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Analytical Study of MHD Thermoconvective Waves Through Its Propagation

Sarvesh Kumar
Department of Mathematics, Ganna Utpadak (P.G.) College, Baheri, Bareilly (U.P.), India

ABSTRACT
Following is the analytical study on the propagation of undamped thermoconvective waves, an electrically conducting viscous fluid is hypothesized which has the property of uniform horizontal magnetic field in heating the uniform vertical concentration gradient for a solute. It has seen that undamped thermoconvective waves propagation in a specific order, whereas the heating of fluid, is based on the solute concentration, this decreased vertically or show vertical pattern. If the heating of fluid takes place in upward manner the propagation of waves is highly effected, the above aspect proves hypothetically and has shown that its laboratory demonstration is also possible.

Keywords : MHD Flow, Viscous fluid, Thermoconvective waves, concentration, gradient, Magnetic diffusivity.

Introduction

Many scientists has worked on the hypothesis of thermoconvective waves with MHD. The condition $k_\theta > \eta(k_\theta = 4.5 \times 10^{-2} \mathrm{cm}^2 \mathrm{sec}^{-1}$ and $\eta = 7.6 \times 10^{-3} \mathrm{cm}^2 \mathrm{sec}^{-1})$, does not exist in hydromagnetic stability has been studied by 'Chandrasekhar' (1961). 'Luikov and Berkovsky' (1970) observed that the phenomenon of BENARD convection does not exist for MHD or the propagation of waves showed decreased manner. The waves having this character are known as TCW, which are effected by nature of fluid and gravitational field. The moving property of TCW specially in fluid which have the property of electrical condition shows the propagation of waves uniformly in horizontally magnetic field was investigated by 'Takashima' (1972). 'Bhattacharyya and Gupta' (1985) studied the mechanism of propagation of TCW in the binary mixture situation. The propagation of the undamped MHD thermoconvective waves depend on the temperature and heat, the waves also shows the relationship between thermal and magnetic diffusivity ($k_\theta > \eta$), where k_θ and η represent the thermal and magnetic diffusivity of the fluid, this situation is possible in astrophysical condition.

The present study is based on a solute with a uniform vertical concentration gradient for the study of propagation of undamped MHD thermoconvective waves for viscous fluid. The whole study is the indication for a certain condition of undamped propagation of TCW ($k_\theta > \eta$).

1. **FORMULATION AND SOLUTION OF THE PROBLEM :** The concerned equations of undamped MHD thermoconvective waves can be represented in the form as given below :

$$\operatorname{div} \vec{v} = 0, \tag{2.1}$$

$$\text{div } \vec{H} = 0, \tag{2.2}$$

$$\rho \frac{d\vec{v}}{dt} = -\text{grad } Q + \mu_e \vec{H}.\text{grad } \vec{H} + \mu \nabla^2 \vec{v} + \rho \vec{g}, \tag{2.3}$$

$$\frac{d\vec{H}}{dt} = \vec{H}.\text{grad } \vec{v} + \eta \nabla^2 \vec{H}, \tag{2.4}$$

$$\frac{d\theta}{dt} = k_\theta \nabla^2 \theta, \tag{2.5}$$

$$\frac{dC}{dt} = k_p \nabla^2 C. \tag{2.6}$$

Where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}$$

The equation of state

$$\text{(i.e.)} \quad \rho = \rho_0 [I - \alpha(\theta - \theta_0) - \alpha'(C - C_0)]. \tag{2.7}$$

reduces to

$$\rho = \rho_0 + (\nabla\rho)_\theta + (\nabla\rho)_{p.} \tag{2.8}$$

Here, $(\nabla\rho)_\theta$ and $(\nabla\rho)_p$ are the change in density variation due to the variation of temperature and concentration respectively. Fluid velocity, density, magnetic permeability, coefficient of dynamic viscosity, magnetic field, temperature, concentration of solute and acceleration due to gravitation are denoted by $\vec{v}, \rho, \mu_e, \mu, \vec{H}, \theta$, C and $\vec{g}$ respectively. As described earlier that the magnetic diffusivity indicated by $\eta . \sigma$ is the fluid's electrical conductivity in equation (2.3), the pressure of the fluid indicated by Q, where as $\frac{\mu_e H^2}{2}$ denotes the magnetic pressure. When temperature increamentation takes place the density of fluid decreased, whereas if the density increases, the solute concentration will also be increased.

