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A Part of Oppermann's Conjecture, Legendre's Conjecture and Andrica's Conjecture

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ABSTRACT

In this paper we discuss a part of Oppermann's Conjecture "there is at least two primes between n2 – n to n2 and at least another two primes between n^2 to n^2+ n for
n≥3.5×〖10〗^6 ". A part of Legendre's Conjecture "there is at least two primes between n^2 to 〖(n+1)〗^2 for n ≥3.5×〖10〗^6 " and a part of Andrica's Conjecture states that " √(p_(n+1))– √(p_n) < 1 for every pair of consecutive prime numbers p_n and p_(n+1) (of course, p_n< p_(n+1)) for n≥3.5×〖10〗^6". We propose a conjecture regarding the distribution of prime numbers.

Keywords: Prime numbers, Admissible Sets, Andrica's conjecture, Brocard's conjecture, Legendre's Type equation here. conjecture.

I. INTRODUCTION

Oppermann's conjecture is an unsolved problem in mathematics on the distribution of prime numbers. It is closely related to but stronger than Legendre's conjecture, Andrica's conjecture, and Brocard's conjecture. It is named after Danish mathematician Ludvig Oppermann, who announced it in an unpublished lecture in March 1877.

Recently Yitang Zhang (A Chinese American mathematician primarily working on number theory and a professor of mathematics at the University of California, Santa Barbara since 2015.) submitted a paper to the Annals of Mathematics in 2013 which established the first finite bound on the least gap between consecutive primes that is attained infinitely often.

His statement is

Definition 1.1.

Let $H = \{h_1, h_2, \cdots, h_{k_0}\}$ *be a set composed of distinct non negative integers. We say that H is admissible if* $v_p(H) < p$ *for every prime p, where* $v_p(H)$ *denotes the number of distinct residue classes modulo p occupied by the* h_i.

Another definition of admissible set is H is admissible if for all prime p, there exist an integer n such that

$$\prod_{i=1}^{k}(n + h_i) \quad (1)$$

is co-prime to p.

Theorem 1.2. *Suppose that H is admissible with* $k_0 \geq 3.5 \times 10^6$ *. Then there are infinitely many positive integers n such that the* k_0 *tuple* $\{n + h_1, n + h_2, \cdots, n + h_{k_0}\}$ *contains at least two primes. Consequently, we have*

$\lim_{n\to\infty} inf\,(p_{n+1} - p_n) < 7 \times 10^7$.

The bound results from the fact that the set H is admissible if it is composed of k_0 *distinct primes, each of which is greater than* k_0 *, and the inequality* $\pi(7 \times 10^7) - \pi(3.5 \times 10^6) > 3.5 \times 10^6$ *.*

Remark 1.3. k_0 *tuple is admissible for every prime p larger than* k_0.

II. CONJECTURES

1. Oppermann's Conjecture

Is an unsolved problem in mathematics on the distribution of prime numbers. It is clearly related to but stronger than Legendre's Conjecture, Andrica's Conjecture and Brocard's Conjecture. It is named after danish mathematician Ludvig Oppermann.

The conjecture states that, for every integer $n > 1$ there is at least one prime number between $n^2 - n$ and n^2 and at least another prime between n^2 to $n^2 + n$.

Lemma 2.1. *(A Part of Oppermann's Conjecture) There exist at least two prime number between* $n^2 - n$ *and* n^2 *and at least another two-prime number between* n^2 *and* $n^2 + n$ *for* $n \geq 3.5 \times 10^6$*.*

Proof. 1. Case I-

Let's consider the set

$H = \{h_1 = n^2 - 2n + 1, h_2 = n^2 - 2n + 2, \cdots, h_n = n^2 - n\}$

By Yitang Zhang's statement if the set H is admissible with $n \geq 3.5 \times 10^6$ then there are infinitely many positive integer m such that the n tuple

$$(m + n^2 - 2n + 1)\,(m + n^2 - 2n + 2)\,(m + n^2 - n) \qquad (2)$$

contains at least two primes.

Let's prove H is admissible.

