International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
doi : https://doi.org/10.32628/IJSRST2215546

Study of the Photo gravitational Circular Restricted Three Body Problem

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Article Info
Volume 9, Issue 6
Page Number : 402-409

Publication Issue
November-December-2022

Article History
Accepted : 20 Nov 2022
Published : 07 Dec 2022

ABSTRACT

In this chapter we will discuss the equation of motion of Photogravitational Circular Restricted Three Body Problem (in brief -PCR3BP) in which both the primaries are sources of radiation [1]. The three body problem studies the motion of three masses whose gravitational attraction have an effect on each other. The dynamics of the three-body problem are essentially different from those of two bodies, because in the latter case, an analytical solution may be found that admits orbits in the form of conic sections. This problem has been studied at great length and is the basis of most of today's orbit planning and trajectory design for satellites.

Keywords: RTBP, CRTBP, Three Body Problem.

I. INTRODUCTION

The development of mathematical model of Circular Restricted Three Body Problem is one of the most challenging problems of Celestial Mechanics. The Circular Restricted Problem specifies the motion of a body of infinitesimal mass under the gravitational attraction of two massive bodies revolving around each other in circular orbits. The problem is restricted in the sense that the infinitesimal body does not influence the motion of other two massive bodies [1-4].

This model of Circular Restricted Three Body Problem is an ideal one. In actual situation, there are various examples in which either or both the primaries are sources of radiation. The first, it is quite reasonable to modify the model by taking the radiation pressure of the primaries into account in ideal equation. The modified model is known as Photogravitational Circular Restricted Three Body Problem (in brief PCR3BP) [5-6].

Radzievsky (1950) studied the Restricted Three Body Problem by considering the more massive primary as a source of radiation. Perezhogin (1976), Kunitsyn and Perezhogin (1978), Simmons et. al (1985), Kumar and Chaudhary (1988), Haque (1992) and many authors studied the problem by taking into consideration the radiation effect of either and both the primaries. In the present problem we have considered both the primaries as sources of radiation [7]. The force of radiation is taken as

$$F = F_g - F_p = F_g\left(1 - \frac{F_p}{F_g}\right) = qF_g$$

where

F_g = the gravitational attraction force,

F_p = the radiation pressure force,

and q = the mass reduction factor.

q_1 and q_2 are the factors characterizing the radiation effects of the two primaries [8].

To simplify the calculation, we have used later on

$q_1 = 1 - \delta_1$

and $q_2 = 1 - \delta_2$

$\delta_{1,2} = 0$ i.e., $q_1 = q_2 = 1$ represents the classical case;

$0 < \delta_{1,2} < 1$ i.e. ($1 > q_1, q_2 > 0$) represents the reduction of the gravitational forces by radiation and $\delta_{1,2} \geq 1$ i.e. ($q_1, q_2 \leq 0$) implies that the radiation is overwhelmed gravity.

The complete range of physically possible q_1, q_2 is $-\infty < q_1, q_2 \leq 1$ i.e., $o \leq \delta_{1,2} < \infty$. We have considered the case when the gravitation prevails i.e., when $0 < \delta_{1,2} < 1$, i.e., when $1 > q_1, q_2 > 0$.

II. DERIVATION OF EQUATIONS OF MOTION OF PCR3BP

We consider three gravitationally interacting bodies of masses m_1, m_2 and m in which two bodies m_1 and m_2, are so-called primaries, are sources of radiation, revolving around their centre of mass in a circular orbit under the influence of their mutual gravitational attraction and the third body m is much smaller than m_1 and m_2. Intuitively the third body m does not affect the motion of the primaries [9]. Thus, the Circular Restricted Problem of Three Bodies is to be described the motion of the third body.

Consider an inertia frame of reference $\xi\eta\zeta$ whose origin lie at the centre of mass O of the two bodies m_1 and m_2 (as shown in fig. 1). Let the ξ- axis lie along the line from m_1 to m_2 at time t = 0 with the η- axis perpendicular to it and also in the orbital plane of the two bodies and ζ- axis perpendicular to the $\xi\eta$- plane, along the angular momentum vector [10].

Let (ξ_1, η_1, ζ_1) and (ξ_2, η_2, ζ_2) be the coordinates of masses m_1 and m_2 respectively in this frame of reference. Let R be the constant distance between the two orbiting masses m_1 and m_2 and r_1, r_2 be the constant distances between infinitesimal mass m and orbiting masses m_1 and m_2 respectively.

