© 2018 IJSRST | Volume 4 | Issue 10 | Print ISSN : 2395-6011 | Online ISSN : 2395-602X
Themed Section: Science and Technology

Strong Spherical Shock in Azimuthal Magneti Field Self-Gravitating Gas

Anil Kumar
Department of Physics, Bareilly College, Bareilly, India

ABSTRACT : Propagation of shock waves depends on the medium, its' viscosity, conductivity, rotation, radiative heat effect and magnetic field too. The aims of present paper is to study the propagation of strong spherical shock waves in a self-gravitating gas atmosphere in presence of magnetic field. Since the azimuthal component of the magnetic field is more effective in comparison to other components, therefore only azimuthal component of the magnetic field is considered here. The problem is explored for two cases (i) when shock moves freely and (ii) when it moves under the influence of overtaking disturbance. Assuming exponential density distribution, the analytical relations for shock velocity and shock strength have been obtained for both the cases. It is found that magnetic field affects the Shock strength very effectively.

I. INTRODUCTION

Study of mantogasdynamic shock waves and detonations has considerable applications in various astrophysical, geophysical and technological problems, for examples, propagation of a flare produced shock in the solar wind, (Lee and Chen [1968], Summers [1975]), generation if gas ionizing shock waves by magnetic compression to produce high temperature plasma samples in laboratory (Sakurai [1965], Ngayama [1981]) and cylindrical blast wave produced by a wire explosion (Sakurai [1965], Christer and Helliwell [1969]. Similarly solutions for the blast wave phenomena in magnet to gas dynamics have been obtained by a number of authors, for example, Pai [1958]. Cole and Grefinger [1962]. Sakurai [1965], Christer and Helliwell [1969], Summers [1975], Verma, Vishwakarma and Sharan [1982], and Singh and Singh [1995] and others.

Recently, propagation and effect of shock wave on human blood has been carried out experimentally by Yadav et al. (2011). They concluded that the application of shock waves in the human body may be the valuable in the medical field. The aim of the present paper is to study the effect of azimuthal magnetic field on the propagation of shock waves through

self-gravitating gas atmosphere. The problem in explode for two cases viz., (i) when shock moves freely and (ii). When its moves under the influence of overtaking disturbances. Assuming exponentially varying density distribution, the analytical relations for shock velocity and shock strength have been obtained for both the cases. It is found that magnetic field affects the shock velocity and shock strength very effectively.

Basic Equation :- The equation governing of a gas under the influence of its own gravitation and a magnetic field have variable azimuthal magnetic field are,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{\mu}{2\rho}\frac{\partial H_\theta^2}{\partial r} + \frac{\mu}{\rho}\frac{\partial H_\theta^2}{r} + \frac{Gm}{r^2} = 0$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{\alpha u}{r}\right) = 0$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + \gamma\rho\left(\frac{\partial u}{\partial r} + \frac{\alpha u}{r}\right) = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial t}\right)(p\rho^{-\gamma}) = 0 \qquad \text{.....(1)}$$

$$\frac{\partial H_\theta}{\partial t} + u\frac{\partial H_\theta}{\partial r} + H_\theta\frac{\partial u}{\partial r} = 0$$

$$\frac{\partial m}{\partial r} - 4\pi r^2\rho = 0$$

Where u, p, ρ, μ, H_θ, γ and m are respectively the particle velocity, the pressure, the density, azimuthal component of magnetic field, specific heat gas ratio, permeability of gas and mass of a sphere while G is gravitational.

Boundary Conditions :- The magneto dynamic shock condition can be written in term of a single parameter ξ (= ρ/ρ_o) as,

$$\rho = \rho_o\xi,\ H_\theta = H_\theta\xi,\ u = \frac{\xi - 1}{\xi}U$$

$$U^2 = \frac{2\xi}{(\gamma+1)-(\gamma+1)\xi}\left[a_0^2 + \frac{b_0^2}{2}\{(2-\gamma)\xi + \gamma\}\right] \qquad \text{.....(2)}$$

Analytical Relation for variables

Strong shock in strong magnetic field (SSSMF) : using shock condition (3) in to equation (6) we have for strong shock.

