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Compact Mean Labeling

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ABSTRACT
Labeling of graphs is the procedure of assigning numbers to the nodes, lines, or both in accordance with an applicable rule. In this study, we demonstrate that the complete graph K_n (n=3) is a compact mean-labeled graph. We also explored graph K_4 is compact mean-labeled graphs
Keywords: Labeling of graphs, complete graph, Cycle graph, Mean labeled graph, Compact mean labeled graphs
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I. INTRODUCTION

In 1966, Rosa [11] introduced β - valuation of a graph. Golomb subsequently called such a labeling graceful. In 1980, Graham and Sloane [3] introduced the harmonious labeling of a graph. In 2003 [12], Somasundaram and Ponraj introduced the mean labeling of a graph. On similar lines, Maheswari V et al., [6] proved relaxed mean labeling on path, star, bistar graphs.

RESULTS AND OBSERVATIONS OF COMPACT MEAN LABELING OF COMPLETE GRAPHS

Definition 1.1: A complete graph is a graph that has an edge between every single vertex in the graph; we represent a complete graph with n vertices using the symbol Kn.

Definition 1.2:[6] A graph G = (V, E) with p vertices and q edges is said to be a relaxed mean graph if there exists a function f from the vertex set of G to {0, 1, 2, 3, …, q − 1, q + 1} and relaxing the vertex label q such that the induced map f* from the edge set of G to {1, 2, 3, …, q}
defined by

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd, then} \end{cases}$$

the resulting vertex labels and edge labels are distinct.

Main Definition1.3: A graph G with P vertices and Q edges is said to be a compact mean labeled graph if it is possible to label the vertices $x \epsilon v$ with distinct elements $f(x)$ from $\{0, 1, 2, \dots .2q - p\}$ in such a way that when each edge $e = uv$ is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edge label $\{0, 1, 2, \dots . q\}$ are distinct. Then f is called a compact mean labeling of G.

Theorem 1.4: Prove that the graph K_n is a compact mean labeled graph for $n = 3$.
Proof: Let K_n be a complete graph with distinct vertices u_1, u_2, u_3 and edgese_1, e_2, e_3.
Define a mapping $f: V(K_n) \rightarrow \{0, 1, 2, \dots .2q - p\}$
by $f(u_i) = \begin{cases} i - 1 \; for \; i = 1, 3 \; (odd) \\ 2i - 1 \; for \; i = 2 \; (even) \end{cases}$
The vertex labels are,
When $i = 1, f(u_1) = 0$,
$i = 2, f(u_2) = 3$,
$i = 3, f(u_3) = 2$,

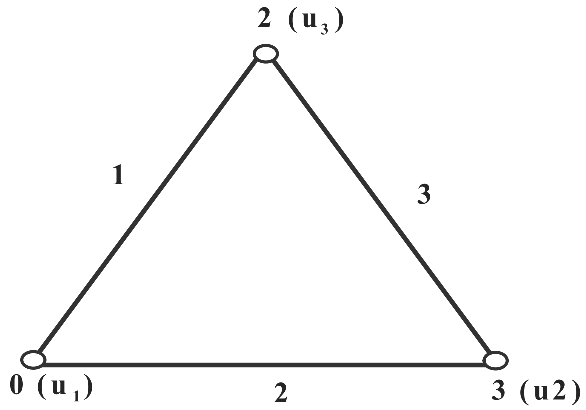

Figure 1

The label of the edge $u_1u_3 = 1$
The label of the edge $u_1u_2 = 2$
The label of the edge $u_2u_3 = 3$
The label of the edge $u_{n-2}u_n = 1$
The label of the edge $u_{n-2}u_{n-1} = 2$
The label of the edge $u_{n-1}u_n = 3$
All the edge values are distinct. Hence K_3 is a complete graph.

Theorem 1.5: Prove that the graph K_4 admits the compact mean labeling.
Proof: Let K_4 be the complete graph with vertices u_1, u_2, u_3, u_4 and edges $e_1, e_2, e_3, e_4, e_5, e_6$.
Define mapping $f: V(K_n) \rightarrow \{0, 1, 2, \dots .2q - p\}$
Such that $f(u_i) = \begin{cases} 2i - 2 \quad \forall \quad i = 1, 2, 3 \\ i + 4 \quad when \quad i = 4 \end{cases}$

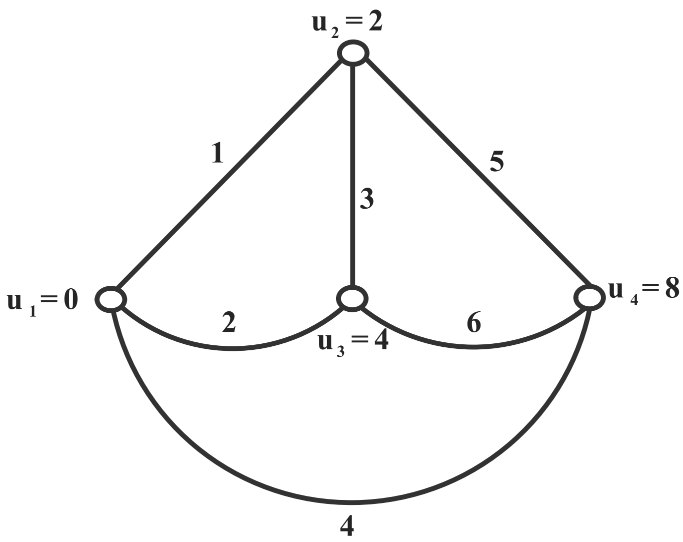

Figure:2

From the mapping $f: V(K_n) \to \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$.

The assigned vertex label are

When $i = 1, f(u_1) = 0$,

$i = 2, f(u_2) = 2$,

$i = 3, f(u_3) = 4$,

$i = 4, f(u_4) = 8$,

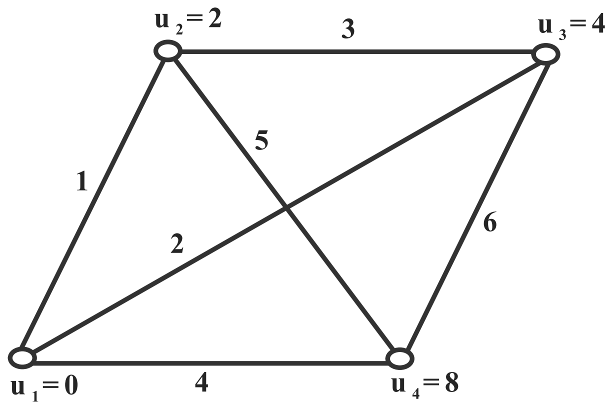

Figure:3

The induced edge labels are,

$u_{n-3}u_{n-2} = u_1u_2 = 1$

$u_{n-3}u_{n-1} = u_1u_3 = 2$

$u_{n-2}u_4 = u_2u_4 = 5$

$u_{n-3}u_n = u_1u_4 = 4$

$u_{n-2}u_{n-1} = u_2u_3 = 3$

$u_{n-1}u_4 = u_3u_4 = 6$

$\therefore$ All the values of edges are distinct from $\{1, 2, \dots q\}$.

$\therefore$ Then graph K_4 admits compact mean labeling.

II. Conclusion

In this research article, the exploration of compact mean labeling for the complete graphs was verified. Identified the distinct edge values using the appropriate labeling rule gives unique weights to the vertices and edges. In further research, we review the bounds for the complete graphs.

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