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Periodic Solution of A MHD Mixed Convention Flow and Heat Transfer Through Porous Medium

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ABSTRACT

The flow through porous boundaries has many applications in science and technology such as water waves over a shallow beach, mechanics of the cochlea in the human ear, aerodynamic heating, flow of blood in the arteries, and petroleum industry. Several authors have studied theoretically the laminar flow in porous channels. Berman [1] considered the viscous fluid and analyzed the flow characteristics when it passed through the porous walls. Later the same problem for different permeability was studied by Terril and Shrestha [2]. The theory of micropolar fluids was introduced by Eringen [3] which are considered as an extension of generalized viscous fluids with microstructure. Examples for micropolar fluids include lubricants, colloidal suspensions, porous rocks, aerogels, polymer blends, and microemulsions. The same Berman problem with micropolar fluid was discussed by Sastry and Rama Mohan Rao [4]. The flow and heat transfer of micropolar fluid between two porous parallel plates was analyzed by Ojjela and Naresh Kumar [5]. Srinivasacharya et al. [6] obtained an analytical solution for the unsteady Stokes flow of micropolar fluid between two parallel plates. The effect of buoyancy parameter on flow and heat transfer of micropolar fluid between two vertical parallel plates was investigated by Maiti [7].

Keywords : MHD, Visco-elastic, Porous Medium.

I. INTRODUCTION

The study of MHD heat and mass transfer through porous boundaries has attracted many researchers in the recent past due to applications in engineering and science, such as oil exploration, boundary layer control, and MHD power generators. The steady incompressible free convection flow and heat transfer

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of an electrically conducting micropolar fluid in a vertical channel was studied by Bhargava et al. [8]. The laminar incompressible magnetohydrodynamic flow and heat transfer of micropolar fluid between porous disks was analyzed numerically by Ashraf and Wehgal [9]. Islam et al. [10] obtained a numerical solution for an incompressible unsteady magnetohydrodynamic flow through vertical porous medium. Nadeem et al. [11] discussed the unsteady MHD stagnation flow of a micropolar fluid through porous media. The effects of Hall and ion slip currents on micropolar fluid flow and heat and mass transfer in a porous medium between parallel plates with chemical reaction were considered by Ojjela and Naresh Kumar [12]. The MHD heat and mass transfer of micropolar fluid in a porous medium with chemical reaction and Hall and ion slip effects by considering variable viscosity and thermal diffusivity were investigated by Elgazery [13]. The mixed convection flow and heat transfer of an electrically conducting micropolar fluid over a vertical plate with Hall and ion slip effects was analyzed by Ayano [14]. When heat and mass transfer occur simultaneously in a moving fluid, the energy flux caused by a concentration gradient is termed as diffusion thermoeffect, whereas mass fluxes can also be created by temperature gradients which is known as a thermal diffusion effect. These effects are studied as second-order phenomena and may have significant applications in areas like petrology, hydrology, and geosciences [15].

In the present study the effects of chemical reaction on two-dimensional mixed convection flow and heat transfer of an electrically conducting micropolar fluid in a porous medium between two parallel plates with Soret and Dufour have been considered. The reduced flow field equations are solved using the quasi-linearization method. The effects of various parameters such as Hartmann number, inverse Darcy's parameter, Schmidt number, Prandtl number, chemical reaction rate, Soret and Dufour numbers on the velocity components, microrotation, temperature distribution, and concentration are studied in detail and presented in the form of graphs.

II. Formulation of the Problem and Solution

Here we assume between two parallel infinite horizontal plates, unsteady two-dimensional mixed convection and incompressible viscous, electrically conducting fluid flow through a porous medium. The upper plate is oscillatory with time parameter about a constant mean velocity where as lower plate is stationary. The temperature of lower plate is T0 and upper one is TW. The upper and lower plates are subjected respectively, to a suction velocity V0 (> 0) and constant injection. A uniform strength B_0 of a magnetic field is applied perpendicular to the insulated horizontal plate. Now by Boussinesq's approximation, the equations for the flow are

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + \nu\frac{\partial^2 u'}{\partial y'^2} + -\frac{\sigma B^2{}_0}{\rho}(u' - U') - \frac{\nu}{K'}(u' - U') + g\beta(T' - T_0) \quad (2)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

