International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
doi : https://doi.org/10.32628/IJSRST

Some String Cosmological Models with Time Dependent Bulk Viscosity

S. S. Charjan
Department of Mathematics, K. Z. S. Science College, Bramhani Kalmeshwar, Dist. Nagpur, India

Article Info
Volume 7, Issue 6
Page Number: 548-556
Publication Issue :
November-December-2020
Article History
Accepted : 10 Dec 2020
Published : 24 Dec 2020

ABSTRACT

Some string cosmological models with bulk viscous fluid for massive string are investigated. To get the determinate model of the universe, we have assumed that the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ in the model and expansion θ in the model is proportional to the shear σ and $\zeta\theta$ = constant. The behavior of the model in presence and absence of bulk viscosity, is discussed. The physical implications of the models are also discussed in detail.

Keywords Bimetric theory; viscous models; cosmic string, magnetic field, Bianchi Type-I

Mathematics Subject Classification 2020: 83D-XX, 83F-XX, 83F05

I. INTRODUCTION

In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble [1]). Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al. [2]; Kibble [1, 3]; Everett [4]; Vilenkin [5]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies (Zel'dovich [6]). These cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings.

The general relativistic treatment of strings was initiated by Letelier [7, 8] and Stachel [9]. Letelier [7] has obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi I and Kantowski-Sachs space-times. Benerjee et al. [10] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [11], Tikekar and Patel [12]. Patel and Maharaj [13] investigated stationary rotating world model with magnetic field. Ram and Singh [14] obtained some new exact solutions of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [15]. Exact solutions of string cosmology for Bianchi type II, V I0, VIII and IX space-times have been studied by Krori et al. [16] and Wang [17]. Singh and Singh [18] investigated string cosmological models with magnetic field in the context of space-time with G3 symmetry. Lidsey, Wands and Copeland [19] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Baysal et al. [20] have investigated the behaviour of a string in the cylindrically symmetric inhomogeneous universe. Bali et al. [21 - 25] have obtained Bianchi types IX, type-V and type-I string cosmological models in general relativity. Yavuz [26] have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Recently Kaluza-Klein cosmological solutions are obtained by Yilmaz [27] for quark matter coupled to the string cloud in the context of general relativity. On the other hand, the matter distribution is satisfactorily described by perfect fluids due to the large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than the perfect fluid. It is well known that when neutrino decoupling occurs, the matter behaves as a viscous fluid in an early stage of the universe. Viscous fluid cosmological models of early universe have been widely discussed in the literature.

Though the Einstein theory of relativity is one of the successful theory of gravitation and is consistent with experimental data and observations, it has some lacunas that it is not free from singularities which were appearing in big-bang in cosmological models. The several theories of gravitation have been proposed as alternative to the theory of general relativity. The most important one amongst them is Rosen's (1974) bimetric theory of gravitation. The bimetric theory of gravitation based on two matrices viz: the fundamental metric tensor g_{ij} and flat metric γ_{ij}. The fundamental metric tensor g_{ij} describes the gravitational potential whereas background metric γ_{ij} describes the inertial forces associated with the acceleration of the frame of reference. The interpretation of these two metric tensors in bimetric theory of relativity is not unique. The metric tensor g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and describes the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. One can regard the γ_{ij} as a flat space time having no physical and geometrical significance but the physical metric tensor g_{ij} considered as a gravitational potential tensor which is determined by field equations or by interactions between matter and gravitation. Therefore flat space time metric interacts with Riemannian metric but not directly with matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$ satisfying both principles of covariance and equivalence and the formation of general relativity. Thus at every point of space–time in bimetric theory of gravitation, there are two metrics

$$ds^2 = g_{ij}\, dx^i\, dx^j \tag{1}$$

$$d\eta^2 = \gamma_{ij}\, dx^i\, dx^j \tag{2}$$

The Rosen's field equations (1973) in bimetric theory of gravitation are

$$N_i^j - \frac{1}{2}\, N\, \delta_i^j = -8\pi T_i^j \tag{3}$$

where $N_i^j = \frac{1}{2}\, \gamma^{pr}\, \left(g^{sj}\, g_{si|p}\right)_{|r}$, $N = g^{ij}\, N_{ij}$ is the Rosen scalar. Here the vertical bar $(\,|\,)$ stands for $\gamma-$ covariant differentiation where $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$.