$$\frac{dU^2}{dr} + A_S \mu U^2 + \alpha B_S \frac{U^2}{r} + C_s \frac{Gm}{r^2} = 0 \tag{8}$$

where

$$A_S = \frac{\chi}{\gamma\left[\chi + \frac{1}{2}(\xi - 1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]} \qquad B_S = \frac{(\xi - 1)\chi}{\left[\frac{\chi}{\xi} + \frac{1}{2}(\xi - 1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]\left[(\xi - 1) + (\chi\xi)^{\frac{1}{2}}\right]}$$

$$C_S = \left[\frac{\xi(\chi\xi)^{\frac{1}{2}}}{(\xi - 1) + (\chi\xi)^{\frac{1}{2}}} - 1\right] \Bigg/ \left[\frac{\chi}{\gamma} + \frac{1}{2}(\xi - 1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]$$

on Intigration, (equation 8) yields

$$U^2 = e^{-A_s \mu r} r^{-\alpha B_S}\left[K - C_S G \int m\, e^{A_s \mu r} r^{\alpha B_S - 2} dr\right] \tag{9}$$

where K is the intigration constant.

Now we assume that the intial density distribution law is

$$\rho_0 = \rho' e^{\mu r} \tag{10}$$

the mass inside a sphere of radius r and unit cross-section is written as

$$m = 4\pi\rho'\left[\frac{e^{\mu r}}{\mu} r^2 - \frac{2e^{\mu r}}{\mu^2} r + \frac{2e^{\mu r}}{\mu^3}\right]$$

$$= \frac{8\pi\rho'}{\mu^3} e^{\mu r}\left[\frac{1}{2}(\mu r)^2 - (\mu r) - 1\right] \tag{11}$$

using equation (11), we have from equation (9),

$$\frac{U^2}{\rho' G} = e^{-A_s \mu r} r^{-\alpha B_S}\left[\frac{K}{\rho' G} - 8\pi\xi C_S \int e^{\mu(1 + A_S)} \frac{r^{(\alpha B_S - 2)}}{\mu^3}\left\{\frac{1}{2}(\mu r)^2 - (\mu r) - 1\right\} dr\right] \tag{12}$$

the expression shock velocity freely propagation shock solving equation (5) yields

$$\frac{P_0}{\rho' G} = 4\pi\left(\frac{e^{2\mu r}}{2\mu} r^2 - \frac{5}{4}\frac{e^{2\mu r}}{\mu^3}.r - \frac{5}{8}\frac{e^{2\mu r}}{\mu^4}\right) - \frac{3}{2}\frac{\beta^2 \gamma p^1 r}{\rho' G} \tag{13}$$

$$a_0^2 = G\rho' 4\pi\gamma\left[\frac{e^{\mu r}}{2\mu^2} r^2 - \frac{5}{4}\frac{e^{\mu r}}{\mu^3} r - \frac{5}{8}\frac{e^{2\mu r}}{\mu^4}\right] - \frac{3}{2}\frac{\beta^2 \gamma p^1 r}{\rho'} e^{-\mu r} \tag{14}$$

The expression from freely propagation shock strength can be written as

$$\left(\frac{U}{a_0}\right)^2 = e^{-A_S\mu r}e^{-\alpha B_S}\left[\frac{K}{\rho' G} - 8\pi\xi C_S \int e^{\mu(1+A_S)}\frac{r^{(\alpha B_S-2)}}{\mu^3}\left\{\frac{1}{2}(\mu r)^2 - (\mu r) - 1\right\}dr\right] /$$

$$4\pi\gamma\left[\frac{e^{\mu r}}{2\mu^2}r^2 - \frac{5}{4}\frac{e^{\mu r}}{\mu^3}r - \frac{5}{8}\frac{e^{2\mu r}}{\mu^4}\right] - \frac{3}{2}\frac{\beta^2\gamma p^1 r}{\rho'}e^{-\mu r} \qquad (15)$$

for C_+ distribution generated by the shock the particle velocity increment by use of equation (3) and (12)

$$du_+ = \frac{\xi-1}{\xi}\frac{1}{2U}\frac{d}{dr}[e^{-A_S\mu r}r^{-\alpha B_S}\left(K - 8\pi\rho' G\xi C_S \int e^{\mu(1+A_S)}\frac{r^{\alpha B_S-2}}{\mu^3}\left\{\frac{1}{2}(\mu r)^2 - (\mu r) - 1\right\}dr\right) \qquad (16)$$

