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Compact Mean Labeling on Bipartite Graphs

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ABSTRACT
Labeling of graphs is the procedure of assigning numbers to the nodes, lines, or both in accordance with an applicable rule. In this study, we demonstrate that the bipartite graph $K_{n,n}$ is a compact mean-labeled graph. We also explored graphs $K_{2,2}$ and $K_{2,3}$, $K_{2,4}$ are compact mean-labeled graphs

Keywords : Labeling of graphs, complete graph, Bipartite graph ,complete bipartite graph graph, Mean labeled graph, Compact mean labeled graphs
2020 Mathematical subject classification Number: 05C78.

I. INTRODUCTION

In 1966, Rosa [11] introduced β - valuation of a graph. Golomb subsequently called such a labeling graceful. In 1980, Graham and Sloane [3] introduced the harmonious labeling of a graph. In 2003 [12], Somasundaram and Ponraj introduced the mean labeling of a graph. On similar lines, Maheswari V et al., [6] proved relaxed mean labeling on path, star, bistar graphs.

RESULTS AND OBSERVATIONS OF COMPACT MEAN LABELING OF COMPLETE BIPARTITE GRAPHS

Definition 1.1: Bipartite Graph:

A graph G=(V, E) is called a bipartite graph if its vertices V can be partitioned into two subsets V_1 and V_2 such that each edge of G connects a vertex of V_1 to a vertex V_2.It is denoted by Kmn, where m and n are the numbers of vertices in V_1 and V_2 respectively.

Definition 1.2: Complete bipartite graph

A graph G = (V, E) is called a complete bipartite graph if its vertices V can be partitioned into two subsets V_1 and V_2 such that each vertex of V_1 is connected to each vertex of V_2. The number of edges in a complete bipartite graph is mn as each of the m vertices is connected to each of the n vertices.

Theorem 1,3: Prove that the graph $K_{2,2}$ admits compact mean labeling

Proof: Let V_1 , V_2 be the bipartition of $K_{2,2}$
$where\ V_1 = \{u, v\}$, $V_2 = \{u_1, u_2\}$

Define the mapping $f: V(K_{2,n}) \rightarrow \{0,1,\ldots..2q-p\}$

By $f(ui) = \begin{cases} 2i-2, \forall\, i = 1,2,3 \\ i-1 \text{ when } i = 4 \end{cases}$

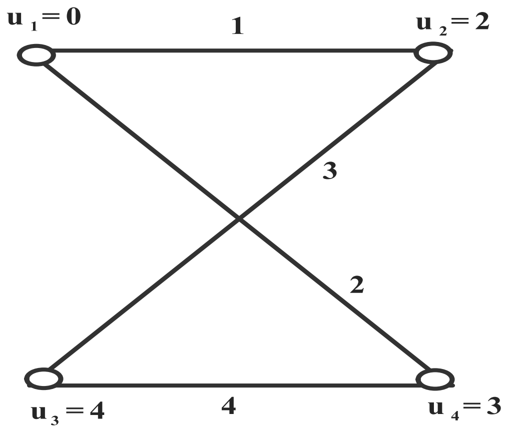
By the mapping $f: V(K_{2,n}) \rightarrow \{0,1,2,3,4\}$

When $i = 1 \quad f(u_1) = 2i - 2 = 0$

$i = 2 \quad f(u_2) = 2i - 2 = 2$

$i = 3 \quad f(u_3) = 2i - 2 = 4$

$i = 4 \quad f(u_4) = i - 1 = 3$



Compact mean labeling of $K_{2,2}$

Figure 1

The corresponding edge labels are as follows

$u_1u_2 = 1 \ (ie) \ 1 \leq i \leq 3$

$U_{n-3}\, U_{n-2} = 1$

$U_{n-3}\, U_n = 2$

$U_{n-2}\, U_{n-1} = 3$

$U_{n-1}\, U_n = 4$

Theorem 2: Prove that the graph $K_{2,3}$ is a compact mean labeled graph

Proof: Let V_1, V_2 be the partition of $K_{2,3}$ where $V_1 = \{u, v\}$, $V_2 = \{u_1, u_2, u_3\}$