Here ρ is the density, ν is the kinematics viscosity, σ is the electric conductivity and velocity of oscillating plate is denoted by U' (t'). Now Boundary conditions for the problem are:

u' = 0, T ' = T0 at y' = 0

u' = U' (t'), T ' = Tw at y' = d (4)

The unsteady velocity U' (t') is given by

$$U'(t') = U_0\left(1 + \frac{1}{2}\varepsilon\left(e^{i\omega' t'} + e^{-i\omega' t'}\right)\right) \tag{5}$$

Where U0 is the constant mean velocity, ω' is the frequency and ε is a positive constant.

We now introduce the following non-dimensional

$$\left.\begin{aligned} \eta &= \frac{y'}{V_0 d}, & \mathrm{u} &= \frac{\mathrm{u'}}{\mathrm{U_0}}, & \mathrm{v} &= \frac{\mathrm{v'}}{\mathrm{V_0}} \\ t &= \frac{\omega' t'}{V_0^2}, & \omega &= \frac{\omega' \mathrm{d}^2}{\nu}, & \theta &= \frac{\mathrm{T'} - \mathrm{T_0}}{\mathrm{T_W} - T_0} \end{aligned}\right\} \tag{6}$$

In equations (1), (2) and (3), this gives:

$$\frac{\partial v}{\partial \eta} = 0 \tag{7}$$

$$\frac{\omega}{R_e}\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + v\frac{\partial u}{\partial \eta} = \frac{\omega}{R_e}\frac{\partial \mathrm{U}}{\partial t} + \frac{1}{R_e}\frac{\partial^2 \mathrm{u}}{\partial \eta^2} + -\frac{1}{R_e}\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)(\mathrm{u} - \mathrm{U}) + \frac{Gr}{\mathrm{Re}}\theta \tag{8}$$

$$\frac{\omega}{R_e}\frac{\partial \theta}{\partial \mathrm{t}} + v\frac{\partial \theta}{\partial \eta} = \frac{1}{\mathrm{Pr}\, R_e}\frac{\partial^2 \theta}{\partial \eta^2} \tag{9}$$

Here Re $=V^2{}_0 d/v$, is the suction/ injection parameter

Prandtl number $\mathrm{Pr} = \frac{\mu Cp}{\kappa}$

Hartmann number $M = \frac{B_0^2 d^2 \sigma}{V_0^2 \mu}$

Grashof number $Gr = \frac{g\beta v(T_w - T_0)}{U_0 V_0^2}$

Permeability parameter $K = \frac{v}{d^2}\frac{K'}{V_0^2}$

Now boundary conditions become

$$\left.\begin{aligned} &u = O, & &\theta = 0, & &\text{at} & &\eta = 0 \\ &u = U(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right) & &\theta = 1 & &\text{at} & &\eta = 1 \end{aligned}\right\} \tag{10}$$

Integration of the equation (4.1.7) gives

v = V0 = 1 (11)

In view of (11), equation of the motion and energy (8) and (9) becomes:

$$\frac{\omega}{R_e}\frac{\partial \mathrm{u}}{\partial \mathrm{t}} + \frac{\partial u}{\partial \eta} = \frac{\omega}{R_e}\frac{\partial \mathrm{U}}{\partial t} + \frac{1}{R_e}\frac{\partial^2 \mathrm{u}}{\partial \eta^2} + -\frac{1}{R_e}\left(\mathrm{M} + \frac{1}{\mathrm{K}}\right)(\mathrm{u} - \mathrm{U}) + \frac{Gr}{\mathrm{Re}}\theta \tag{12}$$

$$\frac{\omega}{R_e}\frac{\partial \theta}{\partial \mathrm{t}} + \frac{\partial \theta}{\partial \eta} = \frac{1}{\mathrm{Pr}\, R_e}\frac{\partial^2 \theta}{\partial \eta^2} \tag{13}$$

