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The Effect of Variable Heat Flux on Unsteady Laminar MHD Boundary Layer Flow and Heat Transfer Due to a Stretching Sheet

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ABSTRACT

The purpose of this research is to look into the solution technique for obtaining MHD velocity and temperature profiles. In the presence of a changing heat flux, the unsteady laminar boundary layer flow and heat transfer of a viscous incompressible fluid across a stretching sheet are numerically investigated. The unsteadiness is thought to be generated by a sudden increase in the surface temperature and a time-dependent stretching velocity. The flow and heat transfer partial differential equations were numerically solved using an implicit finite difference scheme and a quasi-linearization technique. Both velocity and temperature rise with time and magnetic field, according to the findings. The computed results are compared to previous work that has been published. Variable heat flux (VHF) conditions have also been taken into account.

Keywords: Boundary layer, Heat Transfer, MHD, Stretching Sheet, Unsteady Flow.

I. INTRODUCTION

In extrusion operations, the description of fluid dynamics due to a stretching sheet is critical. Sheeting material is created in a variety of industrial manufacturing methods and can be made of metal or polymer. The rate of heat transmission at the stretching surface determines the final product's quality. Many essential applications in manufacturing processes in industry, such as the description of flow and heat transfer in the boundary layer generated by a stretched surface Extrusion of a polymer in a melt-spinning process, metals and plastics, cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, glass blowing, cooling and/or drying of papers, drawing plastic films, continuous casting and spinning of fibres are just a few examples. The rate of heat transmission at the stretching surface determines the quality of the final product in these circumstances. The ultimate product in all of these applications is determined by the rate of cooling and boundary layer flow near the stretching sheet.

The physical situation was recognized as a backward boundary layer problem by Crane [1] studied the boundary-layer flow due to a moving stretching surface with a constant surface temperature in an ambient fluid. He gave a similarity solution in closed analytical form for steady two dimensional incompressible boundary layer flows. The study considered the case when velocity varies linearly with distance from a fixed point. Dutta [2], Grubka and Bobba[3], discussed the temperature field in the flow over a stretching surface when a uniform heat flux is exerted to the surface. Lin and Chen [4] presented an exact solution of heat transfer from a stretching surface with a variable heat flux Furthermore. Kumari et al. [5] studied the unsteady free convection flow over a continuous moving vertical surface in an ambient fluid, Elbashbeshy [6] conducted a numerical study of steady heat transfer over a stretching surface with a variable surface heat flux and uniform heat flux subjected to injection and suction. Sharidan et al. [7] investigated the unsteady boundary layer flow and heat transfer due to stretching sheet for the especial distribution of the stretching velocity and surface temperature.

Moreover, Ishak et al. [8] extended the dimension of the problem of heat transfer due to stretching sheet to unsteady laminar mixed convection boundary layer flow and heat transfer due to a stretching vertical surface. They discussed the effects of unsteadiness parameter, buoyancy parameter and Prandtl number on the flow characteristic. They found that the heat transfer rate at the surface increases with unsteadiness parameter, buoyancy parameter and Prandtl number.

The study of magneto hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable curiosity in modern metallurgical and metal-working processes. There has been a great attentiveness in the study of magneto hydrodynamic flow and heat transfer in any medium due to the effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluids.

All the above investigators restricted their analysis to flow and heat transfer in the absence of magnetic field. In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. Pavlov [9] investigated the flow of an electrically conducting fluid caused solely by the stretching of an elastic sheet in the presence of a uniform magnetic field. Chakrabarti and Gupta [10] considered the flow and heat transfer of electrically conducting fluid past a porous stretching sheet and presented analytical solution for the flow and the numerical solution for the heat transform problem. The flow is caused by sheet stretching according to a power law velocity in the presence of a transverse magnetic field.