Many authors like Rosen[28,31], Karade [32], Israelit [33], Reddy D. R. K. and Venkateswaralu [34], Katore and Rane[35] , Khadekar and Tade[36] , Borkar et al.[37-44] & [46] , Gaikwad et.al. [49] have developed many bianchi type models in bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that occur in general relativity that was appearing in the big–bang in cosmological models of the universe and therefore, recently, there has been a lot of interest in cosmological models in related to Rosen's bimetric theory of gravitation.

In this paper, Bianchi type I string cosmological models with bulk viscous fluid for massive string are investigated. To get the determinate model of the universe, we have assumed that the coefficient of bulk viscosity ξ is inversely proportional to the expansion θ in the model and expansion θ in the model is proportional to the shear σ and ζθ = constant. The behavior of the model in presence and absence of bulk viscosity, is discussed. The physical implications of the models are also discussed in detail.

II. The Metric and Field Equations

We consider the space-time of general Bianchi I type with the metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \qquad (4)$$

where A, B and C are functions of t alone.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \qquad (5)$$

The energy momentum tensor T_i^j for string dust is given by

$$T_i^j = \in \upsilon_i\upsilon^j - \lambda x_i x^j - \zeta\upsilon^{\ell}_{;\ell}(g_i^j + \upsilon_i\upsilon^j) + E_i^j \qquad (6)$$

with

$$\upsilon_i\upsilon^i = -x_i x^i = -1 \qquad (7)$$

and

$$\upsilon^i x_i = 0 \qquad (8)$$

In this model $\in$ is the rest energy density for a cloud of strings and is given by $\in = \in_p + \lambda$ where $\in_p$ and λ denote the particle density and the string tension density of the system of strings respectively, x^i is the direction of strings and ζ is the coefficient of bulk viscosity.

The electromagnetic field E_{ij} is (given by Lichnerowicz (1967) [14]

$$E_{ij} = \bar{\mu}\left[|h|^2\left(\upsilon_i\upsilon_j + \frac{1}{2}g_{ij}\right) - h_i h_j \right] \qquad (9)$$

where four velocity vector υ_i is given by

$$g_{ij}\upsilon^i\upsilon^j = -1 \qquad (10)$$

and $\bar{\mu}$ is the magnetic permeability and the magnetic flux vector h_i defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl}\upsilon^j \qquad (11)$$

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levi Civita tensor density.

Assume the commoving coordinates system, so that $\upsilon^1 = \upsilon^2 = \upsilon^3 = 0, \upsilon^4 = 1$.

Further we assume that the incident magnetic field is taken along x-axis so that

$$h_1 \neq 0 \, and \; h_2 = h_3 = h_4 = 0$$

The first set of Maxwell's equation

$$F_{[ij,k]} = 0 \quad (12)$$

yield

F_{23} = constant H (say)

Due to the assumption of infinite electrical conductivity, we have $F_{14} = F_{24} = F_{34} = 0$.