If we consider the mass transfer equation as described in equation (2.6), the Fick's law is used, according to which, diffusion flux is proportional to the concentration gradient, actually diffusion flux is the total amount of solute which is transported by diffusion through a single unit area with a single unit time, the diffusion flux $\hat{i}$ showed correlation variance with ΔC and $\Delta\theta$. If we consider the heat flux $\vec{v}$, this is also depends on ΔC and $\Delta\theta$.

We have the equation

$$\vec{i} = -\rho k_p \left[\vec{\nabla} C + \left(\frac{k_D}{\theta}\right) \vec{\nabla} \theta \right]. \tag{2.9}$$

Which indicates Fick's law with the addition of mass transfer due to the changes in concentration gradient, the temperature gradient is also responsible for the mass transfer as described above. This is associated with soret effect and denotes the thermal diffusivity in equation (2.9), where the coefficient k_D is the ratio of thermal diffusion. There is also the phenomenon of heat transfer with the variation in concentration gradient is highly effected in binary mixture, in addition to heat transfer due to temperature gradient. This another heat transfer due to ΔC is defined as "diffusion-thermo effect" or 'Dufour'. In the present study soret and Dufour effect are not considered due to neglectivity of these two effect and these laws are important for mixture of the gas (incompressible binary mixture in the studies). The density variation $(\nabla\rho)_p$ denotes in the L.H.S. of equation (2.3). Here ρ is displaced by equation (2.8) and is negligible. The inertial effects of density due to the concentration of the solute are neglected. The equations (2.7) and (2.8) represent the concentration in the basic state, there is the variation in $\vec{v}, \rho, \theta, \vec{H}$, Q and C can be represented in the following manner when the state is undisturbed.

$$\vec{v} = 0, Q = Q_B(x), \quad \rho = \rho_B(x). \tag{2.10}$$

$$\theta = \theta_0 - \beta x, \qquad C = C_0 - \beta' x, \qquad \vec{H} = \vec{H}_0,$$

Where β and β' may be either positive or negative. Comparing equations (2.1) to (2.8),
we have

$$\frac{d\vec{Q}_B}{dx} + \rho_B \vec{g} = 0, \;\; \rho_B = \rho_0(1 + \alpha\beta x + \alpha'\beta' x). \tag{2.11}$$

When the transverse plane waves are considered for propagation along y-axis, the variables are considered as :

$$\vec{v} = (\nu_1, 0), \qquad Q = Q_B(x) + Q_1, \qquad \rho = \rho_B(x) + \rho_1, \tag{2.12}$$

$$\vec{H} = (h, H_0), \quad \theta = \theta_0 - \beta x + \theta_1, \qquad C = C_0 - \beta' x + \phi_1, \tag{2.13}$$

The functions of y and t are variable with perturbation quantities $\nu_1, Q_1, \rho_1, h, \theta_1$ and ϕ_1. $\vec{H}$ and h both arises due to the expansion of the undisturbed horizontal magnetic line of forces with the vertical movability of fluid by propagation of waves.