The overtaking disturbance an independent C– characteristic is taken as and solving use get condition (3) & equation (8).

$$du_- = \frac{\xi-1}{\xi}\frac{1}{2U}\frac{d}{dr}[e^{-D_S\mu r}r^{-\alpha E_S}\left(K - 8\pi\rho' G\xi F_S \int e^{\mu(1+D_S)}\frac{r^{\alpha E_S-2}}{\mu^3}\left\{\frac{1}{2}(\mu r)^2 - (\mu r) - 1\right\}dr\right) \qquad (17)$$

where

$$D_S = \frac{\frac{\chi}{\gamma}}{\left[\frac{\chi}{\gamma} - \frac{1}{2}(\xi-1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]} \qquad E_S = \frac{(\xi-1)\chi}{\left[\frac{\chi}{\gamma} - \frac{1}{2}(\xi-1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]\left[(\xi-1) - (\chi\xi)^{\frac{1}{2}}\right]}$$

$$F_S = \left[\frac{\xi(\chi\xi)^{\frac{1}{2}}}{\left[(\xi-1) - (\chi\xi)^{\frac{1}{2}}\right]} + 1\right] \Bigg/ \left[\frac{\chi}{\gamma} - \frac{1}{2}(\xi-1)\left(\frac{\chi}{\xi}\right)^{\frac{1}{2}}\right]$$

Now in presence of both C_+ & C_- disturbance the field velocity behind the shock will be realted as

$$du_+ + du_- = \frac{\xi-1}{\xi}\ dU \qquad (18)$$

substituting the value du_+ & du_- inelement will shock velocity.

$$\frac{U^{*2}}{\rho' G}=\Big[\Big(e^{-A_S\mu r}r^{-\alpha B_S}+e^{-D_S\mu r}r^{-\alpha E_S}+1\Big)\frac{K}{\rho' G}$$

$$-8\pi\xi C_S\int e^{\mu(1+A_S)r}\frac{e^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr$$

$$+8\pi\xi F_S\int e^{\mu(1+D_S)r}\frac{r^{(\alpha E_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\Big] \qquad (19)$$

and Shock strength (modified)

$$\frac{U^{*2}}{a_0}=\Big[\Big(e^{-A_S\mu r}r^{-\alpha B_S}+e^{-D_S\mu r}r^{-\alpha E_S}+1\Big)\frac{K}{\rho' G}$$

$$-8\pi\xi C_S\int e^{\mu(1+A_S)r}\frac{e^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr$$

$$+8\pi\xi F_S\int e^{\mu(1+D_S)r}\frac{r^{(\alpha E_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\Big]\ /$$

$$4\pi\gamma\left[\frac{e^{\mu r}}{2\mu^2}r^2-\frac{5}{4}\frac{e^{\mu r}}{\mu^3}r-\frac{5}{8}\frac{e^{\mu r}}{\mu^4}\right]-\frac{3}{2}\frac{\beta^2\gamma p^1 e^{-\mu r}}{\alpha^1} \qquad (20)$$

The freely propagation and modified expression for the particle velocity and pressure immediately behind the shock canbe written as

$$\frac{u}{\sqrt{\rho' G}}=\frac{\xi-1}{\xi}\left[e^{-A_s\mu r}r^{-\alpha B_S}\left\{\frac{K}{\rho' G}-8\pi\xi C_S\int e^{\mu(1+A_S)}\frac{r^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\right\}\right]^{\frac{1}{2}} \qquad (21)$$

$$\frac{P}{P_0}=\frac{1+\chi\left[e^{-A_s\mu r}r^{-\alpha B_S}\left\{\frac{K}{\rho' G}-8\pi\xi C_S\int e^{\mu(1+A_S)}\frac{r^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\right\}\right]^{\frac{1}{2}}}{4\pi\gamma\left[\frac{e^{\mu r}}{2\mu^2}r^2-\frac{5}{4}\frac{e^{\mu r}}{\mu^3}r-\frac{5}{8}\frac{e^{\mu r}}{\mu^4}\right]-\frac{3}{2}\frac{\beta^2\gamma p^1 e^{-\mu r}}{\alpha^1}} \qquad (22)$$