The only non-vanishing competent of F_{ij} is F_{23}. So that

$$h_1 = \frac{AH}{\bar{\mu}BC} \quad (13)$$

and

$$|h|^2 = \frac{H^2}{\bar{\mu}^2 B^2 C^2} \quad (14)$$

From equation (9) we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\bar{\mu}B^2C^2} \quad (15)$$

From equation (6) we obtain

$$T_1^1 = \left(-\lambda - \frac{H^2}{2\bar{\mu}B^2C^2} - \zeta\upsilon^{\ell}_{;\ell}\right),\ T_2^2 = T_3^3 = \left(\frac{H^2}{2\bar{\mu}B^2C^2} - \zeta\upsilon^{\ell}_{;\ell}\right),\ T_4^4 = -\left(\in + \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (16)$$

Using (16) the Rosen's field equations (3) gives

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(\lambda + \frac{H^2}{2\bar{\mu}B^2C^2} + \zeta\upsilon^{\ell}_{;\ell}\right) \quad (17)$$

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(-\frac{H^2}{2\bar{\mu}B^2C^2} + \zeta\upsilon^{\ell}_{;\ell}\right) \quad (18)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi ABC\left(-\frac{H^2}{2\bar{\mu}B^2C^2} + \zeta\upsilon^{\ell}_{;\ell}\right) \quad (19)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(\in + \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (20)$$

where $A_4 = \frac{dA}{dt}, B_4 = \frac{dB}{dt}, C_4 = \frac{dC}{dt}$ *etc.*

equations (17) to (20) are four equations in the unknowns $A, B, C, \lambda\ and \in$ therefore to deduce a determinate solution. We assume two supplementary conditions,

the shear (σ) is proportional to the scalar of expansion θ i.e., we assume the condition

$$A = (BC)^n, \quad where\ n > 0 \quad (21)$$

and $\in = \lambda$ (22)

i.e., the rest energy density is equal to the string tension density.

From equations (19) and (20), we obtain

$$2\frac{C_4^2}{C^2} - 2\frac{C_{44}}{C} = 16\pi ABC\left(\zeta\upsilon^{\ell}_{;\ell} - \in - \frac{H^2}{\bar{\mu}B^2C^2}\right) \quad (23)$$

Adding equation (17) and (23) together and using the condition ε =λ ,we get

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi ABC\left(2\zeta\upsilon^{\ell}_{;\ell} - \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (24)$$

From equations (21) and (24), we write

$$(n-1)\frac{B_4^{\ 2}}{B^2}+(n+1)\frac{C_4^{\ 2}}{C^2}+(1-n)\frac{B_{44}}{B}-(n+1)\frac{C_{44}}{C}=-16\pi K(BC)^{n-1}+32\pi(BC)^{n+1}\zeta\upsilon^{\ell}_{;\ell}$$

where

$$K=\frac{H^2}{2\bar{\mu}}$$

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C}-\frac{B_{44}}{B}=\frac{C_4^{\ 2}}{C^2}-\frac{B_4^{\ 2}}{B^2} \qquad (26)$$

On simplifying above equation ,we get

$$\frac{(CB_4-BC_4)_4}{(CB_4-BC_4)}=\frac{(BC)_4}{BC} \qquad (27)$$

which on integrating yield

$$C^2\left(\frac{B}{C}\right)_4=LBC \qquad (28)$$

where L is the constant of integration.

Using assumptions $BC=\mu$ and $\frac{B}{C}=v$, the above equation (28) leads to

$$\frac{v_4}{v}=L \qquad (29)$$

Now using equation (21) and the condition $BC=\mu$ and $\frac{B}{C}=v$, the equation (25) gives

$$-\frac{n}{\mu^{(n+1)}}\left(\frac{\mu_{44}}{\mu}\right)+\frac{n}{\mu^{n+1}}\frac{\mu_4^2}{\mu^2}=-\frac{16\pi K}{\mu^2}+32\pi\zeta\upsilon^{\ell}_{;\ell} \qquad (30)$$

