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Study of Fuzzy Integer Linear Programming Problems (IFLPP) and Simplex Method

Kumari Tanu Mittal[1], Dr. Ashok Kumar Sah[2]

[1]Research Scholar, University Department of Mathematics, B. R. A. Bihar University, Muzaffarpur, Bihar, India

[2]Assistant Professor, Department of Mathematics, Dr. R. M. L. S. College, Muzaffarpur, B. R. A. Bihar University, Muzaffarpur, Bihar, India

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ABSTRACT

In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy linear programming problems with triangular fuzzy numbers. A computational method for solving fully fuzzy linear programming problems (FFLPP) is proposed, based upon the new Ranking function. The proposed method is very easy to understand and to apply for fully fuzzy linear programming problems occurring in real life situations as compared to the existing methods. To illustrate the proposed method numerical examples are solved.

Keywords : Fuzzy Integer Linear Programming (IFLP), Linear Programming Problem (LPP), Triangular Fuzzy Number (TFN).

I. INTRODUCTION

The concept of Fuzzy logic was first conceived by Loft Zadeh, a professor at the university of California at Berkley. But as a way of processing data by allowing partial set membership rather than crisp set membership or nonmembership. Allahviranloo et al., (2008) [1] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function. Kumar et al.,(2010) [2] proposed a new method for solving fully fuzzy linear programming problems with inequality constraints. Kumari Tanu Mittal & Ashok Kumar Sah,(2022) [3] proposed a new method for finding the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints. Bellman & Zadeh, (1970) [4] proposed the concept of decision making in fuzzy environment. Cadenas & Verdegay, (1997) [5] studied a linear programming problem in which all its elements are defined as fuzzy sets. (Chanas, 1983) [6] Proposed the possibility of the identification of a complete fuzzy decision in fuzzy linear Programming by use of the parametric programming technique. Fang et al., (1999) [7] presented a method for solving Linear programming problems with fuzzy coefficients in constraints. Jimenez et al., (2007) [8] proposed a method for solving linear programming problems where all the coefficients are fuzzy numbers and used a fuzzy ranking method to rank the fuzzy objective values and to deal with the inequality relation on constraints. Mittal & Sah (2022) [9] proposed a new method for solving exact fuzzy optimal solution of fully fuzzy linear programming problems with unrestricted fuzzy variables Kolman & Hill, (1984) [10] proposed a Fuzzy variable linear programming problems by use of a certain linear ranking function. Lotfi et al., (2009) [11] discussed fully fuzzy linear programming problems by representing all parameters and variables as triangular fuzzy numbers. Maleki et al., (2000) [12] solved the linear programming problems in which all decision parameters are fuzzy numbers by the comparison of fuzzy numbers. Nagoor Gani & Mohamed Assarudeen, (2013) [13] proposed a new operation on Triangular fuzzy number for solving fuzzy linear programming problem. Nasseri &Alizadeh, (2011) [14] proposed a method for solving fuzzy linear programming problems by solving the classical linear programming. Nehi et al., (2004) [15] defined the concept of optimality for linear programming problems with fuzzy parameters by transforming fuzzy linear programming problems into multiobjective linear programming problems. (Ramik,2005) [16]proposed the fuzzylinear programming problems based on fuzzy relations. (Werners,1987) [17] Introduced an interactive system which supports a decision maker in solving programming models with crisp or fuzzy constraints or fuzzy goals. Zhang et al., (2003) [18] proposed a method for solving fuzzy linear programming problems which involve fuzzy numbers in coefficients of objective functions. (Zimmerman,1978) [19] proposed the though fuzzy logic has been applied to many fields, from at control theory to artificial intelligence, it still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic. Today, Fuzzy logic concept used widely in many implementations like automobile engine and automatic gear control systems, air conditioners, automatic focus control, video enhancement in TV sets, washing machines, behavior based mobile robots, sorting and handling data, Information Systems, traffic control systems and so on.

II. FUZZY NUMBER

Fuzzy Number: A fuzzy number is a fuzzy set of real line with normal, convex and continuous membership function of bounded support[10-13].

Triangular Fuzzy Number: A fuzzy number A is called triangular number with peak point 'a' left width α >0 and right width β >0 if its membership

functions has the following for$A(x)$=

$$\begin{cases} 1-\frac{a-x}{\alpha}\ ; & a-\alpha \le x \le a \\ 1-\frac{x-a}{\beta}\ ; & a \le x \le a+\beta \\ 0\ ; & otherwise \end{cases}$$

We use this notation as A=(a,α,β). This fuzzy number A=(a,α,β)is called nonnegative if if and only if a>0,α>0,β>0

Trapezoidal Fuzzy Number: A fuzzy set A is called trapezoidal fuzzy number with tolerance interval [a, b], left width α and right width β if its membership has the following form $A(x)$=

$$\begin{cases} 1-\frac{a-x}{\alpha}\ ; & a-\alpha \le x \le a \\ 1\ ; & a \le x \le b \\ 1-\frac{x-b}{\beta}\ ; & b \le x \le b+\beta \\ 0\ ; & otherwise \end{cases}$$

We use this notation as =(a,b,α,β) . This fuzzy number A=(a,b,α,β)is called non-negative if if and only if a>0,b>0 ,α>0,β>0.