$$\frac{u^*}{\sqrt{\rho' G}}=\frac{\xi-1}{\xi}\Big[\Big(e^{-A_S\mu r}r^{-\alpha B_S}+e^{-D_S\mu r}r^{-\alpha E_S}+1\Big)\frac{K}{\rho' G}$$

$$-8\pi\xi C_S\int e^{\mu(1+A_S)r}\frac{e^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr \qquad (23)$$

$$+8\pi\xi F_S \int e^{\mu(1+D_S)r} \frac{r^{(\alpha E_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\Bigg]^{\frac{1}{2}}$$

$$\frac{P^*}{P_0}=1+\chi\Bigg[\left(e^{-A_S\mu r}r^{-\alpha B_S}+e^{-D_S\mu r}r^{-\alpha E_S}+1\right)\frac{K}{\rho' G}$$

$$-8\pi\xi C_S \int e^{\mu(1+A_S)r} \frac{e^{(\alpha B_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr$$

$$+8\pi\xi F_S \int e^{\mu(1+D_S)r} \frac{r^{(\alpha E_S-2)}}{\mu^3}\left(\frac{1}{2}(\mu r)^2-(\mu r)-1\right)dr\Bigg]^{\frac{1}{2}}$$

$$\Big/\ 4\pi\gamma\left[\frac{e^{\mu r}}{2\mu^2}r^2-\frac{5}{4}\frac{e^{\mu r}}{\mu^3}r-\frac{5}{8}\frac{e^{\mu r}}{\mu^4}\right]-\frac{3}{2}\frac{\beta^2\gamma p^1 e^{-\mu r}}{\alpha^1} \qquad (24)$$

RESULT AND DISCUSSION :

Expression (12) and (19) represent shock velocity freely propagation shock and under influence of overtaking disturbance. The strong spherical shock in self-gravitating gas atmosphere in presence of magnetic field, specific heat gas ratio (γ) density parameter (μ) and constant ξ. Initially taking shock strength $\frac{U}{a_o}$= 14 at r = 0.01, For γ= 1.45,

μ =0.01, ξ = 4 and ρ' = 1.1, variation of shock velocity with propagation distance r, γ, μ & ξ have been shown in Figure (1, 2, 3 & 4) for freely propagation shock with overtaking disturbance. It is found that the shock velocity decrease in both cases with propagation distance r (Figure-1), decreases very slowly with specific heat gas ratio (γ) (Figure-2) and with density distribution constant (μ) (Figure-3). It is found that shock velocity continuously decreases in case of freely propagation and decreases them increases in case of overtaking disturbance with magnetic constant ξ (Figure-4) The expression (15) and (20), representing, shock strength freely propagation and influence of overtaking disturbance in case of variable azimuthally component magnetic field, show that it depends on the propagation distance, specific heat gas ratio γ and other parameter μ & ξ, shock strength decreases in both cases with propagation distance r. (Figure 5) On increasing the value of specific heat gas ratio, shock strength for freely & overtaking disturbances decreases (Figure 6). Shock strength increases with increase in density parameter (μ) (Figure-7). Shock velocity and shock strength initially decrease as well as increases with magnetic constant (Figure-8) The expression (21) and (23), representing the particle velocity depends on the different parameters.