The studies reported above deal with steady flows. However, the flow problem will become unsteady due to impulsive change in the surface velocity of a moving stretching surface. The unsteady flow on a stretching surface is an important problem, since it is not always possible to maintain steady state conditions. Pop and Na [11] and Nazar et al. [12] have considered the time dependent boundary layer flow due to an impulsively stretching surface. Recently, Srinivasa et al. [13] have investigated unsteady MHD laminar boundary layer flow due to an impulsive stretching surface in the presence of a transverse magnetic field.

The objective of the present study is to find numerical solutions for unsteady boundary layer flow and heat transfer due to stretching sheet with an applied magnetic field by Quasiliearization technique along with finite difference method. We also present results for velocity and temperature profiles graphically.

II. MATHEMATICAL FORMULATION

Consider an unsteady two-dimensional MHD boundary layer flow and heat transfer over a continuous stretching sheet embedded in a moving viscous, incompressible, electrically conducting fluid in the region y > 0, as shown in Figure 1.

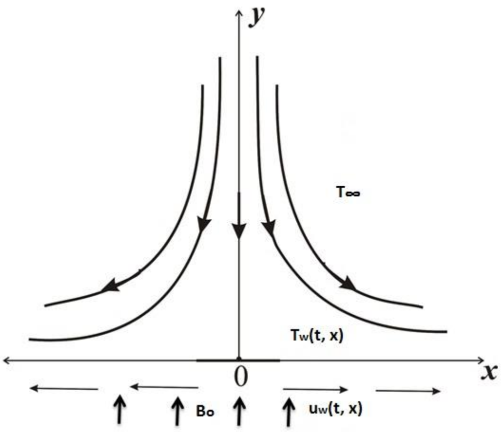


Figure.1. Physical model and coordinate system.

Keeping the origin fixed, two equal and opposite forces are suddenly applied along the *x*-axis, which results in stretching of the sheet and hence, flow is generated. At the same time, the wall temperature $T_w(t, x)$ of the sheet is suddenly raised from T_∞ to $T_w(t, x)$ (> T_∞). Here the fluid is under the influence of the magnetic field B_0, which acts in the y - direction normal to the stretching sheet. The induced magnetic field is negligible, which is valid under the assumption of small magnetic Reynolds number. It is also assumed that the external electric field is zero. Under these assumptions, the basic unsteady boundary layer equations governing for momentum and heat transfer in the presence of MHD are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial y^2} - \frac{\sigma {B_0}^2 u}{\rho} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Subject to the initial conditions

$$t < 0; \quad u = v = 0,\ T = T_\infty \quad \text{for any } x, y$$

$$t \geq 0; \quad u = u_w(t,x)\ ,\ v = 0 \tag{4}$$

$$T = T_w(t,x) \quad u \to 0,\ T \to \infty \text{ at } y \to \infty$$

Here u and v are velocity components along x and y-directions, respectively, where t *is* the time, σ, ρ and ϑ denote, respectively, electrical conductivity, density and kinematic viscosity. T is the temperature, α is the thermal diffusivity and k is the thermal conductivity. Here, we assume that, the velocity of the sheet is $u_w(t,x)$ and the sheet temperature $T_w(t,x)$ have the following form.

$$u_w(t,x) = \frac{cx}{1-\gamma t},$$

$$T_w(t,x) = T_\infty + \frac{c}{2\vartheta x^2(1-\gamma t)^{3/2}} \tag{5}$$

Where c is the stretching rate being a positive constant, γ is a positive constant, which measures the unsteadiness of heat transfer quantity. We introduce now the following new variables

$$\eta = \sqrt{\frac{c}{\vartheta(1-\gamma t)}}\,y,$$

$$\psi = \sqrt{(c\vartheta/((1-\gamma t)\,))}\,xf(\eta)$$

$$T = T_\infty + \frac{c}{2\vartheta x^2(1-\gamma t)^{3/2}}G(\eta)$$

$$T = T_\infty + \left(\frac{q_w}{k}\right)\frac{c}{(1-\gamma t)^{3/2}}G(\eta)\ \text{(VHF)} \tag{6}$$

Where ψ is the stream function which is defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$