In this inertial frame, it is required to balance between the gravitational and centrifugal forces

i.e. $G\frac{m_1 m_2}{R^2} = m_2 a n^2 = m_1 b n^2$ ....(1)

Where G is the Gaussian constant of gravitation, n is the mean angular velocity (in Celestial Mechanics is called mean motion) of masses m_1 and m_2, *a* and *b* are the distances of masses m_1 and m_2 from O respectively.

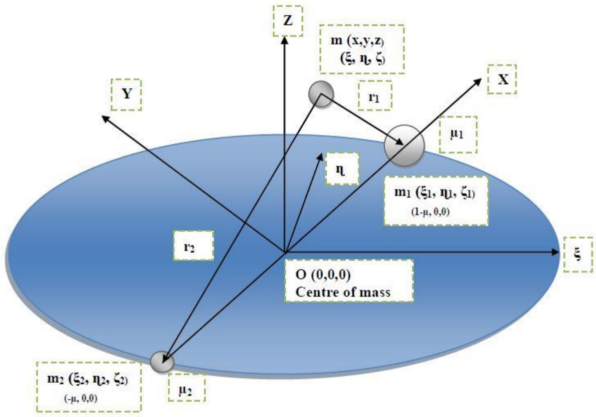

Fig. 1

The primaries m_1 and m_2 are moving around their centre of mass O as shown in figure 1.

$$\therefore\ G\frac{m_1 m_2}{R^2} = m_2 a n^2 \quad \text{and} \quad G\frac{m_1 m_2}{R^2} = m_1 b n^2$$

$$\text{Or,} \quad Gm_1 = R^2 a n^2 \quad \text{and} \quad Gm_2 = R^2 b n^2$$

$$\therefore G(m_1+m_2)=R^2(a+b)n^2 \quad \text{Or,} \quad G(m_1+m_2)=R^2Rn^2$$

$$\text{Or,} \quad G(m_1+m_2)=R^3n^2 \qquad \text{................(2)}$$

This equation is known as Kepler's Law.

Also $\quad a=\frac{m_1}{M}R \quad$ and $\quad b=\frac{m_2}{M}R \qquad$ ....(3)

Where, $M=m_1+m_2$

It is convenient choose our unit of length such that R=1, and our unit mass such that $\mu=G(m_1+m_2)=1$. It follows, from equation (2), the mean angular velocity n=1. However, we shall continue to retain n in our equations, for the sake of clarity [11]. If we assume that $m_1>m_2$ and define

$$\mu=\frac{m_2}{m_1+m_2} \qquad \text{..................................(4)}$$

The force of radiation is taken as

$$F=F_g-F_p=F_g\left(1-\frac{F_p}{F_g}\right)=(1-q)F_g$$

Where, F_g = gravitational attraction force, F_p = radiation pressure force and, q = mass reduction factor. Here, it is assumed that gravitation prevails. Poynting– Robertson drag effect is ignored. Then in this system of units of two masses with sources of radiation are

$$\mu_1=Gm_1q_1=(1-\mu)q_1 \text{ and } \mu_2=Gm_2q_2=\mu q_2 \qquad \text{........... (5)}$$

where, $\mu\le\frac{1}{2}$, q_1 and q_2 are the radiation effect of the primaries m_1 and m_2.

Let (ξ, η, ζ) be coordinate of the infinitesimal mass m in the inertial (sideral) coordinate system [12]. Then the equations of motion of the infinitesimal mass 'm' are

$$\left.\begin{aligned} m\ddot{\xi}&=-Gm_1q_1m\frac{\xi-\xi_1}{r_1^3}-Gm_2q_2m\frac{\xi-\xi_2}{r_2^3}\\ m\ddot{\eta}&=-Gm_1q_1m\frac{\eta-\eta_1}{r_1^3}-Gm_2q_2m\frac{\eta-\eta_2}{r_2^3}\\ m\ddot{\zeta}&=-Gm_1q_1m\frac{\zeta-\zeta_1}{r_1^3}-Gm_2q_2m\frac{\zeta-\zeta_2}{r_2^3}\end{aligned}\right\} \quad \text{.... (6)}$$

i.e.,

$$\left.\begin{aligned} \ddot{\xi}&=Gm_1q_1\frac{\xi_1-\xi}{r_1^3}+Gm_2q_2\frac{\xi_2-\xi}{r_2^3}\\ \ddot{\eta}&=Gm_1q_1\frac{\eta_1-\eta}{r_1^3}+Gm_2q_2\frac{\eta_2-\eta}{r_2^3}\\ \ddot{\zeta}&=Gm_1q_1\frac{\zeta_1-\zeta}{r_1^3}+Gm_2q_2\frac{\zeta_2-\zeta}{r_2^3}\end{aligned}\right\} \quad \text{...(7)}$$