The particle velocity decrease both case with propagation distance (r) (Figure (9) and with specific heat gas ratio (γ) (Figure 10). It is found that the particle velocity decrease in case of freely propagation and initial decreases after sometime increase in this case of overtaking disturbance with magnetic constants.(Figure 11) Expression (22) and (24) representing pressure of the shock waves, pressure decrease both cases with propagation distance (r) (Figure 12), with specific heat gas ratio (γ) (Figure 13). It is found that the shock strength in case of freely propagation. It is found that the pressure increase both case with density distribution constant (μ) (figure14) and magnetic field constant (ξ).(Figure 15)

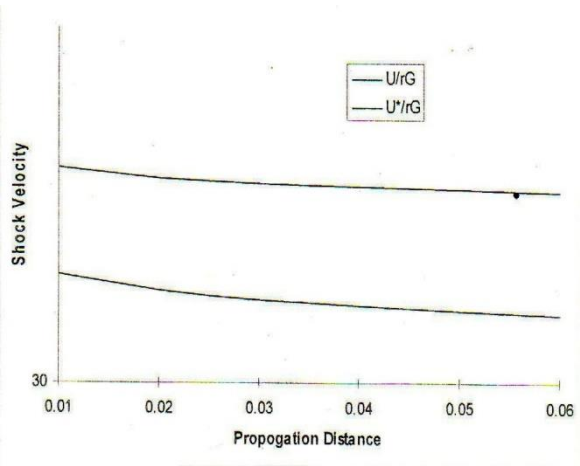


Figure-1

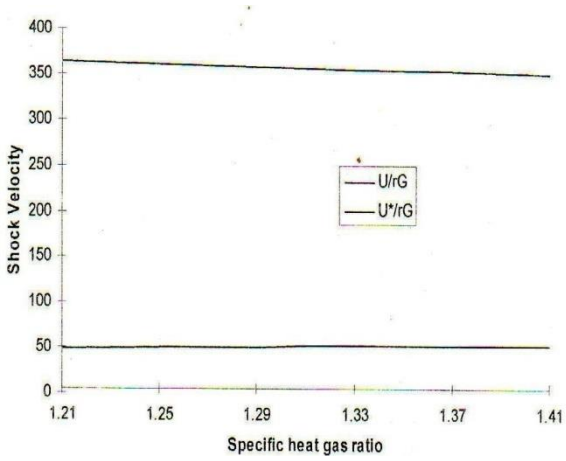
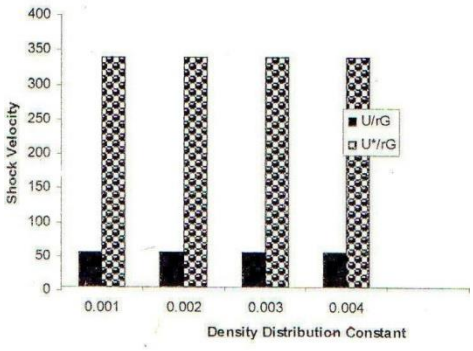


Figure-2



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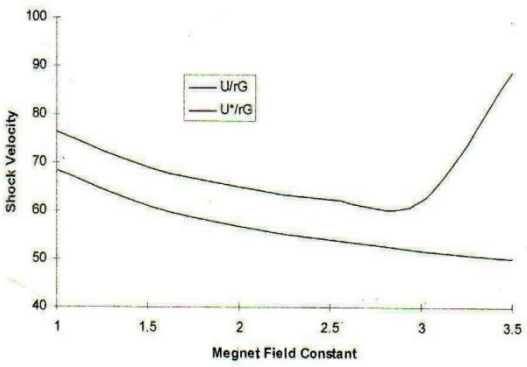

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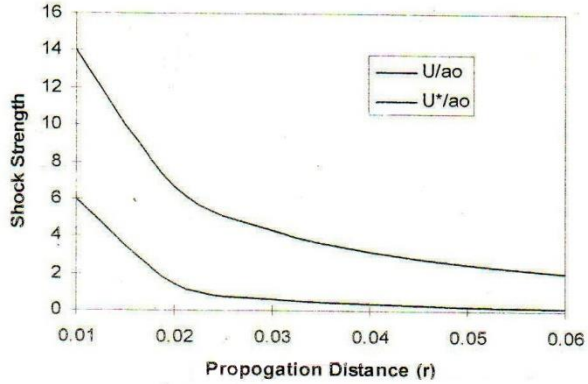