Substituting variables Equations (6) into Equations (2) and (3), they reduce to the following ordinary differential equations

$$f''' + ff'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) - Mf' = 0 \tag{7}$$

$$\frac{1}{Pr}G'' + fG' + 2f'G - \frac{1}{2}A(3G + \eta G') = 0 \tag{8}$$

Subject to the boundary conditions (4), which becomes,

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) = 0$$

$$G(0) = 1 \quad \text{or} \quad G(\infty) = 0 \qquad (9)$$

Here η be the transformed dimensionless independent variables, f is the dimensionless stream function; f' is the dimensionless velocity. Where Pr is the Prandtl number, $A = \frac{\gamma}{c}$ is a non-dimensional constant which measures unsteadiness of the flow and heat transfer and prime denoting the differentiation with respect to the similarity variable η. M is the dimensionless magnetic field parameter, G is dimensional temperature. The parameter of engineering interest is the skin friction coefficient (c_f) and heat transfer coefficients in terms of local Nusselt number(N_{ux}) is given by

$$c_f\sqrt{(R_{ex})} = \frac{\tau_w}{\rho(u_w)^2} = \frac{\mu(\frac{\partial u}{\partial y})_{y=0}}{\rho(u_w)^2} = f''(0) \qquad (10)$$

where μ being the dynamic viscosity, τ_w be the skin friction and q_w be the heat transfer from the sheet.

$$\frac{N_{ux}}{\sqrt{R_{ex}}} = \frac{xq_w}{k(T_w - T_\infty)} = \frac{-xk(\frac{\partial u}{\partial y})_{y=0}}{k(T_w - T_\infty)} = -G'(0) \qquad (11)$$

$$\frac{N_{ux}}{\sqrt{R_{ex}}} = \frac{xq_w}{k(T_w - T_\infty)} = \frac{-xk(\frac{\partial u}{\partial y})_{y=0}}{k(T_w - T_\infty)} = \frac{1}{G(0)}\ \text{(VHF)} \qquad (12)$$

where $R_{ex} = u_w x/\vartheta$ is the local Reynolds number.

III. RESULTS AND DISCUSSION

The transformed Equation (7) and (8) subjected to the boundary condition (9) are solved numerically by using a stable finite difference method along with quasilinearization technique. Since the method described in Inouye and Tate [14] and Ajaykumar et al. [15] for the sake of brevity, its description is omitted here. The results obtained are presented through the graphs for velocity and temperature profiles in different values of unsteadiness parameter (A) and Prandtl number (Pr) along with magnetic field (M) is as shown from fig.2 - 4. The computations are carried out for the different values of magnetic parameter (M).The computed results have been compared with those of Sharidan et al. [7], for skin friction and heat transfer coefficient in Table 1, Pr =0.01, 0.1 and 1.0. Our results are found to be in good agreement with those of [7] correct to four decimal places of accuracy.

Table 2. Shows the effect skin friction and temperature on the wall coefficients for fixed magnetic field M. Here, both skin friction and temperature on the wall increases, for different Pr, with time parameter A. The percentage of increase skin friction coefficient is 13.72 % and temperature on the wall is about 5.95 % for a time parameter A = 0.8 to 1.2.

Table 1. Comparison results for the values of the heat transfer [$G'(0)$] and skin friction coefficient [$f''(0)$] for various values of A and Pr [7]

A	0.8				1.2				2			
	Present	Previous	Present	Previous	Present	Previous	Present	Previous	Present	Previous	Present	Previous
Pr	$G'(0)$	$G'(0)$	$-f''(0)$	$-f''(0)$	$G'(0)$	$G'(0)$	$-f''(0)$	$-f''(0)$	$G'(0)$	$G'(0)$	$-f''(0)$	$-f''(0)$
0.0	0.2091	0.2092	1.2591	1.2610	0.2175	0.2174	1.37	1.3777	0.2338	0.2331	1.58	1.5873

1							47				16	
0.1	0.26 30	0.2629	1.2591	1.2610	0.33 05	0.3306	1.37 47	1.3777	0.44 57	0.438 7	1.58 16	1.5873
1	0.47 22	0.4712	1.2591	1.2610	0.78 90	0.7882	1.37 47	1.3777	1.24 45	1.243 7	1.58 16	1.5873

