reduces to

$$R_1 = \frac{(1+x)}{\lambda x}\left[\{F_1(1+x) + 1\}(1+x)^2 + Q_1 + \frac{T_{A_1}(1+x)}{[\{F_1(1+x)+1\}](1+x)^2+Q_1}\right]$$

(23)

To study the effect of suspended particles, rotation, couple-stress and magnetic field, we examine the nature of $\frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dF_1}$ *and* $\frac{dR_1}{dQ_1}$ analytically. Equation (23) gives

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{1+x}{x}\right)\left\{\frac{1+x}{[1+F_1(1+x)](1+x)^2+Q_1}\right\} \quad (24)$$

which shows that rotation has a stabilizing effect on the system.

Also, from equation (3.1.23), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{x}, \left\{1 - T_{A_1}\frac{1+x}{\{[1+F_1(1+x)](1+x)^2+Q_1\}^2}\right\} \quad (25)$$

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{x}\right).\left\{T_{A_1}\frac{1+x}{\{[1+F_1(1+x)](1+x)^2+Q_1\}^2} + 1\right\} \quad (26)$$

which shows that couple-stresses and magnetic field have a stabilizing or destabilizing effect on the system according to $T_{A_1}(1+x) < or > \{[1+F_1(1+x)](1+x)^2 + Q_1\}^2$

In the absence of rotation ($T_{A_1} = 0$), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{x} \quad (27)$$

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{x}\right) \quad (28)$$

which shows that couple-stresses and magnetic field clearly have a stabilizing effect on the system.

In the absence of magnetic field ($Q_1 = 0$), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{x} \cdot \frac{1}{H}\left\{1 - T_{A_1}\frac{1}{[1+F_1(1+x)]^2(1+x)^3}\right\} \quad (29)$$

which shows that couple-stress has a stabilizing (destabilizing) effect on the system according as $T_{A_1} < or > (1+x)^3[1+F_1(1+x)]^2$

The dispersion relation (22) is also analyzed numerically. In figure 2, R_1 is plotted against x for $T_{A_1} = 100, 150, 200, F_1 = 0.5$ *and* $Q_1 = 10$. In figure 3, R_1 is plotted against notation parameter T_{A_1} for various values of wave number x. In both the figures, it is found that rotation postpones the onset of convection as the Rayleigh number increases with the increase in rotation parameter. In figure 4, R_1 is plotted against x for $Q_1 = 20,40,60$; $F_1 = 0.2, T_{A_1} = 70$ and in figure 5 R_1 is plotted against Q_1 for x =12,4,8,10. Here it is observed that the magnetic field hastens the onset of convection for small wave numbers as the Rayleigh number decreases with an increase in the magnetic field parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number decreases with the increase in couple-stress parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number increases with the increase in couple-stress parameter.

The critical Rayleigh numbers listed in tables 1 to 3 and illustrated in figures 8-10 are obtained from figures 2 to 7 by locating the minimum numerically. From table 1 and figure 8, it is clear that rotation has stabilizing effect on the system. From the table 2 and figure 9, it is observed that the magnetic field has stabilizing effect in the absence of rotation, destabilizing effect for $T_{A_1} = 6000$ and for $T_{A_1} = 2000$, the value of critical Rayleigh number first decreases and then increases for the increase in the value of magnetic field parameter. In figure 10, critical Rayleigh number $R_c$is plotted against couple-stress parameter F_1. In the absence of rotation, couple-stresses have stabilizing effect whereas in the presence of rotation, the value of critical Rayleigh number R_c decreases and then increases for the increase in value of couple-stress parameter F_1. Table 3 confirms these results numerically.

