



Static Flexural Analysis of Thick Isotropic Beams Using Higher Order Shear Deformation Theory

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ABSTRACT

For the static flexure analysis of thick isotropic beams, a higher order shear deformation theory is established in the current study. Axially displaced cantilever beam is examined. The theory's governing differential equation and boundary conditions are generated using the virtual work concept. The numerical findings were calculated for different length-to-thickness ratios of the beams, and the outcomes were compared with those of the Elementary, Timoshenko, trigonometric, and other hyperbolic shear deformation theories as well as with the solution that was published in the literature.

Keywords: Higher order shear deformation theory, Isotropic beam, virtual work, Shear deformation, thick beam, static flexure etc.

I. INTRODUCTION

When thick beams are subjected to transverse loads, shear deformation effects are more noticeable than they are when thin beams are subjected to the same loading. The consequences of shear deformation are more pronounced in thick beams. W. J. M. Rankine Bresse J. A. C. [1] The rotatory inertia and shear flexibility effects were the first dynamical effects in beam theory to be included by [2]. However, refers to this theory in the literature as the Timoshenko beam theory, Rebello C. A. et al. [3] and is known as first-order shear deformation theory. It is based on kinematics (FSDT). The rotatory inertia effect was incorporated by Rayleigh Lord [4], and Timoshenko S. P. later added the shear stiffness effect. Timoshenko demonstrated that shear has a significantly bigger impact on transverse vibration of prismatic beams than rotatory inertia does.

The Displacement Field

Based on the above mentioned assumptions, the displacement field of the present beam Theory can be expressed as follows.

$$u(x, z) = -z \frac{dw}{dx}(x) + \left[\frac{z}{2} \left(\frac{h^2}{4} - \frac{z^2}{3} \right) \right] \phi(x)$$

$$w(x, z) = w(x)$$

Where, u = Axial displacement in x direction which is function of x and z .

w = Transverse displacement in z direction which is function of x .

ϕ = Rotation of cross section of beam at neutral axis which is function of x .

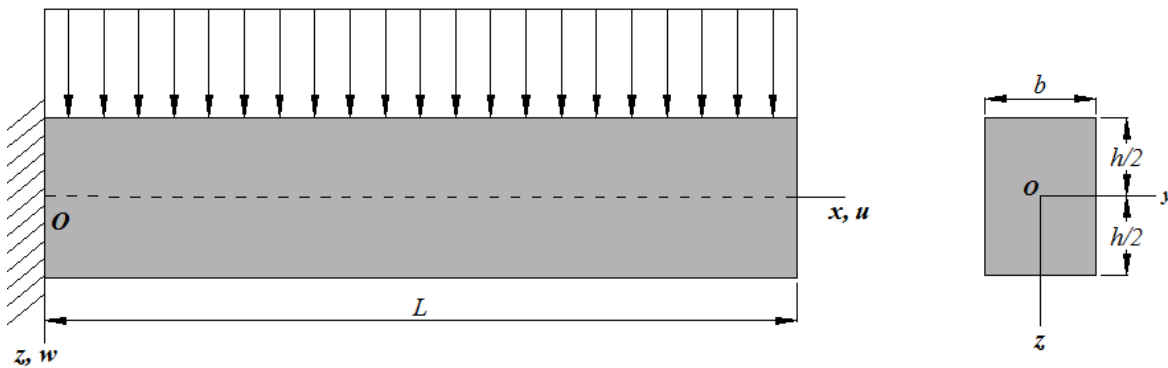
Example-1: A Cantilever beam with uniformly distributed load

$$q(x) = q_0$$

The beam has its origin on left hand side fixed support at $x = 0$ and free at $x = L$. The beam is subjected to varying load, $q(x)$ on surface $z = +h/2$ acting in downward z direction with minimum intensity of load q_0