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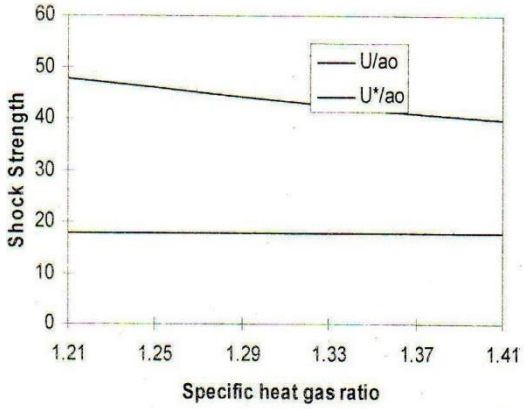


Figure-6

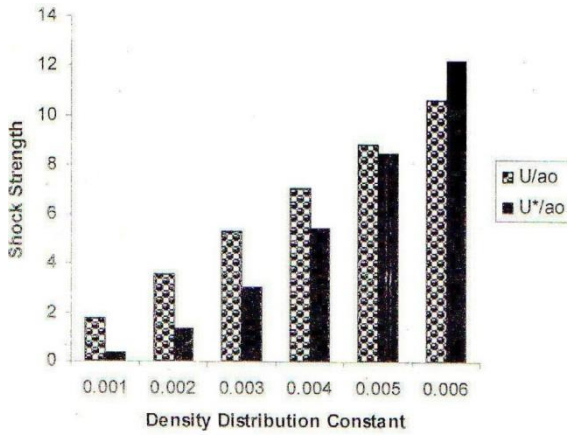


Figure-7

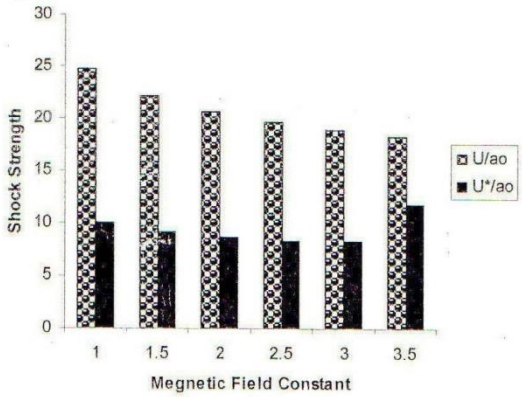
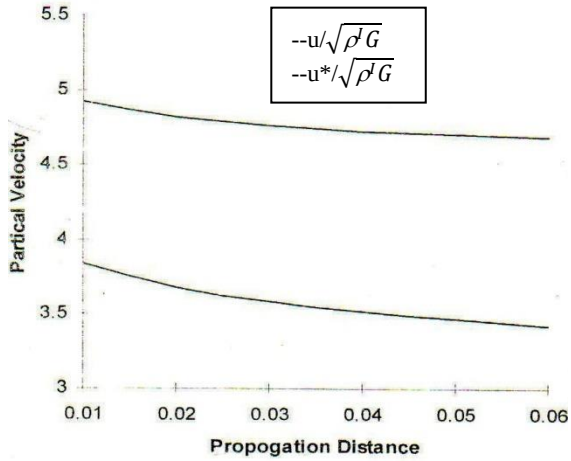


Figure-8



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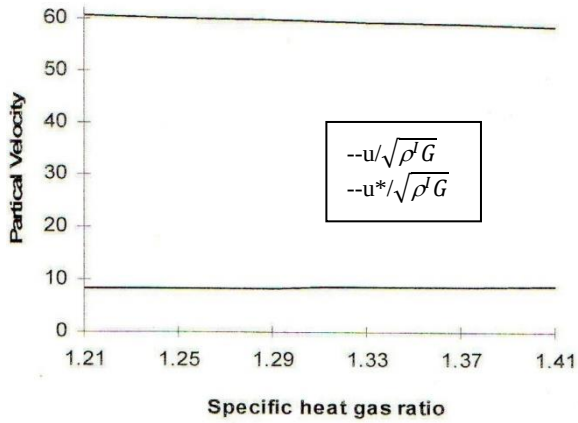