i.e.,

$$\left.\begin{aligned} \ddot{\xi}&=\mu_1\frac{\xi_1-\xi}{r_1^3}+\mu_2\frac{\xi_2-\xi}{r_2^3}\\ \ddot{\eta}&=\mu_1\frac{\eta_1-\eta}{r_1^3}+\mu_2\frac{\eta_2-\eta}{r_2^3}\\ \ddot{\zeta}&=\mu_1\frac{\zeta_1-\zeta}{r_1^3}+\mu_2\frac{\zeta_2-\zeta}{r_2^3}\end{aligned}\right\} \quad \text{...(8)}$$

i.e.,

$$\left.\begin{aligned} \ddot{\xi}&=(1-\mu)q_1\frac{\xi_1-\xi}{r_1^3}+\mu q_2\frac{\xi_2-\xi}{r_2^3}\\ \ddot{\eta}&=(1-\mu)q_1\frac{\eta_1-\eta}{r_1^3}+\mu q_2\frac{\eta_2-\eta}{r_2^3}\\ \ddot{\zeta}&=(1-\mu)q_1\frac{\zeta_1-\zeta}{r_1^3}+\mu q_2\frac{\zeta_2-\zeta}{r_2^3}\end{aligned}\right\} \quad \text{... (9)}$$

$$\left.\begin{aligned} \text{Where}\quad r_1^2 &= (\xi_1-\xi)^2+(\eta_1-\eta)^2+(\zeta_1-\zeta)^2 \\ r_2^2 &= (\xi_2-\xi)^2+(\eta_2-\eta)^2+(\zeta_2-\zeta)^2 \end{aligned}\right\} \ldots\ldots (10)$$

If the ζ axis is perpendicular to the plane of the two massive bodies (primaries) then

$$\zeta_1 = \zeta_2 = 0.$$

If the two masses are moving in circular orbit, then the distance between them is fixed and they move about their common centre of mass at a fixed angular velocity i.e., mean motion n. In these circumstances it is natural to consider the motion of the particle in a rotating (synodic) reference frame in the locations of the two masses are also fixed [14]. Consider a new set of coordinate axes X, Y, Z having the same origin and the X and Y axes are rotating with angular velocity unity i.e., n=1 about Z- axis which coincides with the **ζ**-axis perpendicular to the plane of the primaries as shown in Fig-1.The direction of the X- axis can be chosen such that two massive bodies always lie along it, having coordinates $(x_1, y_1, z_1) = (-\mu, 0{,}0)$ and $(x_2, y_2, z_2) = (1-\mu, 0{,}0)$.

Hence

$$\left.\begin{aligned} r_1^2 &= (x+\mu)^2+y^2+z^2, \\ r_2^2 &= (x-1+\mu)^2+y^2+z^2, \\ r^2 &= x^2+y^2+z^2 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (11)$$

Where (x, y, z) are the coordinates of the infinitesimal mass m with respect to rotating or synodic system [15].
We now discuss about Rotation Matrices in three dimensions Coordinate system.
Let us take a fixed coordinate axes χ, ψ, ω and the coordinates of any point P with respect to this coordinate axes be (χ, ψ, ω).
Also, consider a new coordinate axis χ', ψ', ω' having the same origin and $\chi'$and ψ' axes are rotating about ω' axis which coincides with ω' axis and let (χ', ψ', ω') be coordinates of the particle of P with respect to this coordinate axes.

$$\left.\begin{aligned} \chi &= \chi'\cos\theta - \psi'\sin\theta \\ \psi &= \chi'\sin\theta + \psi'\cos\theta \\ \omega &= \omega' \end{aligned}\right\} \ldots\ldots\ldots\ldots (12)$$

In Matrix form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \chi' \\ \psi' \\ \omega' \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

This is called the rotation matrix.
In this way, for our problem the coordinates (x, y, z) are connected to the old coordinates (ξ, η, ζ) by the relation as above is to be found out.

$$\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(14)$$

Differentiating (14) twice, we have

$$\begin{bmatrix} \dot\xi \\ \dot\eta \\ \dot\zeta \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot x - y \\ \dot y + x \\ \dot z \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

And $$\begin{bmatrix}\ddot{\xi}\\ \ddot{\eta}\\ \ddot{\zeta}\end{bmatrix}=\begin{bmatrix}\cos t & -\sin t & 0\\ \sin t & \cos t & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\ddot{x}-2\dot{y}-x\\ \ddot{y}+2\dot{x}-y\\ \ddot{z}\end{bmatrix} \qquad \ldots\ldots\ldots (16)$$