Applying the condition ζθ = constant to the above equation, we get

$$\mu_{44}-\frac{\mu_4^2}{\mu}+\beta\mu^{n+2}=\frac{16\pi K}{n}\mu^n \qquad (31)$$

where

$$\beta=\frac{32\pi}{n}\zeta\upsilon^{\ell}_{;\ell}$$

which reduces to

$$\frac{d}{d\mu}\left[f^2\right]+(-\frac{2}{\mu})f^2=2\left[\frac{16\pi K}{n}-\beta\mu^2\right]\mu^n \qquad (32)$$

where $\mu_4=f(\mu)$

The differential equation (32) has solution

$$f^2=\frac{32\pi K}{n(n-1)}\mu^{n+1}-\frac{2\beta}{n+1}\mu^{n+3}+P\mu^2 \qquad (33)$$

where P is the constant of integration.
From equation (29) we write

$$\log v=\int\frac{L\,d\mu}{\sqrt{\frac{32\pi K}{n(n-1)}\mu^{n+1}-\frac{2\beta}{(n+1)}\mu^{n+3}+P\mu^2}}+\log b \qquad (34)$$

Using $\mu_4=f(\mu)$ and expression (33), the metric (4) will be

$$ds^2=-\frac{d\mu^2}{\left[\frac{32\pi K}{n(n-1)}\mu^{(n+1)}-\frac{2\beta}{(n+1)}\mu^{(n+3)}+P\mu^2\right]}+\mu^{2n}dx^2+\mu v dy^2+\frac{\mu}{v}dz^2 \qquad (35)$$

where v is determined by equation (34).

After suitable transformation of coordinates i.e., putting $\mu = T$, $x = X$, $y = Y$, $z = Z$
the above metric (35) takes the form

$$ds^2 = -\frac{dT^2}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)} - \frac{2\beta}{(n+1)}T^{(n+3)} + PT^2\right]} + T^{2n}dX^2 + T\,vdY^2 + \frac{T}{v}dZ^2 \qquad (36)$$

This is the Bianchi Type-I bulk viscous fluid string dust magnetized cosmological model with magnetic field in bimetric theory of relativity.

In the absence of magnetic field i.e., K=0, the metric (36) have the form

$$ds^2 = \frac{-dT^2}{\left[PT^2 - \frac{2\beta}{(n+1)}T^{(n+3)}\right]} + T^{2n}dX^2 + T\,vdY^2 + \frac{T}{v}dZ^2 \qquad (37)$$

In the absence of viscosity i.e., $\beta = 0$, we write

$$ds^2 = \frac{-dT^2}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)} + PT^2\right]} + T^{2n}dX^2 + T\,vdY^2 + \frac{T}{v}dZ^2 \qquad (38)$$

III. *Physical and Geometrical* features

The density $(\in)$, the string tension density (λ), for the model (36) is given by

$$\in = \lambda = \left[\frac{(5n - 4n^2 - 1)}{n(n-1)}\frac{K}{T^2} + \frac{(2n^2 + n - 1)\beta}{16\pi(n+1)}\right] \qquad (39)$$

Now the expansion θ is given by $\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$

which has the value

$$\theta = \frac{(n+1)f}{T}$$

or

$$\theta = (n+1)\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} \qquad (40)$$

The components of shear tensor σ_i^j are given by

$$\sigma_1^1 = \frac{1}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right]$$

or

$$\sigma_1^1 = \frac{(2n-1)}{3}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} \qquad (41)$$

Likewise, we obtain the other components of σ_i^j as

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2} \qquad (42)$$

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right]^{\frac{1}{2}} - \frac{L}{2} \qquad (43)$$

$$\sigma_4^4 = 0 \qquad (44)$$

Thus

$$\sigma^2 = \frac{1}{2}(\sigma_{ij}\sigma^{ij}) = \frac{(2n-1)^2}{12}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)} - \frac{2\beta}{(n+1)}T^{(n+1)}\right] + \frac{L^2}{4} \tag{45}$$

And the spatial volume is

$$R^3 = \alpha T^{(n+1)}, \tag{46}$$

where

$$\alpha = \frac{1}{f} = \frac{1}{\left[\frac{32\pi K}{n(n-1)}T^{(n+1)} - \frac{2\beta}{(n+1)}T^{(n+3)} + PT^2\right]^{\frac{1}{2}}}$$