Table 2. Results for the values of the temperature on the wall $[G(0)]$ and skin friction coefficient $[f''(0)]$ for various values of A and Pr with M= 0.5

M = 0.5				
A	0.8		1.2	
Pr	$G(0)$	$-f''(0)$	$G(0)$	$-f''(0)$
0.1	1.0508	1.0598	1.0666	1.1970
0.72	1.0959	1.0598	1.1554	1.1970

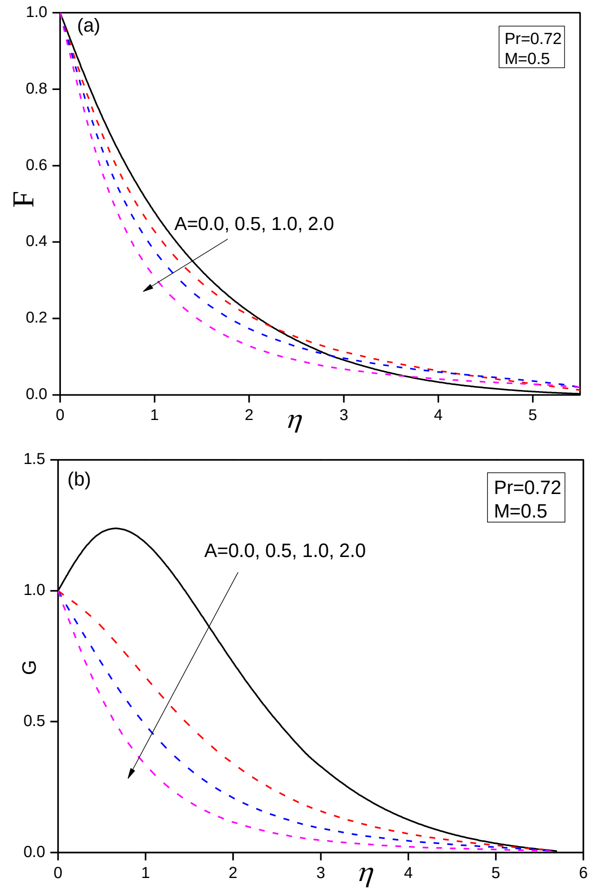


Figure 2 (a) The velocity and (b) temperature profiles for different values of A for M=0.5

Figure 2 shows the effect of unsteady parameter (A) for a fixed Pr =0.72 with M=0.5. The variation of velocity with respect to similarity variable η is decreases with increase of A. Hence the boundary layer thickness decreases with the increase of A see in [Figure 2 (a)]. On the other hand the temperature with respect similarity variable η shows that decreases monotonically with the increase of A, except A = 0. In this case (A=0) the temperature overshoot its value at the surface of the sheet, as seen in Figure 2(b). It is possible to see that the temperature at the surface of the sheet is invariant with respect to A. Physically the unsteadiness increases the sheet loses more heat which causes decreases in temperature.