Now consider the solution of the equations (12) and (13) in the form

$$u(\eta, t) = F_0(\eta) + \frac{\varepsilon}{2}\left(F_1(\eta)e^{it} + \overline{F}_1(\eta)e^{-it}\right) \tag{14}$$

$$\theta(\eta,t)=\theta_0(\eta)+\frac{\varepsilon}{2}\left(\theta_1(\eta)e^{it}+\bar{\theta}_1(\eta)e^{-it}\right) \quad (15)$$

where the tilde denotes a complex conjugate, subjecting for u(η,t) and θ (η,t) from equations (14) and (15) and for U(t) from equation (10) into equations (12) and (13) and then equating steady and periodic terms separately to zero, we get

$$F_0''-\mathrm{Re}\,F_0'-\left(M+\frac{1}{K}\right)F_0=-\left(M+\frac{1}{K}\right)-Gr\theta_0 \quad (16)$$

$$F_1''-\mathrm{Re}\,F_1'-\left(M+\frac{1}{K}+i\omega\right)F_1=-\left(M+\frac{1}{K}+i\omega\right)-Gr\theta_1 \quad (17)$$

$$\bar{F}_1''-\mathrm{Re}\,\bar{F}_1'-\left(M+\frac{1}{K}+i\omega\right)\bar{F}_1=-\left(M+\frac{1}{K}+i\omega\right)-Gr\bar{\theta}_1 \quad (18)$$

$$\theta_0''-\Pr\mathrm{Re}\,\theta_0'=0 \quad (19)$$

$$\theta_1''-\Pr\mathrm{Re}\,\theta_1'-i\omega\Pr\theta_1=0 \quad (20)$$

$$\bar{\theta}_1''-\Pr\mathrm{Re}\,\bar{\theta}_1'-i\omega\Pr\bar{\theta}_1=0 \quad (21)$$

Where the prime denotes differentiation with respect to η Notice here that there for terms of order ε, we collect separately coefficient of e it and e-it since these quantities vary independently with time. The equations (17), (18) and equations (20), (21) are complex conjugate each other respectively. Hence, we derived the solution from equations (17) to (20). The corresponding boundary conditions are:

$$\theta_0(\eta)=\theta_1(\eta)=\bar{\theta}_1(\eta)=0,\quad F_0(\eta)=F_1(\eta)=\bar{F}_0(\eta)=0 \text{ at } \eta=0$$

$$\theta_0(\eta)=1,\ \theta_1(\eta)=\bar{\theta}_1(\eta)=0,\ F_1(\eta)=F_1(\eta)=\bar{F}_1(\eta)=1 \text{ at } \eta=1 \quad (22)$$

Solving equations (16), (17), (20) and (21) with the help of boundary condition (22), we get

$$\theta_0=A\left(1-e^{\Pr\mathrm{Re}\,\eta}\right),\qquad \theta_1=0, \quad (23)$$

$$F_0=B\,e^{m_1\eta}+C\,e^{m_2\eta}+1+GrA\left(\frac{1}{N}+m_3e^{\Pr\mathrm{Re}\,\eta}\right) \quad (24)$$

$$F_1(\eta)=1+\frac{e^{n_1\eta+n_2}-e^{n_1+n_2\eta}}{e^{n_1}-e^{n_2}} \quad (25)$$

$$A=\frac{1}{1-e^{\Pr\mathrm{Re}}} \quad (26)$$

$$N=M+\frac{1}{K} \quad (27)$$

$$m_1 = \frac{1}{2}\left[\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(M + \frac{1}{K}\right)}\right]$$

$$m_2 = \frac{1}{2}\left[\mathrm{Re} + \sqrt{\mathrm{Re}^2 - 4\left(M + \frac{1}{K}\right)}\right]$$

$$m_3 = \frac{1}{\mathrm{Re}^2\ \mathrm{Pr}(\mathrm{Pr} - 1) - N}$$

$$n_1 = \frac{1}{2}\left[\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\left(M + \frac{1}{K} + i\omega\right)}\right]$$

$$n_2 = \frac{1}{2}\left[\mathrm{Re} - \sqrt{\mathrm{Re}^2 + 4\left(M + \frac{1}{K} + i\omega\right)}\right]$$

$$B = \frac{1}{e^{m_1} - e^{m_2}}\left[e^{m_2} - GrA\left(\frac{1 - e^{m_2}}{N} + m_3\left(e^{\mathrm{Pr}\,\mathrm{Re}} - e^{m_2}\right)\right)\right]$$