Equations (2.1) and (2.2) elaborate the condition of magnetic solenoidel and continuity of equation these are identical with reference to $\vec{v}$ and $\vec{H}$ as shown in equations (2.12) and (2.13). By using equations (2.12) and (2.13) in equations (2.3) to (2.6) and taking the help of equations (2.7), (2.8), (2.10) and (2.11), we get the following equations :

$$\nabla_\nu \nu_1 - \vec{g}(\alpha\theta_1 + \alpha'\phi_1) - \frac{\mu_e \vec{H}_0}{\rho_0}\frac{\partial h}{\partial y} = 0, \tag{2.14}$$

$$\nabla_\eta h - \vec{H}_0 \frac{\partial \nu_1}{\partial y} = 0, \tag{2.15}$$

$$\nabla_{k_\theta} \theta_1 - \beta \nu_1 = 0, \tag{2.16}$$

$$\nabla_{k_r} \phi_1 - \beta' \nu_1 = 0, \tag{2.17}$$

where

$$\nu = \frac{\mu}{\rho_0}, \quad \nabla_x \equiv \frac{\partial}{\partial t} - x\frac{\partial^2}{\partial y^2}.$$

The equations (2.14) to (2.17) represent the phenomenon of natural convection with the help of equations (2.7) and (2.8), we can find the approximation of Oberbeck – Boussinesq, when the small oscillation takes place. Here we can explain a specific force for a single fluid pressure can be denoted by p,{$\rho = \rho\,(\theta, p)$}. An algebraic equation of state is connected with ρ, θ and p can be intergrodifferential equation of state as below :

$$d\rho = \left(\frac{\partial\rho}{\partial\theta}\right)_p d\theta + \left(\frac{\partial\rho}{\partial p}\right)_\theta dp, \tag{2.18}$$

this equation is the derivative extended part of $\alpha = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\theta}\right)_P$ and$\beta_\theta = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_\theta$.

Equation for ρ with density distribution ρ_θ can be solved at equilibrium state as below :

$$\rho=\rho_0[1-\alpha\,(\theta-\theta_0)+\beta_\theta\,(p-p_0)] \tag{2.19}$$

If we consider horizontal temperature difference for a gravitational field and is no forced convection $p-p_0=0$, the value of $p-p_0$ based on solution for vertical temperature differences, this cannot be shown as priori. The natural convection can be shown slow motion with minute rates of deformation, if pressure of all particles can be assumed as hydrostatic in nature. So we can take $p-p_0 \tilde{}\, \rho$ gL, here L is the vertical length, then the equation (2.19) takes the form

$$\frac{\text{density variation due to compressiblity}}{\text{density variation due to thermal expansion}} \sim \frac{\beta_\theta \rho g L}{\alpha\Delta\theta} \tag{2.20}$$

Here $\Delta\theta=\theta-\theta_0$ if L=2 m and $\Delta\theta \tilde{}\, 20^0$C for a water, the ratio can be 2.5×10^{-3}. Here effect of pressure on density variation can be neglected. According to 'Arpaci and Larsen' for atmospheric air L= 2m and $\Delta\theta \tilde{}\, 20^0$C, the ratio should be 3.5×10^{-3}. Here in both cases the effect of compressibility on density change can be neglected or ignored. In equation (2.20), the effect of compressibility on density change can ignored for thick layers ($\vec{H}$) and the temperature difference is small. So the equation of state (2.19) can be summarized in the form of equation

$$\rho=\rho_0[1-\alpha\,(\theta-\theta_0)]. \tag{2.21}$$

The phenomenon of compressibility effect on density change has proved in above equation. α, the volume expansion coefficient having the range of 10^{-3} to $10^{-4}$for most of the fluid, the variations in density are 1% at 20^0C for small variation in temperature, here density is constant in terms of natural convection except in the buoyancy force $\rho_0\vec{g}\alpha\Delta\theta$, which is proved by equation (2.21), the Oberbeck-Boussinesq approximation has proved by mathematical justification if the following conditions are governed:

i) Movement is buoyancy – driven and no forced convection is seen.
ii) The thickness of layer is not large when natural convection takes place.
iii) θ is small compared with fluid layer.