Define the mapping $f: V(K_{2,3}) \rightarrow \{0,1,\ldots..2q-p\}$

$f: V(K_{2,3}) \rightarrow \{0,1,2,3,4,5,6,7\}$

By $f(u) = 0, f(v) = n+3$

The label of vertices V_2 is

$f(V_i) = \{2i \quad 1 \leq i \leq n-2$

The values of vertices V_1, V_2, V_3 are given below

$i = 1 \rightarrow f(V_1) = 2$

$i = 2 \rightarrow f(V_2) = 4$

$i = 3 \rightarrow f(V_3) = 6$

The corresponding edge labels are as follows

The label of the edge uv_1 is 1

The label of the edge uv_{i+1} is $i+1$ for $1 \leq i \leq n-1$

The label of the edge $VV_1 = 4$

The label of the edge $V_2 = 5$, $VV_3 = 6$

Hence $K_{2,3}$ is a compact mean labeled graph

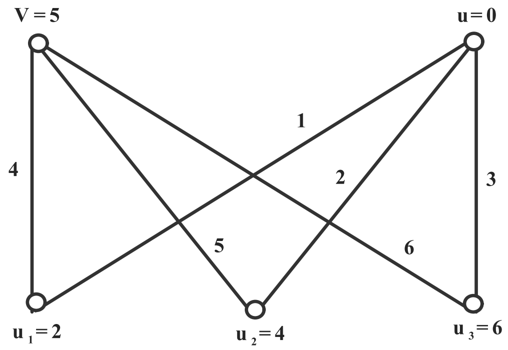


Figure2

Theorem 3 : Prove that the graph $K_{2,4}$ is compact mean labeled graph

Proof : Let V_1, V_2 be the partition of $K_{2,4}$ where $V_1 = \{v, u\}$, $V2 = \{u_1, u_2, u_3, u_4\}$

Define of mapping $f: V(K_{2,4}) \rightarrow \{0,1,2,\ldots..2q-p\}$

$f: V(K_{2,4}) \rightarrow \{0,1,2,3,4,5,6,7,8,9,10\}$

$f(u) = 0, f(V) = 3n+2$

The label of vertices u_r is

$f(u_i) = \{2i-1 \ for\ 1 \leq i \leq n$

The values of the vertex labels $f(u_1) = 1$

$f(u_2) = 3, \ f(u_3) = 5, \ f(u_4) = 7$

(or)

$F(u_i) = 1, f(u_i+1) = 3, f(u_i+2) = 5, f(u_i+3) = 7$

The corresponding edge labels are as follows

The label of the edge $Vu_1 = n+1$

The label of the edge $Vu_2 = n+2$

The label of the edge $Vu_3 = n+3$

The label of the edge $Vu_4 = n+4$

Hence k2,4 is a compact mean labeling of $K_{2,4}$

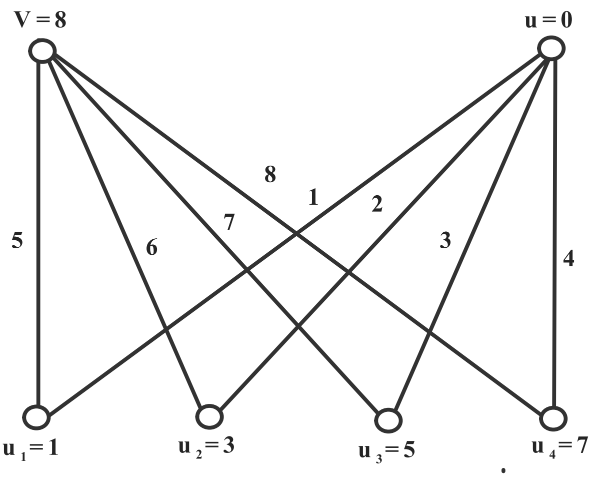


Figure3

$$u_{n-5}u_{n-8} = u_5u_2 = 8$$
$$u_{n-7}u_{n-5} = u_3u_5 = 9$$
$$u_{n-6}u_{n-5} = u_4u_5 = 10$$

∴ All the values of edges are distinct values of the set $\{1, 2, 3, 4, 5, 6 \dots 10\}$.

∴ Then graph K_5 admits compact mean labeling graph.

Conclusion

In this research article, the exploration of compact mean labeling for the bipartite graphs was verified. Identified the distinct edge values using the appropriate labeling rule gives unique weights to the vertices and edges. In further research, we review the bounds for the complete bipartite graphs.

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