Substituting the values of $\xi,\ \eta,\ \zeta,\ \ddot{\xi},\ \ddot{\eta},\ \ddot{\zeta}$ in equation (9), we have

$$(\ddot{x}-2\dot{y}-x)\cos t-(\ddot{y}+2\dot{x}-y)\sin t$$
$$=(1-\mu)q_1\left\{\frac{(x_1-x)\cos t-(y_1-y)\sin t}{r_1^3}\right\}+\mu q_2\left\{\frac{(x_2-x)\cos t-(y_2-y)\sin t}{r_1^3}\right\}$$
$$(\ddot{x}-2\dot{y}-x)\sin t+(\ddot{y}+2\dot{x}-y)\cos t$$
$$=(1-\mu)q_1\left\{\frac{(x_1-x)\sin t+(y_1-y)\cos t}{r_1^3}\right\}+\mu q_2\left\{\frac{(x_2-x)\sin t+(y_2-y)\cos t}{r_1^3}\right\}$$
$$\ddot{z}=(1-\mu)q_1\left\{-\frac{z}{r_1^3}\right\}+\mu q_2\left\{-\frac{z}{r_1^3}\right\}$$

i.e.,

$$(\ddot{x}-2\dot{y}-x)\cos t-(\ddot{y}+2\dot{x}-y)\sin t$$
$$=\left\{(1-\mu)q_1\frac{(x_1-x)}{r_1^3}+\mu q_2\frac{(x_2-x)}{r_2^3}\right\}\cos t-\left\{(1-\mu)q_1\frac{(y_1-y)}{r_2^3}+\mu q_2\frac{(y_2-y)}{r_2^3}\right\}\sin t$$
$$(\ddot{x}-2\dot{y}-x)\sin t+(\ddot{y}+2\dot{x}-y)\cos t$$
$$=\left\{(1-\mu)q_1\frac{(x_1-x)}{r_1^3}+\mu q_2\frac{(x_2-x)}{r_2^3}\right\}\sin t+\left\{(1-\mu)q_1\frac{(y_1-y)}{r_2^3}+\mu q_2\frac{(y_2-y)}{r_2^3}\right\}\cos t$$
$$\ddot{z}=-(1-\mu)q_1\left\{\frac{z}{r_1^3}\right\}-\mu q_2\left\{\frac{z}{r_1^3}\right\}$$
$$\ldots\ldots\ldots\ldots(17)$$

Multiplying the first two equations of (18) by cost and sint respectively and again by –sint and cost and adding them, we obtain

$$\ddot{x}-2\dot{y}-x=(1-\mu)q_1\frac{(x_1-x)}{r_1^3}+\mu q_2\frac{(x_2-x)}{r_2^3}$$
$$\ddot{y}+2\dot{x}-y=1-\mu)q_1\frac{(y_1-y)}{r_2^3}+\mu q_2\frac{(y_2-y)}{r_2^3}$$
$$\ddot{z}=-(1-\mu)q_1\frac{z}{r_1^3}-\mu q_2\frac{z}{r_2^3} \qquad \ldots(18)$$

If the X-axis will pass through the centre of the finite bodies, then $y_1=0, y_2=0$ and equation (18) becomes

$$\ddot{x}-2\dot{y}-x=-(1-\mu)q_1\frac{(x-x_1)}{r_1^3}-\mu q_2\frac{(x-x_2)}{r_2^3}$$
$$\ddot{y}+2\dot{x}-y=-(1-\mu)q_1\frac{(y-y_1)}{r_1^3}-\mu q_2\frac{(y-y_2)}{r_2^3}$$
$$\ddot{z}=-(1-\mu)q_1\frac{z}{r_1^3}-\mu q_2\frac{z}{r_2^3}$$

i.e.,

$$\ddot{x}-2\dot{y}=x-(1-\mu)q_1\frac{(x-x_1)}{r_1^3}-\mu q_2\frac{(x-x_2)}{r_2^3}$$
$$\ddot{y}+2\dot{x}=y-(1-\mu)q_1\frac{(y-y_1)}{r_1^3}-\mu q_2\frac{(y-y_2)}{r_2^3}$$
$$\ddot{z}=-(1-\mu)q_1\frac{z}{r_1^3}-\mu q_2\frac{z}{r_2^3} \qquad \ldots(19)$$

These are the equations of motion of the infinitesimal body with respect to the set of rotating (synodic) coordinates, so that the finite bodies always lie on the X-axis and they have important property that they do not involve explicitly the independent variable t.