From above statement the Oberbeck-Boussinesq approximation has proved in the two situations found from above statement:

i) The magnitude of α' in the modified form of equations (2.7) and (2.8), is very small (α'<0).

ii) C which is variation in concentration through out the fluid layer is minutely compared with Q itself.

The Oberbeck-Boussinesq approximation has proved by buoyancy force $\vec{g}(\alpha\theta_1+\alpha'\,\phi_1)$ in the equation (2.14) i.e. momentum equation. The thickness of the layer in which natural convection takes place is not too large in our opinion the length of the tank containing the binary mixture can be compared with height of tank.

$$(\nu_1, \theta_1, \phi_1, h) = (V, \xi, \phi, G)\, e^{i(\omega t - ky)}, \tag{2.22}$$

where V, ξ, ϕ and G are constants, ω is real and wave number is k. Subscription of equation (2.22) in equations (2.14) to (2.17) and elimination of V,ξ,ϕ, and G, thus we get the dispersion relation in the form of non dimensionless condition of the equation in the following manner.

$$\left(a_1+\frac{\ell_1^2}{P}\right)\left(a_1+\frac{\ell_1^2}{R_m}\right)\left[(a_1+\ell_1^2)\left(a_1+\frac{\ell_1^2}{R}\right)-\gamma_1\right]-\left(a_1+\frac{\ell_1^2}{R}\right)\left[\gamma_1'\left(a_1+\frac{\ell_1^2}{R_m}\right)-M\ell_1^2\left(a_1+\frac{\ell_1^2}{P}\right)\right]=0, \tag{2.23}$$

where

$$\omega_1=\left(\frac{\nu}{g^2}\right)^{1/3}\omega,\quad \ell_1=\left(\frac{\nu^2}{g}\right)^{1/3}\ell,\quad R=\frac{\nu}{k_\theta},\quad P=\frac{\nu}{k_p},$$

$$\gamma_1=\left(\frac{\nu^2}{g}\right)^{1/3}\alpha\beta,\quad R_m=\frac{\nu}{\eta},\ \gamma_1'=\left(\frac{\nu^2}{g}\right)^{1/3}\alpha'\beta',$$

$$M=\frac{\mu_e H_0^2}{\rho_0(\nu g)^{2/3}},\quad a_1=i\omega_1.$$

If there is no presence of magnetic field (M=0) and there is neither any type of temperature gradient nor any concentration of solute gradient ($\gamma_1=\gamma'=0$). Equation (2.23) paired in four first order equation of ℓ_1^2 and explain pure viscous diffusion waves (ν-waves), pure mass diffusion waves (k_p − waves), pure thermal diffusion waves (k_θ − waves) and pure magnetic diffusion waves (η − waves). This is known very well that amplitude of waves described by a specific factor of exp (2π)$\simeq$ 540 times per wave length. These type of waves are very strongly damped. In the following situation- $M\neq 0, \gamma_1\neq 0$ and $\gamma'_1\neq 0$, the pure waves joined to produce four types of mode's like modified k_θ- waves, modified k_p- waves, modified $\nu-waves$ and modified η -waves. This waves shows possibilites of undamaped TCW. The value of ℓ_1 should be actual for undamped propagation of TCW, here the speed of dimensionless phase shows $\frac{\omega_1}{\ell_1}$ position. When we solve the imaginary part of equation (2.23), we can find the equations (2.25) and (2.26) , for $\omega_1\neq 0$ and $\ell_1\neq 0$ as given below:

$$\omega_1^4-\left[\left\{\frac{1}{RR_m}+\frac{1}{P}+\left(\frac{1}{P}+1\right)\left(\frac{1}{R}+\frac{1}{R_m}\right)\right\}\ell_1^4+M\ell_1^2-\gamma_1-\gamma_1'\right]\omega_1^2+\frac{\ell_1^8}{RPR_m}$$