IV. Discussion

It is observed that , the expansion (θ) in the model (36) increases as the coefficient

of viscosity β decreases. When $T \to 0, \in = \lambda \to \infty$, and when $T \to \infty$, $\in = \lambda = \frac{(2n^2+n-1)\beta}{16\pi(n+1)}$. When $\beta \to 0, \in = \lambda = \frac{(5n-4n^2-1)}{n(n-1)}\frac{K}{T^2}$ and when $\beta \to \infty$, $\in = \lambda \to \infty$. When $T \to \infty$, the expansion θ, the shear σ, and spatial volume R^3 are all tends to infinity.

Since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$ and $\beta = 0$ yield the maximum expansion is

$(n+1)\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}}$, the model does not approach isotropy for large value of T.

For $n = \frac{1}{2}$, there is shear $\sigma^2 = \frac{L^2}{4}$ hence the model (36) does not approaches isotropy for large values of T.

In the absence of magnetic field, the rest energy density $(\in)$, string tension density (λ) model (36) is given by

$$\in = \lambda = \frac{(2n^2+n-1)\beta}{16\pi(n+1)} \tag{47}$$

The expression for expansion (θ), the component of shear tensor (σ_i^j) and the spatial volume (R^3) are given by

$$\theta = (n+1)\left[P - \frac{2\beta}{(n+1)}T^{n+1}\right]^{\frac{1}{2}} \tag{48}$$

$$\sigma_1^1 = \frac{(2n-1)}{3}\left[P - \frac{2\beta}{n+1}T^{(n+1)}\right]^{\frac{1}{2}} \tag{49}$$

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[P - \frac{2\beta}{n+1}T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L}{2} \tag{50}$$

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[P - \frac{2\beta}{n+1}T^{(n+1)}\right]^{\frac{1}{2}} - \frac{L}{2} \tag{51}$$

$$\sigma_4^4 = 0 \tag{52}$$

$$\sigma^2 = \frac{(2n-1)^2}{12}\left[P - \frac{2\beta}{n+1}T^{(n+1)}\right]^{\frac{1}{2}} + \frac{L^2}{4} \tag{53}$$

$$R^3 = \alpha_1 T^{(n+1)} \tag{54}$$

where $\alpha_1 = \dfrac{1}{\left[PT^2 - \dfrac{2\beta}{(n+1)}T^{(n+3)}\right]^{\frac{1}{2}}}$

In the absence of viscosity the rest energy density $\in$, string tension density λ for model (36) is given by

$$\in = \lambda = \frac{\left(5n-4n^2-1\right)}{n(n-1)}\frac{K}{T^2} \qquad (55)$$

The expression for expansion (θ), the components of shear tensor σ_i^j, and the spatial volume $\left(R^3\right)$

$$\theta = (n+1)\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}}$$

$$\sigma_1^1 = \frac{(2n-1)}{3}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}}$$

(56)

$$\sigma_2^2 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L}{2}$$

(57)

$$\sigma_3^3 = \frac{(1-2n)}{6}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}} - \frac{L}{2}$$

(58)

$$\sigma_4^4 = 0 \qquad (59)$$

$$\sigma^2 = \frac{(2n-1)^2}{12}\left[P + \frac{32\pi K}{n(n-1)}T^{(n-1)}\right]^{\frac{1}{2}} + \frac{L^2}{4} \qquad (60)$$

$$R^3 = \alpha_2 T^{(n+1)} \qquad (61)$$

where $\alpha_2 = \dfrac{1}{\left[PT^2 + \dfrac{32\pi K}{n(n-1)}T^{(n+1)}\right]^{\frac{1}{2}}}$

In the absence of magnetic field, the expansion in the model (36) decreases as T

Increases and in the absence of bulk viscosity, the expansion in the model (36) increases as T increases. Since $\lim_{T\to\infty}\dfrac{\sigma}{\theta} \neq 0$, the model does not approaches for large values of *T.*

The spatial volume

$R^3 \to 0,\ when\ T \to 0\ and\ R^3 \to \infty\ \ when\ \ T \to \infty.$

V. Acknowledgement

The author is grateful to Dr. M. S. Borkar, Ex-Professor, Post Graduate Teaching Department of Mathematics, R. T. M. Nagpur University, Nagpur for his helpful guidance.