Figure-10

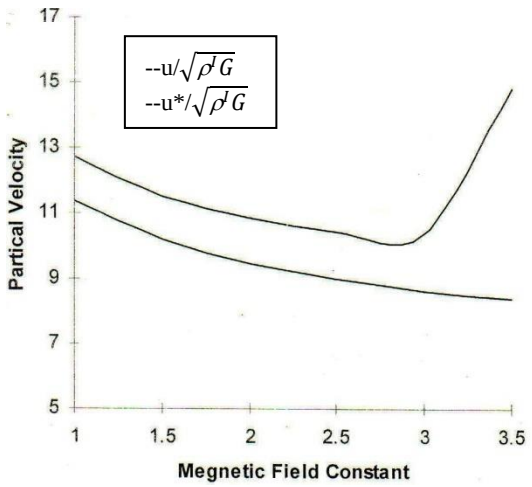


Figure-11

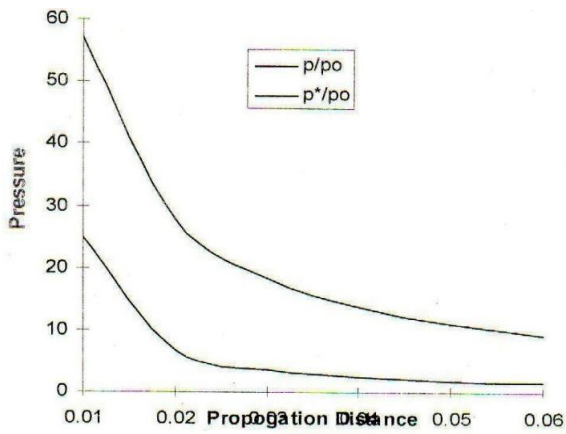
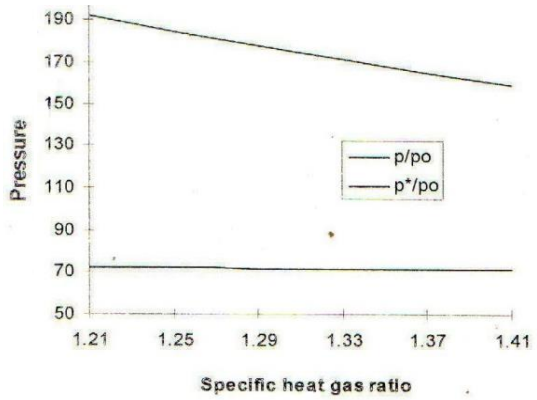


Figure-12



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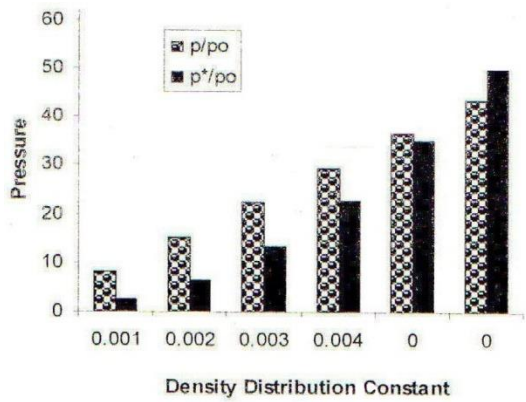

Figure-14

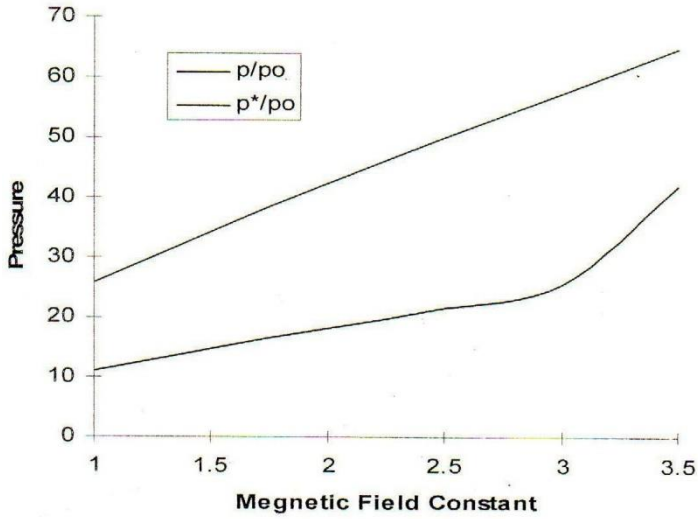

Figure-15

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