$$+\frac{M\ell_1^6}{RP}-\frac{\ell_1^4}{R_m}\left(\frac{\gamma_1'}{R}+\frac{\gamma_1}{P}\right)=0 \tag{2.25}$$

and

$$\omega_1^2\left(\frac{1}{R_m}+\frac{1}{R}+\frac{1}{P}+1\right)=\frac{\ell_1^4}{RP}\left(\frac{1}{R_m}+1\right)+\ell_1^2\left(\frac{1}{R}+\frac{1}{P}\right)\left(\frac{\ell_1^2}{R_m}+M\right)\text{-}\left[\gamma_1\left(\frac{1}{R_m}+\frac{1}{P}\right)+\gamma'\left(\frac{1}{R}+\frac{1}{R_m}\right)\right]. \tag{2.26}$$

Equation (2.26) can be written as

$$\omega_1^2=B_1\ell_1^4+B_2\ell_1^2+B_3, \tag{2.27}$$

where

$$B_3=\frac{\left[\gamma_1\left(\frac{1}{R_m}+\frac{1}{P}\right)+\gamma_1'\left(\frac{1}{R}+\frac{1}{R_m}\right)\right]}{\left(\frac{1}{R_m}+\frac{1}{R}+\frac{1}{P}+1\right)}. \tag{2.28}$$

In the consequences of equations (2.27) and (2.25) takes the form

$$\ell_1^8\left[B_1^2-B_1\left\{\frac{1}{RR_m}+\frac{1}{P}+\left(\frac{1}{P}+1\right)\left(\frac{1}{R}+\frac{1}{R_m}\right)\right\}+\frac{1}{R_mRP}\right]$$
$$+\ell_1^6\left[2B_1B_2-B_1M-B_2\left\{\frac{1}{RR_m}+\frac{1}{P}+\left(\frac{1}{P}+1\right)\left(\frac{1}{R}+\frac{1}{R_m}\right)\right\}+\frac{M}{RP}\right]$$
$$+\ell_1^4\left[2B_1B_3+B_2^2+B_1(\gamma_1+\gamma_1')-B_3\left\{\frac{1}{RR_m}+\frac{1}{P}+\left(\frac{1}{P}+1\right)\left(\frac{1}{R}+\frac{1}{R_m}\right)\right\}-MB_2-\frac{1}{R_m}\left(\frac{\gamma_1}{P}+\frac{\gamma_1'}{R}\right)\right]+\ell_1^2[2B_2B_3+B_2(\gamma_1+\gamma_1')-B_3M]+B_3^2+B_3(\gamma_1+\gamma_1')=0. \tag{2.29}$$

Equation (2.29) is a biquadratic equation in ℓ_1^2 with real coefficients. When we consider the product of $\ell_1^2, \ell_2^2, \ell_3^2$ and ℓ_4^2 assume as four roots as a negative, the equation (2.29) represent at least one positive root. Using the values of B_1, B_2 *and* B_3 from equations (2.26) to (2.28) in equation (2.29), we get

$$(\ell_1\ell_2\ell_3\ell_4)^2=\frac{C}{D}, \tag{2.30}$$

where

$$C=\frac{-\left[\gamma_1\left(\frac{1}{R_m}+\frac{1}{P}\right)+\gamma_1'\left(\frac{1}{R}+\frac{1}{R_m}\right)\right]\left[\gamma_1\left(\frac{1}{R}+1\right)+\gamma_1'\left(\frac{1}{P}+1\right)\right]}{\left(\frac{1}{R_m}+\frac{1}{R}+\frac{1}{P}+1\right)^2} \tag{2.31}$$