III. PRELIMINARIES

We need the following definitions of the basic arithmetic operators on fuzzy triangular numbers based on the function principle which can be found in [6].

Definition 1.

A fuzzy number à is a triangular fuzzy number denoted by (a,a, as) where a_1, a_2 and as are real numbers and its membership function(x) is given below.

$\mu_{\tilde{a}}$(x) =

$$\begin{cases} (x - a_1)/(a_2 - a_1) & \text{for } a_1 \leq x \leq a_2 \\ (a3 - x) / (a_3 - a_2) & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.

Let (a_1,a_2,a_3) and (b_1,b_2,b_3) be two triangular fuzzy numbers. Then

(i) (a_1,a_2,a_3) ⊕ (b_1,b_2,b_3) = (a_1 + b_1,a_2+ b_1,a_3+ b_3)

(ii) (a_1,a_2,a_3) Θ (b_1,b_2,b_3) = (a_1 - b_1,a_2 - b_1,a_3 - b_1)

(iii) k (a_1,a_2,a_3) = (ka_1,ka_2,ka_3), for $k \geq 0$

(iv) k (a_1,a_2,a_3) = (ka_3,ka_2,ka_1), for $k < 0$

Let F(R) be the set all real triangular fuzzy numbers

Definition 3.

Let $\tilde{A}$ (a_1,a_2,a_3) and $\tilde{B}$ (b_1,b_2,b_3) be in F(R). Then,

(i) $\tilde{A}$=$\tilde{B}$ ⇔ $a_i = b_i$, for all for i = 1 to 3 and

(ii) $\tilde{A}$=$\tilde{B}$ ⇔ $a_i \leq b_i$, for all for i = 1 to 3.

Definition 4.

Let $\tilde{A}$ (a_1,a_2,a_3) be in F(R). Then,

(i) $\tilde{A}$ is said to be positive if $a_i \geq 0$, for all for i=1 to 3;

(ii) $\tilde{A}$ is said to be integer if $a_i \geq 0$, ∀i=1 to 3 are integers and

(iii) $\tilde{A}$ is said to be symmetric if $a_2 - a_1 = a_3 - a_2$.

Definition 5.

A real fuzzy vector $\tilde{b}$ = $(\tilde{b})_{m\times1}$ is called nonnegative and denoted by $\tilde{b}$≥0 if each element of $\tilde{b}$ is a nonnegative real fuzzy number, that is $\tilde{b}_I \geq 0$, 1,2,…,m.

Consider the following m × n fuzzy linear system with nonnegative real triangular fuzzy numbers;

$$A\,\tilde{x} \leq \tilde{b} \qquad (1)$$

where a = (a_{ij})m×1 is a nonnegative crisp matrix and $\tilde{x} = (\tilde{x}_j), \tilde{b} = (\tilde{b}_i)$are nonnegative fuzzy vectors $\tilde{x}_j, \tilde{b}_i$ ∈F(R) and for all $1 \leq j \leq n$ and $1 \leq i \leq m$.

Definition 6.

A nonnegative fuzzy vector $\tilde{x}$ is said to be a solution of the fuzzy linear system (1) if $\tilde{x}$ satisfies equation (1).

Using the definitions 3 and 6 and the arithmetic operations on triangular fuzzy number, we obtain the following theorem.

Theorem 1.

Let A$\tilde{x}$ < $\tilde{b}$ be an m × n fuzzy linear system where

A$(a_{ij})_{m\times n}$ is a nonnegative crisp matrix, $\tilde{x} = (\tilde{x}_j)\tilde{b} = (\tilde{b}_i)$ are nonnegative real triangular fuzzy vectors and $(\tilde{x}_j)$ (x_1^j, x_2^j, x_3^j) and $\tilde{b}_i = (b_1^i, b_2^i, b_3^i)$ ∈F (R), for all 1≤ j ≤ n and 1≤ i ≤ m. If $x_2^0 = (x_2^{jo})_{n\times1}$ is a solution of the system

$Ax_1 \leq b_1$, $x_1 \geq 0$, $x_1 - x_2^0$ where $x_1 = (x_1^j)_{n\times 1}$ and $x_3^0 = (x_3^{jo})_{n\times t}$ is a solution of the system $Ax_3 \leq b_3$, $x_3 - x_2^0$ where $x_3 = (x_3^{jo})_{nx1}$ and $b_3 = \left(b_3^i\right)_{m\times 1}$ then $\tilde{x}^0 = \left(\tilde{x}_j^0\right)$ is a solution of the system $A\tilde{x} \leq \tilde{b}$ where $\tilde{x}_j^0 = \left(x_1^{jo}, x_2^{jo}, x_3^{jo}\right)$.