$$C = -B - 1 - GrA\left(\frac{1}{N} + m_3\right)$$

The transient velocity field, we can calculate the skin friction at the lower plate as

$$\tau_w = \frac{\tau_w^* d}{\nu U_0} = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$

$$= Bm_1 + Cm_2\ GrA(\mathrm{Pr}\,\mathrm{Re}\, m_3) + \frac{\varepsilon}{2}|G|\cos(t + \alpha) \qquad (28)$$

where $G = G_r + iG_i = \dfrac{n_1 e^{n_2} - n_2 e^{n_1}}{e^{n_1} - e^{n_2}}$, $|G| = \sqrt{G_r^2 + G_i^2}$ and $\tan\alpha = \dfrac{G_i}{G_r}$,

1. Results & Discussions

Here we analyzed the different physical quantities for different parameters when the flow between two infinite horizontal plates and the heat transfer through porous medium. The exact solution has been derived for viscous flow.

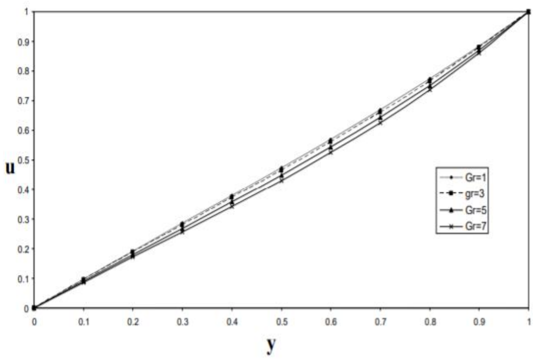


Figure- 1

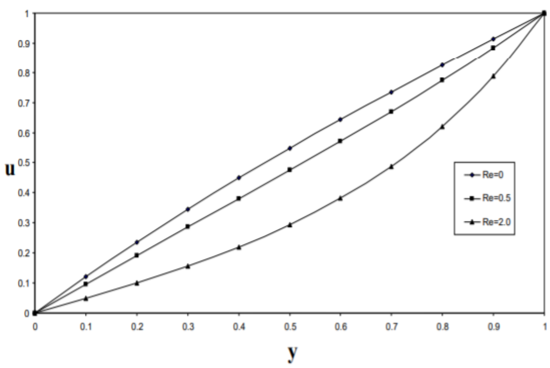
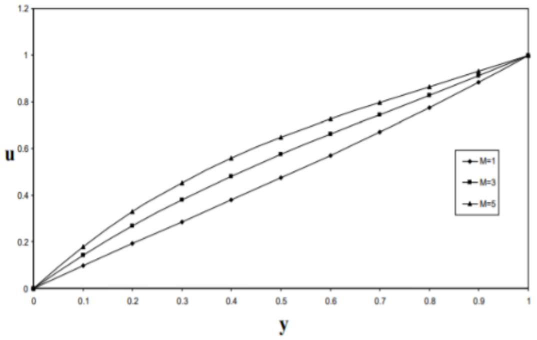


Figure- 2

Figure 1: The transient velocity profile u for different values for Grashof number Gr (M = 1, K = 3, ω = 5, Pr = 0.71, Re = .5, ε = .25 and t = π/2) Figure 2: The transient velocity profile u for different values for Re (M = 1, K = 3, ω = 5, Pr = 0.71, Gr = 2, ε = .25 and t = π/2)



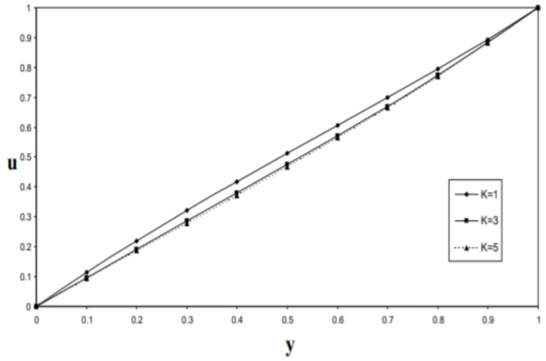
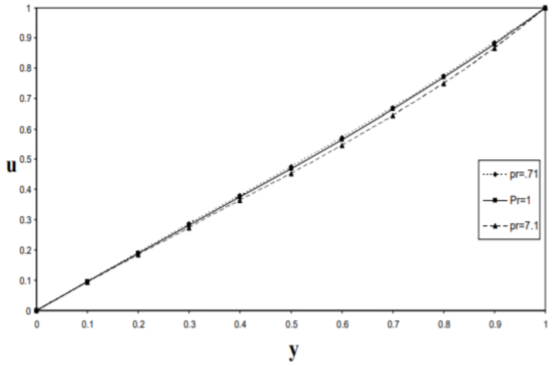