For a special case $m = n = 1$ by equation (1) the product of equation (2) that is

$$(m + n^2 - 2n + 1)\,(m + n^2 - 2n + 2) \cdot \cdots (m + n^2 - n) \qquad (3)$$

for $m = n = 1$ is 1 and is co-prime to every prime number p.

Therefore, H is admissible.

2. Case II-

Let's consider the set

$$G = \{h_1 = n^2 - n, h2 = n^2 - n + 1, \cdots, hn = n^2 - 1\}$$

By Yitang Zhang's statement if the set G is admissible with $n \geq 3.5 \times 10^6$ then there are infinitely many positive integer m such that the n tuple

$\{m + n^2 - n, m + n^2 - n + 1, \cdots, m + n^2 - 1\}$ *(4)*

contains at least two primes.

Let's prove G is admissible.

For a special case $m = n = 1$ by equation (1) the product of equation (4) that is

$$(m + n^2 - n)(m + n^2 - n + 1) \cdots (m + n^2 - 1) \qquad (5)$$

for $m = n = 1$ is 1 and is co-prime to every prime number p.

Therefore, G is admissible.

Therefore by Yitang Zhang's statement there exist at least two prime numbers be- tween n^2 and $n^2 + n - 1$ that is there exist at least two prime numbers between n^2 and $n^2 + n$ for $n \geq 3.5 \times 10^6$.

Hence proved.

2. Legendre's Conjecture

Legendre's conjecture, proposed by Adrien-Marie Legendre, states that there is a prime number between n^2 to $(n + 1)^2$for every positive integer n. The conjecture is one of Landau's problems (1912) on prime numbers; as of 2022, the conjecture has neither been proved nor disproved.

Lemma 3.1. *(A Part of Legendre's Conjecture)* *There exist at least two prime number between n^2 and $(n+1)^2$ for $n \geq 3.5 \times 10^6$.*

Proof. As per the lemma 2.1 There exist at least two prime number between n^2 and $n^2 + n$ for $n \geq 3.5 \times 10^6$.
But obviously There exist at least two prime number between n^2 and $n^2 + n + n + 1$ for $n \geq 3.5 \times 10^6$.
that is There exist at least two prime number between n^2 and $(n+1)^2$ for $n \geq 3.5 \times 10^6$.
Hence proved.

3. Andrica's Conjecture

Andrica's conjecture (named after Dorin Andrica) is a conjecture regarding the gaps between prime numbers, states that $\sqrt{p_{n+1}} - \sqrt{p_n} < 1$ for every pair of consecutive prime numbers p_n and p_{n+1}.

Lemma 4.1. *(A Part of Andrica's Conjecture)* $\sqrt{p_{n+1}} - \sqrt{p_n} < 1$ *for every pair of consecutive prime numbers p_n and p_{n+1} for $n \geq 3.5 \times 10^6$.*

Proof. As per the lemma 2.1 There exist at least two prime number between n^2 and $n^2 + n$ for $n \geq 3.5 \times 10^6$. Suppose that the consecutive primes between n^2 and $n^2 + n$ are p_n and p_{n+1}.

Let $\sqrt{n^2 + n} - \sqrt{n^2} = \sqrt{n}(\sqrt{n+1} - \sqrt{n})$

$$= \frac{\sqrt{n}}{(\sqrt{n+1}+\sqrt{n})} \quad (6)$$

We know $\sqrt{n+1} + \sqrt{n} > \sqrt{n}$

$$\frac{\sqrt{n}}{(\sqrt{n+1}+\sqrt{n})} < 1$$

Equation (6) is

$$\sqrt{n^2 + n} - \sqrt{n^2} < 1$$

As p_n and p_{n+1} lies between n^2 and $n^2 + n$. So $\sqrt{p_n}$ and $\sqrt{p_{n+1}}$ lies between $\sqrt{n^2}$ and $\sqrt{n^2 + n}$.

~~Therefore~~

$\sqrt{p_{n+1}} - \sqrt{p_n} < 1$ *for $n \geq 3.5 \times 10^6$.*

Hence proved.

III.CONCLUSION

We have proved that a Oppermann's Conjecture for for $n \geq 3.5 \times 10^6$ and Legendre's, Andrica's Conjecture follows Oppermann's Conjecture with the help of a paper [1].

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