and

$$D=\frac{-\left[P\left(\frac{1}{R_m}+\frac{1}{R}\right)\left(\frac{1}{R_m}+1\right)\left(\frac{1}{R}+1\right)+\frac{2}{R_m}\left(\frac{3}{R}+\frac{1}{R_m}\right)+2+R\left(\frac{1}{R_m}+\frac{1}{P}\right)\left(\frac{1}{R_m}+1\right)\left(\frac{1}{P}+1\right)+R_m\left(\frac{1}{R}+\frac{1}{P}\right)\left(\frac{1}{R}+1\right)\left(\frac{1}{P}+1\right)+\left(\frac{1}{R}+\frac{1}{P}\right)\left\{6+\frac{1}{RP}+\frac{1}{R_m}+\frac{1}{R_m^2}\right\}+\left(\frac{1}{R}+\frac{1}{P}\right)^2+\frac{4}{R_m}\left(\frac{1}{P}+1\right)\right]}{\left(\frac{1}{R_m}+\frac{1}{R}+\frac{1}{P}+1\right)^2RPR_m}. \tag{2.32}$$

Since R_m ,R and P are all positive, there for from above equation D<0.

Let us consider

$$\gamma_1\left(\frac{1}{R_m}+\frac{1}{P}\right)+\gamma_1'\left(\frac{1}{R_m}+\frac{1}{R}\right)<0, \tag{2.33}$$

$$\gamma_1\left(\frac{1}{R}+1\right)+\gamma_1'\left(\frac{1}{P}+1\right)>0. \tag{2.34}$$

In the above case, equation (2.31) follows that C>0 and equation (2.30) shows the product of the four root of equation (2.29)is negative In this way equation (2.29) allow to entre one root
ℓ_1^2, which is positive that ℓ_1 is real. From equation (2.27), we find $\omega_1^2>0$, here ω_1 is real and the values of B_1 and B_2 are positive and B_3 >0 with reference to equation (2.28) and inequality (2.33). when we consider the inequalities (2.33) and (2.34) the undamped TCW can propagate till both ω_1 and ℓ_1 are real. The following cases may be considered.

I. If $k_\theta > k_p$(or $R < P$).
The undamped TCW can propagate in the hatched region A of $\gamma_1-\gamma_1'$ parameter plane (Fig.1) from above situation such a region exists when $\gamma_1>0$ and $\gamma_1'<0$.

II. If $k_\theta < k_p$(or $R > P$).
The undamped TCW indicate hatched area B of $\gamma_1-\gamma_1'$ parameter plane (Fig.2) from above situation a zone exits when $\gamma_1<0$ and $\gamma_1'>0$.

III. If $k_\theta = k_p$(or $R = P$).
The inequalities (2.33) and 2.34 indicate unsatisfactory answer, it means undamped TCW can not exist. This have no more physical interest.

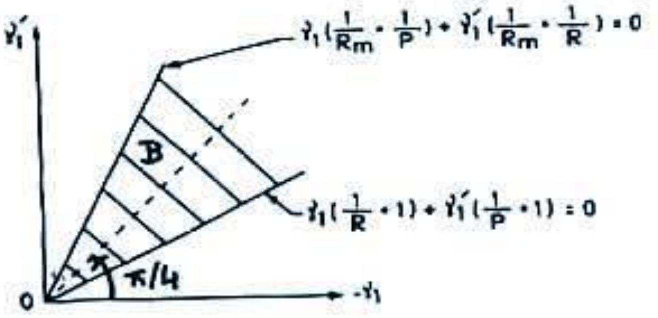

Fig.1: Zone of undamped TCW for $k_\theta > k_p$.

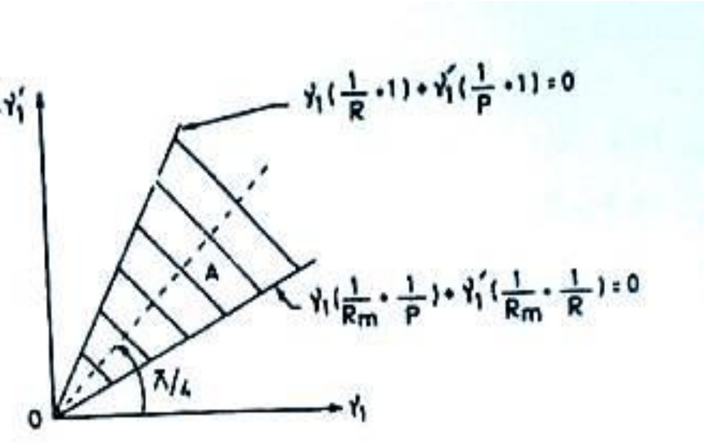

Fig.2: Zone of undamped TCW for $k_\theta < k_p$.