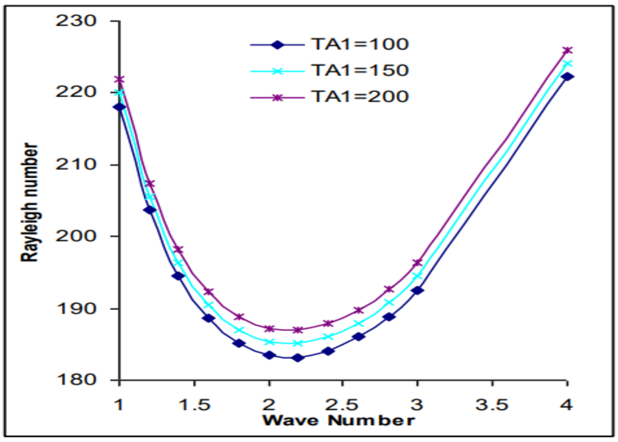

Figure 2: Variation of R_1 with x

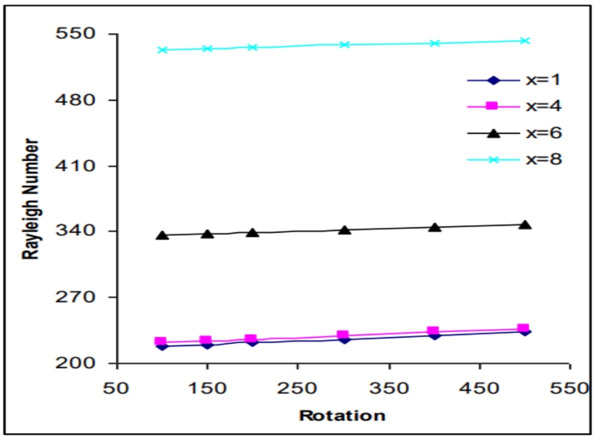

Figure 3: Variation of R_1 with T_{A1}

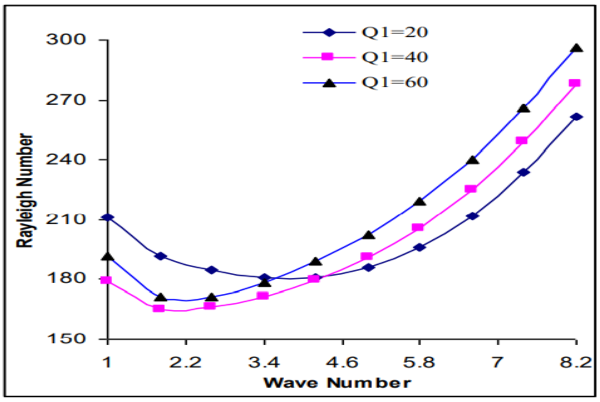

Figure 4: Variation of R_1 with x

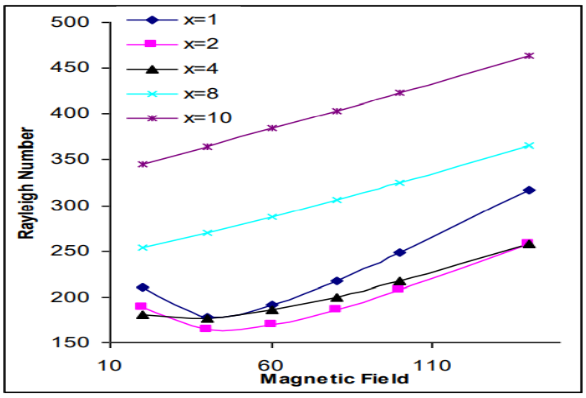

Figure 5: Variation of R_1 with Q_1

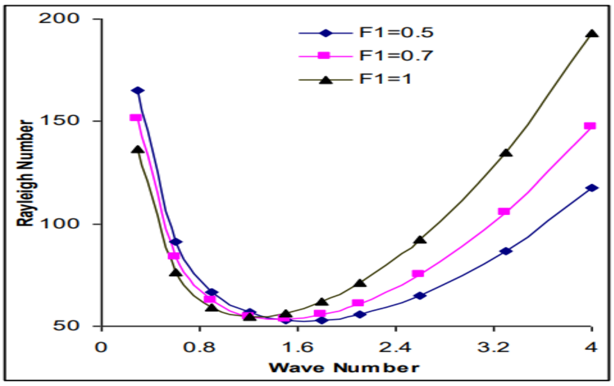

Figure 6: Variation of R_1 with x

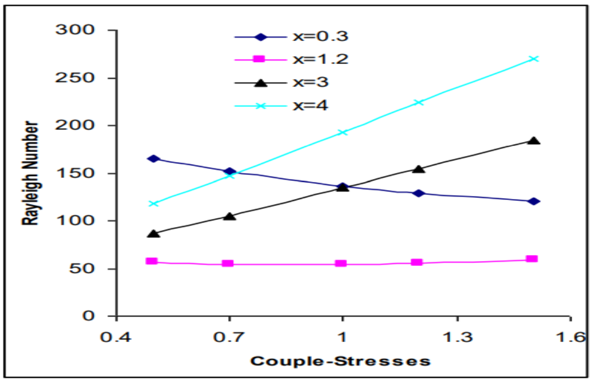

Figure 7: Variation of R_1 with F_1

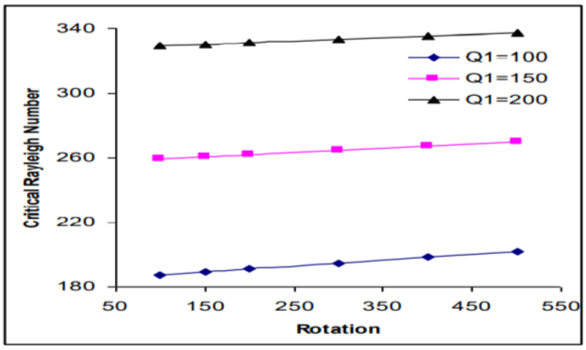

Figure 8: Variation of R_c with T_{A1}

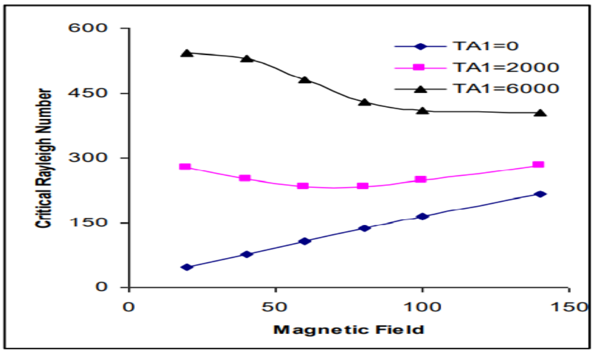

Figure 9: Variation of R_c with Q_1

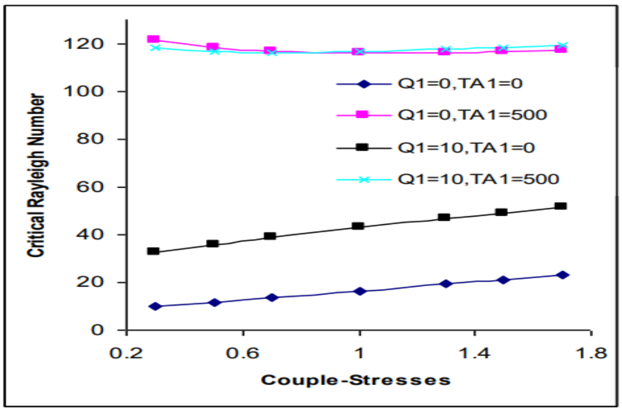

Figure 10: Variation of R_c with F_1

Table 1: The critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection for various values of $\overset{A}{T}$.

T_{A1}	Q_1 100		Q_1 150		Q_1 200	
	x_c	R_c	x_c	R_c	x_c	R_c
100	2.0	187.4235	2.3	259.2703	2.5	329.2593
150	2.0	189.2602	2.3	260.5939	2.5	330.3077
200	2.0	191.0969	2.3	261.9175	2.5	331.3561
250	2.0	194.7704	2.3	264.5647	2.5	333.4530
300	2.0	198.4439	2.3	267.2120	2.5	335.5498
400	2.0	202.1173	2.3	269.8592	2.5	337.6466

Table 2: The critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection for various values of Q_1.

Q_1	T_{A1} 0		T_{A1} 2000		T_{A1} 6000	
	x_c	R_c	x_c	R_c	x_c	R_c
20	1.6	46.3411	5.5	277.2780	8.4	543.9253
40	2.0	77.5500	2.0	251.6312	7.5	529.6470
60	2.3	106.8679	2.0	233.0730	1.6	481.3190
80	2.5	135.1525	2.1	233.5286	1.7	430.0211
100	2.7	162.7387	2.3	246.9746	1.8	408.6656
140	3.1	216.5092	2.7	281.4301	2.1	405.1923

Table 3: Critical Rayleigh numbers and the wave numbers of the associated disturbances for the onset of instability as stationary convection for various values of F1.