Figure- 3 Figure- 4

Figure 3: The transient velocity u profile for different values of M (Gr = 2, K = 3, ω = 5, Pr = 0.71, Re = .5, ε = .25 and t = π/2) Figure 4: The transient velocity u profile for different values of K (Gr = 2, M = 1, Pr = 0.71, ω = 5, Re = .5, ε = .25 and t = π/2)



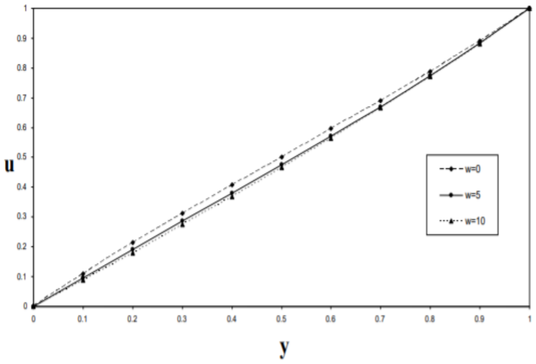
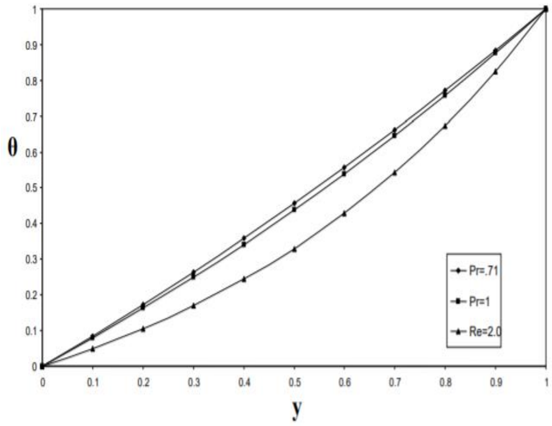


Figure- 5 Figure- 6

Figure 5: The transient velocity u profile for different values of Pr (M = 1, Gr = 2, K = 3, ω = 5, Pr = 0.71, Re = .5, ε = .25 and t = π/2) Figure 6: The transient velocity u profile for different values of ω (M = 1, Gr = 2, K = 3, ω = 5, Pr = 0.71, Re = .5, ε = .25 and t = π/2)



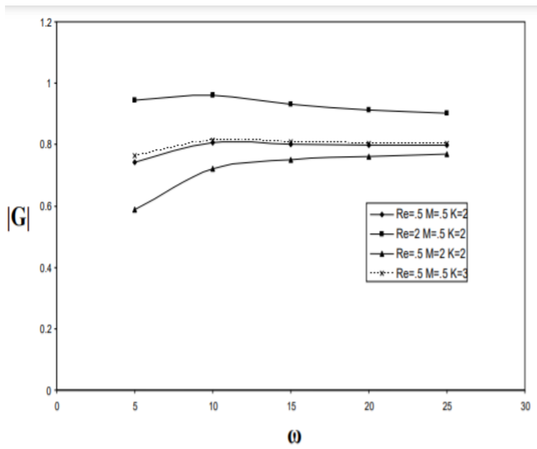


Figure- 7 Figure- 8

Figure 7: The temperature profile θ for different values of Pr and Re (Pr=.71, 1.0, 7.1)