VI. REFERENCES

[1]. Kibble, T. W. B. (1976). J. Phys. A: Math. Gen. 9, 1387.
[2]. Zel'dovich, Ya. B., Kobzarev, I. Yu., and Okun, L. B. (1975). Zh. Eksp. Teor. Fiz. 67, 3; (1975). Sov. Phys.-JETP 40, 1.
[3]. Kibble, T. W. B. (1980). Phys. Rep. 67, 183.
[4]. Everett, A. E. (1981). Phys. Rev. 24, 858.
[5]. Vilenkin, A. (1981). Phys. Rev. D 24, 2082.
[6]. Zel'dovich, Ya. B., (1980). Mon. Not. R. Astron. Soc. 192, 663.
[7]. Letelier, P. S., (1979). Phys. Rev. D 20, 1249.
[8]. Letelier, P. S., (1983). Phys. Rev. D 28, 2414.
[9]. Stachel, J., (1980). Phys. Rev. D 21, 2171.
[10]. Banerjee, A., Sanyal, A. K. and Chakraborty, S., (1990). Pramana-J. Phys. 34, 1

[11]. Chakraborty, S., (1991). Ind. J. Pure Appl. Phys. 29, 31.
[12]. Tikekar, R. and Patel, L. K., (1992). Gen. Rel. Grav. 24, 397; (1994). Pramana-J. Phys. 42, 483.
[13]. Patel, L. K. and Maharaj, S. D., (1996). Pramana-J. Phys. 47, 1.
[14]. Ram, S. and Singh, T. K., (1995). Gen. Rel. Grav. 27, 1207.
[15]. Carminati, J. and McIntosh, C. B. G., (1980). J. Phys. A: Math. Gen. 13, 953.
[16]. Krori, K. D., Chaudhury, T., Mahanta, C. R. and Mazumdar, A., (1990). Gen. Rel. Grav. 22, 123.
[17]. Wang, X. X., (2003). Chin. Phys. Lett. 20, 615.
[18]. Singh, G. P. and Singh, T., (1999). Gen. Relativ. Gravit. 31, 371.
[19]. Lidsey, J. E., Wands, D. and Copeland, E. J., (2000). Phys. Rep. 337, 343.
[20]. Baysal, H., Yavuz, I., Tarhan, I., Camci, U. and Yilmaz, I., (2001). Turk. J. Phys. 25, 283.
[21]. Bali, R. and Dave, S., (2001). Pramana-J. Phys. 56, 513.
[22]. Bali, R. and Upadhaya, R. D., (2003). Astrophys. Space Sci. 283, 97.
[23]. Bali, R. and Dave, S., (2003). Astrophys. Space Sci. 288, 503.
[24]. Bali, R. and Singh, D. K., (2005). Astrophys. Space Sci. 300, 387.
[25]. Bali, R. and Anjali, (2006). Astrophys. Space Sci. 302, 201.
[26]. Yavuz, ˙Ilhami, Yilmaz, ˙Ihsan and Baysal, H¨usn¨u, (2006). Int. J. Mod. Phys. D 8, 1365.
[27]. Yilmaz, I., (2006). Gen. Relativ. Gravit. 38, 1397.
[28]. Rosen, N. Gen. Relativ. Gravit. 4, 435 (1973)
[29]. Rosen, N., Ann. Phys. 84, 455 (1974)
[30]. Rosen, N. Gen. Relativ. Gravit. 6, 259 (1975)
[31]. Rosen, N. Gen. Relativ. Gravit. 9, 339 (1978)
[32]. Karade T.M. Ind. J. Pure. Appl. Math. 11, 1202 (1980)
[33]. M. Israelit, Gen. Relativ. Gravit. 11, 25 (1979)
[34]. Reddy D R K and Venkateswarlu, Astrophys. Space. Sci. 158 : 169 (1989)
[35]. Katore, S.D. and Rane, R.S. (2006), Magnetized cosmological models in biometric theory of gravitation, Pramana J. Phys. 67(2),-237-227.
[36]. G.S. and Tade, S.D. (2007), String cosmological models in five dimensional bimetric theory of gravitation, .Astrophys. Space Sci. DoI 10.1007\S10509-007-9410-2.T.