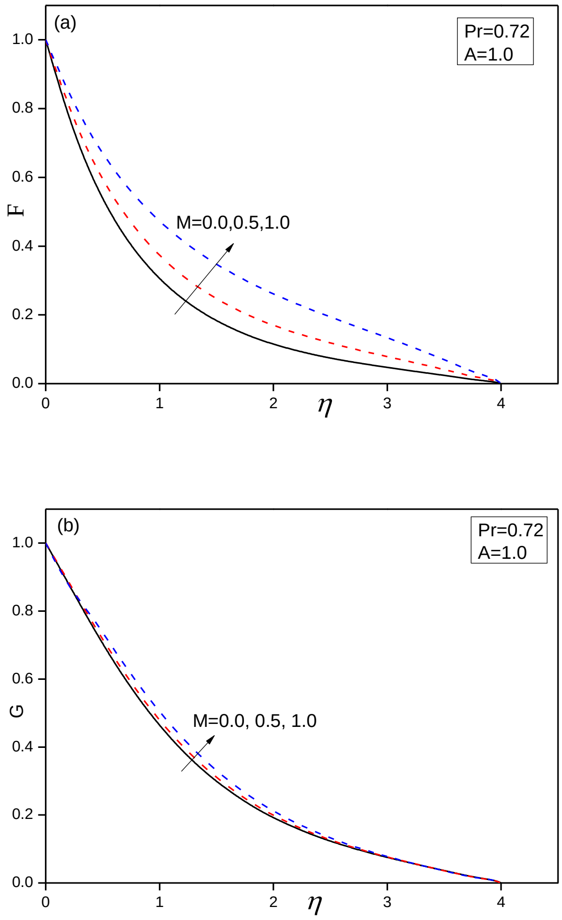
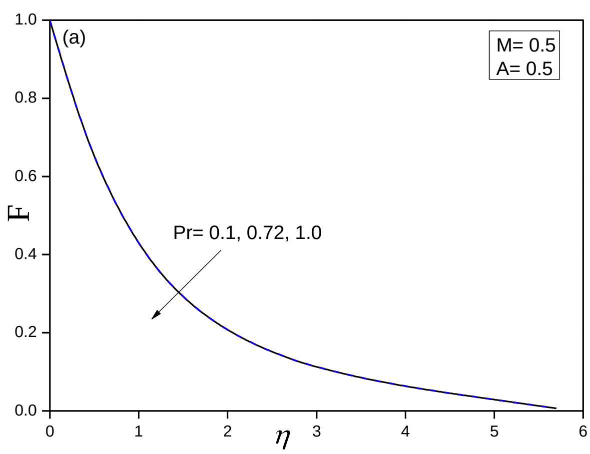


Figure 3 (a) The velocity and (b) temperature profiles for different values of *M* with *A* = 1.0

The effect of magnetic field (*M*) on the corresponding velocity and temperature profiles with for a fixed Pr =0.72 and A = 1.0 as shown in Figure 3. This shows that both velocity and temperature increases with the increase of *M*.



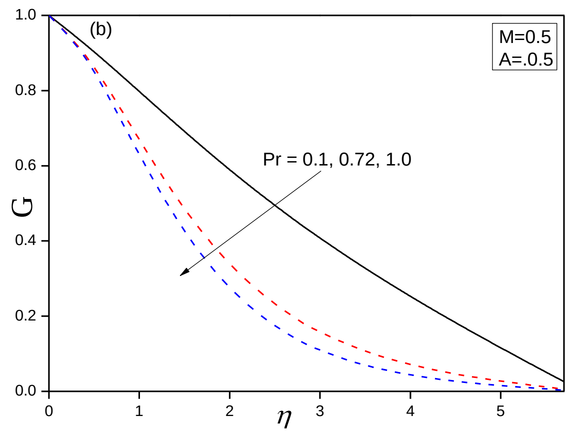


Figure 4 (a) The velocity and (b) temperature profiles for different values of Pr with M=0.5 and A=0.5.

Figure 4 represents the variation of velocity and temperature for different values Prandtl number (Pr) with fixed parameter $A = M = 0.5$. It shows that the temperature decreases as Prandtl number increases for fixed value of η *i.e.* the temperature decreases as the distance away from the sheet increases and it became almost zero at $\eta = 5$ which ends the boundary layer thickness. The temperature decreases with in the boundary layer for all values of Pr. This is consistent with the fact the boundary layer thickness decreases with increases of η [Figure 4 (b)] but less effect in velocity [4(a)].

IV. CONCLUSIONS

From the present study, the effect of magnetic parameter M, the unsteadiness parameter A and Prandtl number Pr on the skin friction and heat transfer coefficient were studied. The numerical results indicate the following.

1) The thickness of velocity and thermal boundary layer decreases with increasing unsteadiness parameter.
2) As magnetic parameter (M) increases both velocity and temperature increases.
3) Increasing the Prandtl number Pr with magnetic parameter leads to a decrease in the velocity and surface temperature.

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