Let a function U be defined by

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} q_1 + \frac{\mu}{r_2} q_2 \quad \dots\dots\dots\dots\dots\dots (20)$$

We have,

$$(1-\mu)r_1^2 + \mu r_2^2 = (1-\mu)\{(x+\mu)^2 + y^2\} + \mu\{(x-1+\mu)^2 + y^2\}$$
$$= (1-\mu)(x+\mu)^2 + (1-\mu)y^2 + \mu\{(x+\mu)^2 - 2(x+\mu) + 1 + y^2\}$$
$$= (1-\mu)(x+\mu)^2 + (1-\mu)y^2 + \mu(x+\mu)^2 - 2\mu(x+\mu) + \mu + \mu y^2$$
$$= (x+\mu)^2 + y^2 - 2\mu x - 2\mu^2 + \mu$$
$$= x^2 + \mu^2 + 2\mu x + y^2 - 2\mu x - 2\mu^2 + \mu$$
$$= x^2 + y^2 - 2\mu^2 + \mu$$
$$= x^2 + y^2 - \mu(\mu - 1)$$
$$\therefore x^2 + y^2 = (1-\mu)r_1^2 + \mu r_2^2 + \mu(\mu - 1)$$

Thus, equation (2.20) can be written a

$$U = \frac{1}{2}\left((1-\mu)r_1^2 + \mu r_2^2 + \mu(\mu-1)\right) + \frac{1-\mu}{r_1} q_1 + \frac{\mu}{r_2} q_2 \quad .\dots (21)$$

Thus equation (2.19) can be written as

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$
$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y}$$
$$\ddot{z} = \frac{\partial U}{\partial z} \quad \dots\dots\dots\dots\dots\dots (22)$$

Multiplying first equation by $\dot{x}$, second by $\dot{y}$ and third by $\dot{z}$ of (22) and adding them, we obtain

$$\dot{x}\ddot{x} - 2\dot{x}\dot{y} + \dot{y}\ddot{y} + 2\dot{x}\dot{y} + \dot{z}\ddot{z} = \dot{x}\frac{\partial U}{\partial x} + \dot{y}\frac{\partial U}{\partial y} + \dot{z}\frac{\partial U}{\partial z} \quad \dots\dots\dots\dots.. (23)$$

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} = \frac{dU}{dt} \quad \dots\dots\dots\dots\dots\dots. (24)$$

Integrating, we have

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2U - C \quad \dots\dots\dots. (25)$$

Where C is the constant of integration.

The L.H.S. of (25) is the square of the velocity of the particle of the infinitesimal mass in the rotating frame and is denoted by V^2 then

$$V^2 = 2U - C \quad \dots\dots\dots\dots (26)$$

This is the Jacobi's Integral and is sometime called the Integral of

Relative Energy. It is the only one that can be obtained in the Circular Restricted Three Body Problem [16]. Now we introduce the perturbations in coriolis and centrifugal forces in terms of the parameters α and β. Thus, in a synodic coordinate system (x, y, z) the equations of motion of the Photogravitational Circular Restricted Three Body Problem (PCR3P) in which both the primaries are sources of radiation and there are perturbations α and β in coriolis and centrifugal forces respectively are

$$\ddot{x} - 2\alpha\dot{y} = \frac{\partial U}{\partial x}$$

$$\ddot{y} + 2\alpha\dot{x} = \frac{\partial U}{\partial y}$$

$$\ddot{z} = \frac{\partial U}{\partial z} \qquad \ldots\ldots\ldots\ldots (27)$$

Where,

$$U = \frac{1}{2}[(1-\mu)r_1^2 + \mu r_2^2]\beta + \frac{1-\mu}{r_1}q_1 + \frac{\mu}{r_2}q_2 + \frac{1}{2}\mu(\mu-1)\beta \quad .. (28)$$

and

$q_1 = 1 - \delta_1; \quad 0 < \delta_1 < 1,$

$q_2 = 1 - \delta_2; \quad 0 < \delta_2 < 1,$

$\alpha = 1 - \varepsilon_1; \quad 0 < |\varepsilon_1| \ll 1,$

$\beta = 1 + \varepsilon_2; \quad 0 < |\varepsilon_2| \ll 1.$

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Cite this article as :

Vikash Kumar, Dr. K. B. Singh, & Dr. M. N. Haque, "Study of the Photo gravitational Circular Restricted Three Body Problem", International Journal of Scientific Research in Science and Technology (IJSRST), Online ISSN : 2395-602X, Print ISSN : 2395-6011, Volume 9 Issue 6, pp. 402-409, November-December 2022. Available at doi : https://doi.org/10.32628/IJSRST2215546
Journal URL : https://ijsrst.com/IJSRST2215546