International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
doi : https://doi.org/10.32628/IJSRST2310131

Fuzzy Mathematical Modelling and Lexicographical Solution Concept in Fuzzy Games

Amit Kumar[1], Dr. K. B. Singh[2], Dr. B. P. Kumar[3]

[1]Research Scholar, University Department of Mathematics, B. R. A. Bihar University, Muzaffarpur, Bihar, India

[2]Assistant Professor, P. G. Department of Physics, L. S. College, Muzaffarpur, B. R. A. Bihar University, Muzaffarpur, Bihar, India

[3]Retd. Professor & Head, University Department of Mathematics, B. R. A. Bihar University, Muzaffarpur, Bihar, India

Article Info

Publication Issue

Volume 10, Issue 1

January-February-2023

Page Number

266-269

Article History

Accepted: 15 Jan 2023

Published: 04 Feb 2023

ABSTRACT

In this paper, we present about the fuzzy mathematical modelling and lexicographical solution concept in fuzzy games. By introducing the concept of topological game over an ideal of Hausdorff space, a game over some special product space is played. Fuzzy set theory has been applied to fuzzify some of the results obtained.

Keywords: Fuzzy Logic, Game Theory, Topological Game.

I. INTRODUCTION

The concept of excess in concerned with a coalition. We here introduce a new concept of an excess of a player by extending the concept of an excess of a coalition in a non-fuzzy game as well as a fuzzy game. By considering a payoff vector which minimizes the excess of a player in the lexicographical order, we propose a new solution concept in a fuzzy game.

II. EXCESS IN FUZZY GAMES

The concept of excess was used to study the dynamics games by M. Davis and M. Maschter. Here we define the excess of a coalition and the excess of a player in a non-fuzzy game as well as fuzzy games.

Let A = {S/S CN} be a family of coalitions

and X = (X_1,X_2,X_3, ….,X_n) be a payoff vectors.

The coalition S can be defined by $\tau^S_{(i)} = \begin{cases} 1, & if\ i \in S \\ 0 \ldots, & if\ i \in S \end{cases}$ /

and represented by $\tau^S = \left(\tau^S_{(1)}, \tau^S_{(2)}, \tau^S_{(3)}, \ldots .\ \tau^S_{(n)}\right)$

DEFINITION 1.

Excess of a Coalition in a Non-Fuzzy Game

The excess of a coalition S W.R. to a payoff vector X is defined by

where $\tau_i^S = \begin{matrix} 1, \\ 0 \ldots, \end{matrix} \begin{cases} if\ i\ \in S \\ if\ i\ \in S \end{cases}$ for any player i ∈ N

DEFINITION 2. EXCESS OF A FUZZY COALITION

For a fuzzy game FG = (N, f)

Let X be a payoff vector and G be a fuzzy game.

Then the excess of the fuzzy coalition τ∈ [0,1] n w. r. to a payoff vector X is defined by e (τ,X)= f (τ) - X τ where f (τ) is a value of the characteristic function representing the gain that the fuzzy coalition τ can obtain from a game alone and X. τ is the amount of payoff of the fuzzy coalition τ for the payoff vector X.

III. EXCESS OF A PLAYER IN A NON-FUZZY GAME

For a game g = (N, U)

Let e(S, X) be a excess of a coalition w.r. to a payoff vector X. Then, an excess of a player w.r. to X is defined by

$$e(i, X) = \sum_{i \in S \in A} e(S, X) = \sum_{i \in S \in A} (V(S) - X.\tau^S)$$

i.e. $e(i, X) = \sum_{S \in A} \tau_i^S\, e(S, X) = \sum_{S \in A} \tau_i^S (V(S) - X.\tau^S)$

where $\tau_{(i)}^S$ is written as $\tau_{(i)}^S$

and hence $\tau_{(i)}^S = \begin{matrix} 1, \\ 0 \ldots, \end{matrix} \begin{cases} if\ i\ \in S \\ if\ i\ \in S \end{cases}$

First, we consider a fuzzy game with a finite number of fuzzy coalitions

A Fuzzy coalition is represented by an n - dimensional vector,

Let a set of finite number of coalitions τ be T.

$$\tau \in [0,1]^n$$

Then an excess e* (i,X) of player I w.r. to a payoff vector X is

$$e * (i, e) = \sum_{\tau \in T} \tau_i\, e(\tau, X)$$

Consequently, an excess e (i,X) of player i in a non-fuzzy game can be viewed as a special case of an excess e*((i,x) of player i in a fuzzy game.

Secondly, we consider a fuzzy game with an infinite number of fuzzy coalitions. The excess e* (i,x) in a fuzzy game, which permits all of the fuzzy coalitions $\tau \in [0,1]^n$, is defined by multiplying an excess e (τ,x) by a rate of participation τ of player i and integrating it from 0 to 1.

i.e. $e * (i, x) = \int_0^1 \tau_i e(\tau, x) d\tau$

where $\int_0^1 d\tau = \int_0^1 \ldots \ldots \int_0^1 d\tau_1 . d\tau_2 . d\tau_3 \ldots .. d\tau_n$

The domain of τ is limited to

$$D = \{\tau / \alpha_i \leq \tau_i \leq \beta_i ,\ 0 \leq \alpha_i \leq \beta_i \leq 1,\ \forall i \in N\} \text{ instead of } [0,1]^n.$$

The excess e* (i,x) can also be

Considered as e* (i,x) = $\int_D \tau_i e(\tau, x) d\tau$

IV. EXCESS OF A PLAYER IN FUZZY GAME

For a fuzzy game FG = (N,f).