Figure 8: The amplitude of skin friction $|G|$ and frequency ω

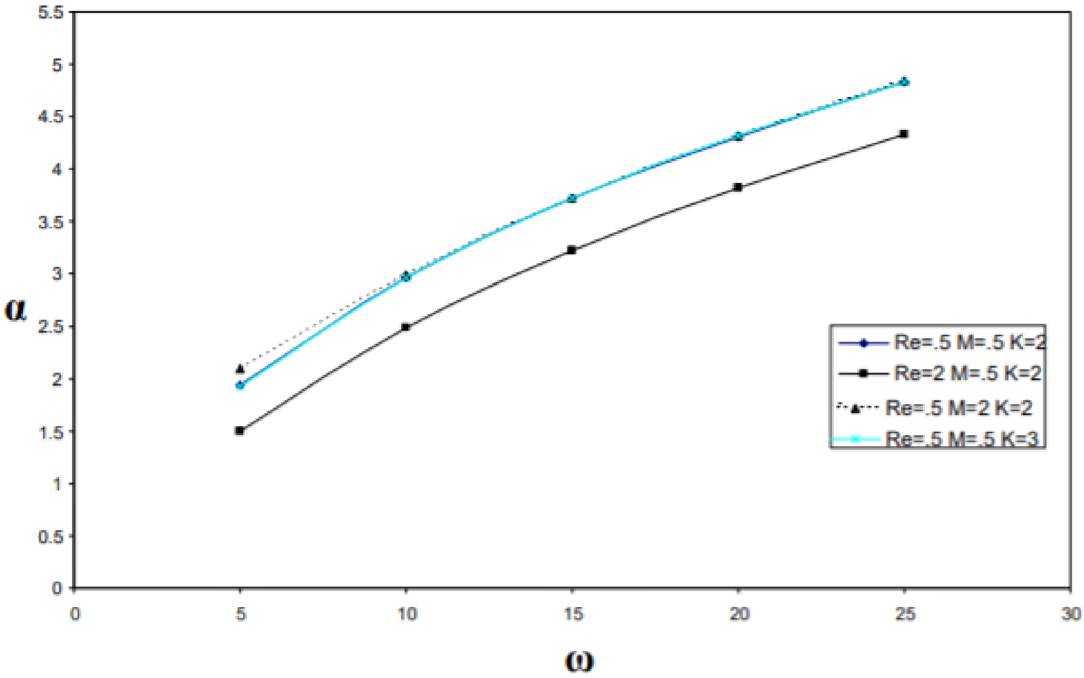


Figure 9: The phase angle of skin friction α and frequency ω

It is clear from figures 1 and 2, that the velocity decrease when the Grashof number Gr and injection/suction parameter Re increase respectively for different parameters.

Figures 3, 4, 5 shows that the velocity profiles again decrease while porosity parameter K and Prandtle number Pr increase respectively for same values of Gr, M, ω, ε etc. Also figure 6 shows that when frequency ω increase then velocity profile again decreases. For different Prandtle number (Pr=.71, 1.0, 7.1) It is evident from the figure 7 that there is increase in Prandtle number then decrease in temperature.

The amplitude |G|, of the friction at the stationary plate has been plotted in figure 8 against the frequency ω. It is obvious from the figure that the amplitude increases with increasing of the Hartmann number but it decreases with increasing of injection/suction parameter Re and permeability parameter K. It also clear that the amplitude increases with increasing plate oscillations. The phase angle α, is presented figure 9, which exhibits a significant decrease in the phase angle with increasing of the Hartmann number M, and the magnetic field strength. However, it increases with increasing of injection/suction parameter Re, and permeability parameter K. The figure also shows that when the increase in phase angle then there is simultaneously increase in frequency of the plate oscillations ω

III. CONCLUSION

Velocity decrease when the Grashof number Gr and injection/suction parameter Re increase respectively for different parameters. Velocity profiles again decrease while porosity parameter K and Prandtle number Pr increase respectively for same values of Gr, M, ω, ε etc. Frequency ω increase then velocity profile again decreases. There is increase in Prandtle number then decrease in temperature. Amplitude increases with increasing of the Hartmann number but it decreases with increasing of injection/suction parameter Re and permeability parameter K. It is also clear that the amplitude increases with increasing plate oscillations. Significant decreases in the phase angle with increasing of the Hartmann number M, and the magnetic field strength. However, it increases with increasing of injection/suction parameter Re, and permeability parameter K. The increase in phase angle then there is simultaneously increased in frequency of the plate oscillations ω.

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