It is very much interesting phenomenon of the propagation of MHD thermoconvective waves in presence of magnetic field which is parallel to the gravitational direction. The another interesting phenomenon is that the thermoconvective waves possibilities appear in the binary fluid layer due to heating effect from below or above in the presence of solute, the waves propagation will not be affected by the magnetic field. During the propagation of transverse waves, fluid particles shows movement to upward and downward in a vertical direction, it is also the property of MHD flow that there is no electromagnetic force which initiates in the fluid due to the direction of fluid flow parallel to the magnetic field. So we can say that the propagation of TCW has not affected by the magnetic field or electric current which is initiated in the flow of propagation.

2. **DISCUSSION**- The inequalities (2.33) and (2.34) implies that the presence of undamped TCW have few physical valuable significance. The conditions of inequalities (2.33) and (2.34) and the conditions used in magnetic prandtl number $R_m \left(= \frac{\nu}{\eta}\right)$ do not represent the restriction of k_θ and η (Takashima). Above described condition does not depend on the strength of the magnetic field (M), which are reverse with the result of 'Takashima'. The presence of undamped waves based on the power of the magnetic field. Conditions are satisfactory in the two cases $k_\theta > k_p$ and $k_\theta < k_p$, where γ_1 and γ_1' so reverse sign $\alpha > 0$ and $\alpha' < 0$ in equation (2.24) which shows β and β' both positive or both negative. We can find very interesting result on **BENARD** convection, if fluid is heated from below (β >0), the **BENARD** convection does not appear. In this condition the solute concentration decreases vertically ($\beta' > 0$), the propagation of undamped TCW show $k_\theta > k_p$, γ_1 and γ_1' behind in the region of A (Fig. 1). The result of $k_\theta < k_p$ that undamped TCW can propagate if the layer is heated above β <0 which provide that solute concentration increases vertically upward $\beta' < 0$ and γ_1 and γ_1' behind in the zone B (Fig.2). If we consider that a small drop of fluid is displaced downward in the new condition of the drop at higher temperatue with higher concentration of solute shows some variation in the condition of variation in k suppose $k_\theta < k_p$. The diffusion of mass should be faster than the heat form the drop for the surrounding area, the drop become less dominant in solute but it is hotter the surrounding associated region, so it increase again. The downward and upward motion is responsible for the propagation of TCW. In both the cases of k, (i.e.) $k_\theta > k_p$ and $k_\theta < k_p$, the relationship between potential energy and density appears. This indicates the propagation with viscous and ohmic destriction of energy.

At last, we can say that the inequalities (2.33) and (2.34) do not put any relationship with M, the effect is possible for the demonstration of TCW in laboratory is possible for outer magnetic field.

REFERENCES

1. Arpaci, S.V. and Larsen, P.S. : Convective Heat Transfer, Prentice Hall, Englewood Cliffs, New Jersey.
2. Bhattacharyya, S.N. and Gupta A.S. : Phys. Fluids, Vol.20, pp. 3215(1985).
3. Chandrasekhar, S. : Hydrodynamic and Hydromagnetic stability, Oxford University Press, 1961.
4. Luikov, A.V. and Berkovsky, B.M. : Int. Jour. Heat Mass Transfer, Vol. 13, pp. 741 (1970).
5. Takashima, M. : Jour. Phys. Soc. Japan, Vol. 33, pp. 1712 (1972).