[37]. Borkar, M. S. and Charjan, S. S. (2009), Bianchi Type I string dust cosmological model with magnetic field in bimetric relativity, IJAM 22(3) , 445-456, Bulgeria.
[38]. Borkar, M. S. and Charjan, S. S. (2010), The charged perfect fluid distribution in bimetric theory of relativity, J. Ind. Acad. Math , 32(1), 147-152.
[39]. Borkar, M. S. and Charjan, S. S. (2010), LRS Bianchi Type I string dust magnetized cosmological models in bimetric theory of relativity, Tensor N.S. 71(2), 155-162.
[40]. Borkar, M. S. and Charjan, S. S. (2010), Bianchi Type I bulk viscous fluid string dust cosmological model with magnetic field in bimetric theory of gravitation An Int. J. AAM 5 (1) , 96-109.
[41]. Borkar, M. S. and Charjan, S. S. (2010), Bianchi Type I magnetized cosmological model in bimetric theory of gravitation, An Int. J. AAM 5 (10), 1660-1671.
[42]. Borkar, M. S. and Charjan, S. S. (2013), Bianchi Type I magnetized cosmological model with - term in bimetric theory of gravitation, An Int. J. AAM 8 (10), 116- 127.
[43]. Borkar, M. S.,Charjan, S. S. and Lepse V.V. (Dec.2013) International Journal Of Applied Mathematics, ISSN 2051-5227, Vol.28(2), Pp 2051-2059.
[44]. Borkar, M. S.,Charjan, S. S. and Lepse V.V. (2014), LRS Bianchi Type I Cosmological Models With Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation,International Organization Of Scientific Research, Journal of Mathematics, Vol. 10, Issue 3, Pp 16-30.
[45]. Charjan, S. S. (2016), Conformally Flat Spherically Symmetric charged Perfect Fluid

distribution in Bimetric Theory of gravitation, International Journal of Scientific research in Science, Engineering and Technology , Vol . 2, (6) , Pp.842-845.

[46]. Borkar, M. S., Charjan, S. S. and Lepse V.V., (2016) Bianchi Type-I hyperbolic Models With Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, Int. J. AAM Vol. 11 (1), Pp. 192-214.

[47]. Charjan S.S. Massive String Cosmological model with perfect fluid distribution with Cosmological Term in Bimeric theory of Gravitation, International Journal of Scientific research in Science, Engineering and Technology ,March-April 2018 Vol 04 ,(4),pp.1602-1610.

[48]. Charjan, S. S., Dhongle P.R., and Borkar, M. S., (2019) Bianchi Type I Bulk Viscous Fluid String Dust Magnetized Cosmological Model with Λ-Term in Bimetric Theory of Gravitation , International Journal of Scientific Research in Mathematical and Statistical Sciences Vol.6, Issue.3, pp.35-40.

[49]. Gaikwad, N. P., Borkar M. S. and Charjan S. S., (2011), Bianchi Type I massive string magnetized baratropic perfect fluid cosmological model in bimetric theory of gravitation, Chin. Phys. Lett. 28(8).