F_1	$Q_1 = 0$				$Q_1 = 10$			
	$T_{A1} = 0$		$T_{A1} = 500$		$T_{A1} = 0$		$T_{A1} = 500$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
0.3	0.4	9.7412	3.3	121.3369	1.0	32.8000	2.7	118.4836
0.5	0.4	11.6620	2.8	118.4081	0.9	35.9723	2.3	116.6769
0.7	0.4	13.5828	2.4	116.9902	0.9	38.8683	2.0	116.2168
1.0	0.4	16.4640	2.1	116.2354	0.8	42.9120	1.9	116.6277
1.3	0.4	19.3452	1.9	116.3986	0.7	46.8153	1.6	117.6984
1.7	0.4	23.1868	1.6	117.1955	0.7	51.5879	1.4	119.7011

III. THERMAL STABILITY OF THE SYSTEM AND OSCILLATORY MODES

Now to determine under what conditions the principle of exchange of stabilities (PES) is satisfied (i.e. is real σ is real and the marginal states are characterized by σ=0) and the oscillations come into play, we multiply equation (14) with W* and integrate over the range of z and making use of equations (15)-(18) together with the boundary conditions (20) and get

$$\sigma I_1 + I_2 + FI_3 - \frac{gaka^2}{\nu\beta}(I_4 + H\sigma^* p_1 I_5) + d^2[\sigma I_6 + I_7 + FI_8] + \frac{\mu_e \eta d^2}{4\pi\rho_o \nu}(I_9 + \sigma^* p_2 I_{10}) + \frac{\mu_e \eta}{4\pi\rho_o \nu}(I_{12} + \sigma^* p_2 I_{11}) = 0$$

where,

$I_1 = \int (|DW|^2 + a^2|W|^2)dz, \qquad I_2 = \int \ (|D^2W^2| + 2a^2|DW|^2 + a^4|W|^2)\text{dz},$

$I_3 = \left(\int |D^2W|^2 + 3a^2|D^2W|^2 + 3a^4 \, |DW|^2 + a^6|W|^2 \right) dz,$

$I_4 = \int (|D\Theta|^2 + a^2|\Theta|^2) \, dz, \qquad I_5 = \int |\Theta|^2 dz,$

$I_6 = \int |Z|^2 dz \qquad I_7 = \int (|DZ|^2 + a^2|Z|^2) \ dz,$

$I_8 = \int (|D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2) \, dz,$

$I_9 = \int (|DX|^2 + a^2|X|^2)dz, \qquad I_{10} = \int \ |X|^2 dz,$

$I_{11} = \int (|DK|^2 + a^2|K|^2)dz, \qquad I_{12} = \int (|D^2K|^2 + 2a^2|DK|^2 + a^4|X|^2)dz.$

and σ^* is complex conjugate of σ. The integrals $I_1 - I_{12}$ are all positive definite. Putting $\sigma = i\sigma_i(\sigma^* = -i\sigma_i)$ in equation (30) and equating imaginary parts, we obtain

$$\sigma_i \left[I_1 + \frac{gaka^2}{\nu\beta}(I_4 + Hp_1I_5) + d^2I_6 - \frac{\mu_e\eta d^2}{4\pi\rho_o\nu}p_2I_{10} - \frac{\mu_e\eta}{4\pi\rho_o\nu}p_2I_{11} \right] = 0 \qquad (31)$$

It is clear from equation (31) that σ_i may be zero or non-zero, which implies that modes may be non-oscillatory or oscillatory. In the absence of magnetic field and rotation, equation (31) reduces to

$$\sigma_i \left[I_1 + \frac{gaka^2}{\nu\beta}(I_4 + Hp_1I_5) + d^2I_6 \right] = 0 \qquad (32)$$

The terms in the bracket are positive definite. Thus $\sigma_i = 0$ which means that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of rotation and magnetic field.

IV. CONCLUSION

In this section, the effect of magnetic field and rotation has been considered on the thermal stability of a couple-stress fluid. The effect of various parameters such as magnetic field, rotation and couple-stresses has been investigated analytically as well as numerically. The main results from the analysis are as follows: In order to investigate the effects of magnetic field, rotation and couple-stresses, we examine the behavior of $\frac{dR_1}{dQ_1}, \frac{dR_1}{dT_{A_1}}$ *and* $\frac{dR_1}{dF_1}$ analytically. It is found that rotation has stabilizing effect on the system. The magnetic field couple-stresses has a stabilizing effect in the absence of rotation whereas in the presence of rotation it has a stabilizing effect if $T_{A_1}(I + x) < \{[I + F_1(I + x)](I + x)^2 + Q_1\}^2$ and destabilizing effect if $T_{A_1}(I + x) < \{[I + F_1(I + x)](I + x)^2 + Q_1\}^2$. The principle of exchange of stabilities is satisfied in the absence of rotation and magnetic field.

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