Note 1. If $(\tilde{x})=(X_1, X_2, X_3$ and $\tilde{b}$ (b_1, b_2, b_3) are symmetric, we can obtain x_3^0, from the relation $x_3^0 = x_2^0 + (x_2^0 - x_1^0)$ without solving the last system $Ax_3 = b_3$, $x_3 - x_2^0 \geq$.

IV. FUZZY INTEGER LINEAR PROGRAMMING PROBLEM (IFLPP)

Consider the following integer linear programming problem with fuzzy variables:

(P) Maximize $\tilde{z} = c\tilde{x}$

subject to $A\tilde{x} < \tilde{x}$, (2)

$\tilde{x} > 0$ and are integers (3)

where the coefficient matrix $A = (a_{ij})_{m\times n}$ is a nonnegative real crisp matrix, the cost vector $c = (c_1, \ldots\ldots c_n)$ is nonnegative crisp vector and $\tilde{x} = \left(\tilde{x}_j\right)_{n\times 1}$ and $\tilde{x}$ $\left(\tilde{b}_i\right)_{m\times 1}$ are nonnegative real fuzzy vectors such that $\tilde{x}_j, \tilde{x}_i \in F(R)$ for all $1 \leq j \leq n$ and $I \leq i \leq m$.

Definition 7.

A fuzzy vector $\tilde{x}$ is said to be a feasible solution of the problem (P) if $\tilde{x}$ satisfies (2) and (3).

Definition 8.

A feasible solution $\tilde{x}$ of the problem (P) is said to be an optimal solution of the problem (P) if there exists no feasible $\tilde{u} = (\tilde{u}_j)_{n\times 1}$ of (P) such that $c\tilde{u} > c\tilde{x}$.

Using the Theorem 1 and the arithmetic operations of fuzzy numbers, we can obtain the following result.

Theorem 2.

A fuzzy vector $\tilde{x}$ $(x_1^0, x_2^0, x_3^0)$is an optimal solution the problem (P) if x_2^0, x_1^0 and x_3^0 are optimal solutions of the following crisp integer linear programming problems (P_2), (P_1) and (P_3) respectively where

(P_2) Maximum $z_2 = cx_2$

subject to $Ax_2 \leq b_2$, $x_2 \geq 0$ are integers;

(P_1) Maximum $z_1 = cx_1$

subject to $Ax_1 \leq b_1$, $x_1 \geq 0$, $x_1 \leq x_2^0$ and are integers

(P_3) Maximum $z_3 = cx_3$

subject to $Ax_3 \leq b_3$, $x_3 \geq 0$, $x_3 \leq x_2^0$ and are integers.

Proof:

Suppose that $\tilde{x}^0 = (x_1^0, x_2^0, x_3^0)$is an optimal solution of the problem (P).

Let $\tilde{x} = (x_1^0, x_2^0, x_3^0)$be a feasible solution of the problem (P).

This implies that

$c_1x_1 \leq c_1x_1^0$; $c_2x_2 \leq c_2x_2^0$; $c_3x_3 \leq c_3x_3^0$.

$Ax_1^0 \leq b_1^0$; $Ax_2^0 \leq b_2^0$; $x_1^0, x_2^0, x_3^0 \geq 0$ (4)

Let $\tilde{z} = (z_1, z_2, z_3)$ be the objective function of the problem (P).

Now from (4) we have,

Max. $z_1 = c_1x_1^0$; Max $z2 = c_2x_2^0$; Max. $z3 = c_2x_3^0$ (5)

Now from (4) and (5) we can conclude that $x_2^0, x_1^0$and x_3^0 are optimal solutions of the crisp integer linear programming problems (P_2), (P_1) and (P3).

Suppose that x_2^0, x_1^0 and x_3^0 are optimal solutions of the crisp integer linear programming problems (P_2), (P_1) and (P_3) with optimal values z_2^0, z_1^0 and z_2^0 respectively. This implies that $\tilde{x}^0 = (x_1^0, x_2^0, x_3^0)$is an optimal solution of the problem (P) with optimal value $\tilde{z}^0 = (z_1^0, z_2^0, z_3^0)$. Hence the theorem.

V. NUMERICAL EXAMPLES

The proposed method is illustrated by the following examples.

Example 1.

Consider the following integer linear programming problem with fuzzy variables.