Cantilever beam with uniformly distributed load

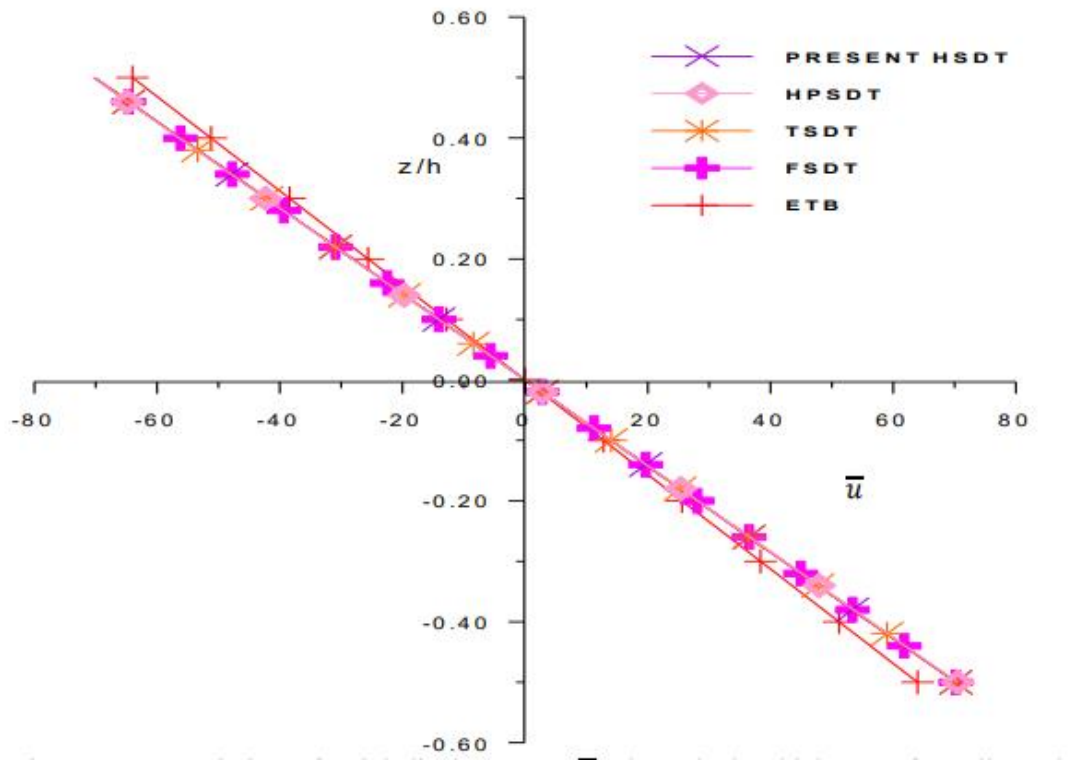
$$q(x) = q_0 x$$



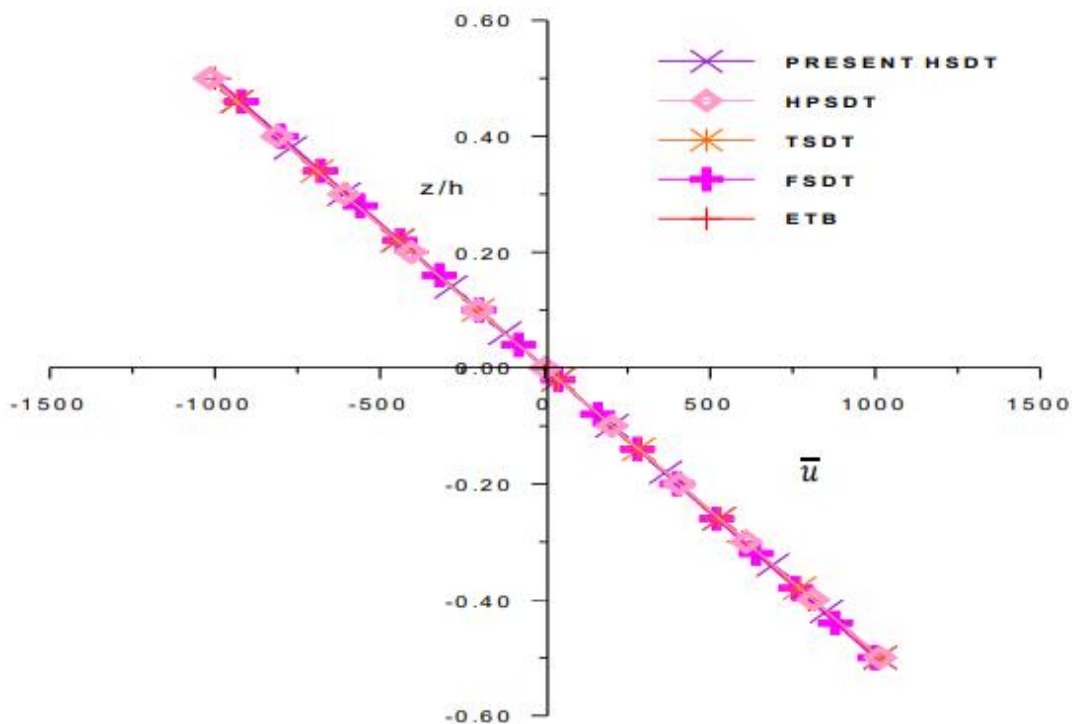
$$\bar{u} = -\frac{z L^3}{h h^3} \left\{ \left[\frac{2x^3}{L^3} - \frac{6x^2}{L^2} + \frac{6x}{L} \right] + \left[\frac{E h^2 A_0^2}{H L^2 C_0} \sinh \lambda x - \cosh \lambda x + 1 \right] \right\} + \left[\frac{A_0 E L}{C_0 G h} \left(\frac{zh}{8} - \frac{z^3}{6h^3} \right) \left(\sinh \lambda x - \cosh \lambda x + 1 - \frac{x}{L} \right) \right]$$

Table No 1: Non-Dimensional Axial Displacement (\bar{u}) at ($x = L$, $z = h/2$), Cantilever Beam Subjected to Uniformly Distributed Load for Aspect Ratio 4 and 10.

Theory	AS -4	AS- 10
Present HSDT	70.24	1015.6
HPSDT	70.49	1016.22
TSDT	70.2	1015.57
FSDT	64	1000
ETB	64	1000



Variation of axial displacement (\bar{u}) through the thickness of cantilever beam at ($x = L, z=0$) when subjected uniformly distributed load for aspect ratio 4



Variation of axial displacement (\bar{u}) through the thickness of cantilever beam at ($x = L, z=0$) when subjected to uniformly distributed load for aspect ratio 10

II. CONCLUSIONS

1. It has been noted that the outcomes of the current hypothesis have a very good agreement with other hypotheses. However, compared to the values provided by other theories, ETB and FSDT produce lower values of this stress. This stress's through-thickness variation, as reported by ETB and FSDT, is linear throughout the thickness of the beam, which highlights the impact of shear deformation's neglect.
2. Due to significant stress concentration, current higher order and other refined theories explain the non-linear fluctuations of axial stress over the thickness at the built-in end. However, lower order theories like ETB and FSDT cannot account for this effect of local stress concentration.

III. REFERENCES

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