Let e (τ,x) be an excess of a fuzzy coalition τ w.r. to a payoff vector x.

Also let D $[0,1]^n$ and T $[0,1]^n$.

Then an excess of a player in any fuzzy game is defined by

$$e^* (i,x) = \int_D \tau_i e(\tau,x) d\tau + \sum_{\tau \in T} \tau_i e(\tau, X)$$

LEXICOGRAPHICAL ORDER: DEFINITION

Let γ(x) be a vector arranged in order of decreasing magnitude i.e.,

i.e., $i < j = \gamma_i (x) > \gamma_j (x)$

Then for any two payoff vectors x and y if x = y or for the first component h in which they differ i.e. $\gamma_n(x) < \gamma_n (y)$, x is smaller than y in the lexicographical order and denoted by $\leq_L$.

Using the concept of lexicographical order, we consider a solution which minimizes an excess of a player e (i,x) or e* (i,x).

V. SOLUTION WHICH MINIMIZES AN EXCESS OF PLAYER

Let H : Rn Rn be a mapping which arranges components of an n-dimensional vector in order of decreasing magnitude. Then for a non fuzzy game G = (N,V) and a fuzzy game FG = (N,f) the lexicographical solutions which minimize the excess e (i,x) or e* (i,x) of a player can be defined as follows:

$L_s(G) = \{x / H (e(1,x), e(2,x), \ldots\ldots e(n,x)$

$\leq_L H (e(1,y), e(2,y), \ldots\ldots e(n,y)$

$\forall y \in X (G)\}$

$L_s(FG) = \{x / H (e(1,x), e(2,x), \ldots\ldots e(n,x)$

$\leq_L H (e(1,y), e(2,y), \ldots\ldots e(n,y)$

$\forall y \in X (FG)\}$

where X (G) and X (FG) are the sets of imputations defined by

$X (G) = \{x / x_i > V (\{i\}), \forall i \in N_i, \sum_{i \in N} X_i = V (N)\}$

$X (FG) = \{x / x_i > f (\tau^i), \forall i \in N_i, \sum_{i \in N} X_i = F (\tau^N)\}$

The solution $E_s(G)$ exists and coincides with the solution $L_s(G)$, if a payoff vector X satisfying.

$$\min_{x \in X(G)} \max_{i \in N} e(i,x) \text{ is unique.}$$

Similarly E_s (FG) exists and coincides with the solutions $L_s(FG)$, if a payoff vector X satisfying

$$\min_{x \in X(FG)} \max_{i \in N} e^*(i,x) \text{ is unique.}$$

The above first conditions is valid if x satisfying the equation.

$$e (1,x) = 2 (2,x) = e (3,x) = \ldots\ldots = e(n,x)$$

Lies in the set of the imputations X (G) and the solution $L_s(G)$ can be computed by solving the linear programming problem. Thus, if the solution $E_s(G)$ exists, $L_s(G)$ and $E_s(G)$ coincide with each other. For a fuzzy game FG = (N,f), the existence of the solution Es(FG) can be proved similarly.

The second conditions of the theorem is

$$2^{n-2}V(N) + n\ \sum_{i \in S} V(S) - \sum_{\substack{j \in N \\ j \neq i}} \sum_{\substack{i \in S \\ i \neq j}} V(S) \geq 0$$

i = 1,2,3,……..,n

The only term

$$\sum_{\substack{j \in N \\ j \neq i}} \sum_{\substack{i \in S \\ i \neq j}} V(S)$$

In this condition, is negative and if the value of coalition to which the player j belongs is not so small compared with others. First conditions of the theorem satisfied and hence the solution Es (G) exists.

VI. REFERENCES

[1]. Kumar, B.P. (1982): Lattice & topological approach to game theory, Ph.D. thesis, Bhagalpur Univ., Bhagalpur (Bihar).

[2]. Kondel,A. (1982) A fuzzy techniques in Pattern recognition, John Wiley, New York.

[3]. M.Davis and (1965) The Kernel of Co-operative game, Nawal Research M. Maschler Logistics Quarterly 12,223-259.

Cite this article as :

Amit Kumar, Dr. K. B. Singh, Dr. B. P. Kumar, "Fuzzy Mathematical Modelling and Lexicographical Solution Concept in Fuzzy Games", International Journal of Scientific Research in Science and Technology (IJSRST), Online ISSN : 2395-602X, Print ISSN : 2395-6011, Volume 10 Issue 1, pp. 266-269, January-February 2023.
Available at doi : https://doi.org/10.32628/IJSRST2310131
Journal URL : https://ijsrst.com/IJSRST2310131