(P) Maximize $\tilde{z} = 10\tilde{x}_1 + 10\tilde{x}_2$

subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq (46,48,60)$

$\tilde{x}_1 + 3\tilde{x}_2 (7,12,20)$

$\tilde{x}_1 + \tilde{x}_2 < 0$ and are integers

Let $\tilde{z} = (z_1, z_2, z_3)$ $\tilde{x}_1 = (y_1, x_1, t_1)$ $\tilde{x}_2 = (y_2, x_2, t_2)$

Now, the problem (P_2) is given below:

(P_2) Maximize $z_2 = 10x_1 + 20x_2$

Subject to

$6x_1 + 8x_2 < 48$; $x_1 + 3x_2 \leq 12$

0 1 2 x x $\geq$ and are integers.

Now, using an algorithm for ILP problem, the solution of the problem (P_2) is $x_1 = 5$, x_2 and $z_2 = 90$.

Now, the problem (P_1) is given below:

(P_1) Maximize $z_1 = 10y_1 + 20y_2$

Subject to

$6y1 + 8y2 \leq 46$; $y1 + 3y2 \leq 7$; $y1 \leq 5$; $y2 \leq 2$.

$y_1, y_2 \geq 0$ and are integers.

Now, using an algorithm for ILP problem, the solution of the problem (P_1) is $y_1 = 4$, $y_2 = 1$ and $z_1 = 60$.

Now, the problem (P_3) is given below:

(P_3) Maximize $z_3 = 10t_1 + 20\, t_2$

subject to

$6t_1 + 8t_2 \leq 60$; $t_1 + 3t_2 \leq 20$; $t_1 \geq 5$; $t_2 \geq 2$

$t_1, t_2 > 0$ and are integers.

Now, using an algorithm for ILP problem, the solution of the problem (P_3) is $t_1 = 6$, $t_2 = 3$ and $z_3 = 120$.

Therefore, the solution for the given fuzzy integer linear programming problem is

$\tilde{x}_1 = (y_1, x_1, t_1) = (4,5,6)$; $\tilde{x}_2 = (y_2, x_2, t_2) = (1,2,3)$ and $\tilde{z} = (60,90,120)$.

Example 2.

Consider the following integer linear programming problem with fuzzy variables:

(P) Maximize $\tilde{z} = 4\tilde{x}_1 + \tilde{x}_2$

subject to $\tilde{x}_1 + \tilde{x}_2 \leq (4,8,12)$

$2\tilde{x}_1 + \tilde{x}_{12} (6,9,12)$

$\tilde{x}_1 + \tilde{x}_2 < 0$ and are integers

Let $\tilde{z} = (z_1, z_2, z_3)$ $\tilde{x}_1 = (y_1, x_1, t_1)$ and $\tilde{x}_2 = (y_2, x_2, t_2)$ and also, $\tilde{x}_1$.

and $\tilde{x}_1$ be symmetric.

Now, the problem (P_2) is given below:

(P_2) Maximize $z_2 = 4x_1 + 3x$

subject to $x_1 + 2x_2 < 8;' \ 2x_1 + x_2 < 9;$

$x_1, x_2 > 0$ and are integers.

Now, by using an algorithm for ILP problem, the solution of the problem (P_2) is $z_2 = 19$, $x_1 = 4$ and $x_2 = 1$.

Now the problem (P_1) is given below:

(P_1) Maximize $z_1 = 4y_1 + 3y_2$

subject to $y_1 + 2y_2 \leq 4$; $2y_1 + y_2 \leq 6$; $y_1 \leq 4$; $y_2 \leq 1$

$y_1, y_2 \geq 0$ and are integers.

Now, using the algorithm for ILP problem, solution of the problem (P1) is $z_1 = 12$, $y_1 = 3$ and $y_2 = 0$.

Now, since $x_1 = (y_1, x_1, t_1)$ and $x_2 = (y_2, x_2, t_2)$ are symmetric, we have $t_1 = 5$, $t_2 = 2$ and $z_3 = 26$.

Therefore, the solution for the given fuzzy integer linear programming problem (IFLPP) is $\tilde{x}_1 = (y_1, x_1, t_1) = (3,4,5)$; $\tilde{x}_2 = (y_2, x_2, t_2) = (0,1,2)$ and $\tilde{z} = (12,19,26)$.

VI. CONCLUSION

In this paper a new method is proposed for solving the fuzzy optimal solution of IFLP problem transform into FVLP problems. The IFLP problem is converted into FVLP problem using new Ranking function. Ranking function is reasonable and effective for calculating the triangular weights of criteria. But the proposed method is less time consumption compared to previous method. Proposed method requires a smaller number of iterations. Therefore, it is easier to solve fully fuzzy linear programming problem.

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