International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)

Spherically Symmetric Quantities in Biometric Geometry

S. S. Charjan
Department of Mathematics, K. Z. S. Science College, Bramhani Kalmeshwar, Dist. Nagpur, India

Article Info
Volume 8, Issue 6
Page Number : 603-607
Publication Issue
November-December-2021
Article History
Accepted : 15 Dec 2021
Published : 30 Dec 2021

ABSTRACT

A geometry based on two metrics at each point of space is proposed purely from mathematical point of view. Defining the bimetrically Spherically Symmetric Scalars, Vectors and Tensors of order -2 are deduced in bimetric geometry.

Keywords: Bimetric Geometry, Spherical Symmetry

Mathematics Subject Classification 2020: 51-XX 83D-XX, 83F-XX

INTRODUCTION

As far as our knowledge goes there are some attempts of introducing two metric at each point of space; Rosen (1940), Eisenhart (1960) and Logunov and Mestvirishvili (1989). The geometry to that effect was utilized by Rosen (1940)in proposing theory Known as bimetric theory of gravitation or bimetric theory of relativity of Rosen, which differs from the general theory of relativity of Einstein (1912,1915) and which was later explored by many workers [Falik and Rosen (1980,1981), De Liebscer (1975), Yilmaz (1977), Goldman and Rosen (1977), Rosen and Rosen (1977), Israelit and Rosen (1983),Israelit (1979,1981), Rosen (1973,1974,1975,1979,1980,1983)]. This theory has some advantages over the general relativity: the quantities such as Chriostffel symbols and others becomes tensors which otherwise in Riemannian geometry they are not. Also on physical grounds there is no space for black holes. Our approach in constructing the bimetric geometry is rather different- it is purely mathematical. However, we shall be studying the impact of Rosen's bimetric theory with regard to our geometric ideas and the relevance of various bimetric quantities constructed in due course.

The concept of spherical symmetry (s. s.) in general relativity have been adopted from that of Euclidean geometry .Takeno (1966) has put the concept of spherical symmetry purely on a mathematical stand point. In his work the behavior of the spherically symmetric quantities is relegated to vanishing of Lie derivative and in consequence of which the forms of physical quantities (scalar, vector, tensor of rank 2) have been investigated.

In this paper , the attempt have been made to review this work of Takeno from the geometry of a bimetric theory of relativity proposed by Rosen (1940), and the related geometry contemplated by Karade and Nahatkar (2001).

DEFINITION.

A geometric quantity Ω is spherically symmetric with regard to bimetric geometry (or a quantity Ω is said to be bimetrically spherically symmetric) if

$$\nabla_{m(\xi)}\ \Omega = 0, \tag{1}$$

for $\xi = \xi^{ai}$, $(a = 1, 2, 3)$, where $\nabla_{m\,(\xi)}$ is the bimetric covariant derivative with respect to ξ and is given by $\nabla_{m(\xi)} = (g_m - f_m)_{(\xi)}$, $g_m = g$ -covariant derivative with respect to metric g_{ij} and $f_m = f$ -covariant derivative with respect to metric f_{ij} . In this definition, ξ^{ai} are the Killing vectors given by

$$\xi^{1i} = (0, \sin\phi, \cot\theta\cos\phi, 0),\quad \xi^{2i} = (0, -\cos\phi, \cot\theta\sin\phi, 0),\quad \xi^{3i} = (0, 0, -1, 0). \tag{2}$$

SPHERICALLY SYMMETRIC QUANTITIES OF THE SPHERICALLY SYMMETRIC SPACE-TIME V_4.

Scalar.

Following (1), the scalar A is bimetrically spherically symmetric if $\nabla_{m\,(\xi)}A = (g_m - f_m)_{(\xi)}\ \ A = 0$

$$\nabla_{m\,(\xi)}A = \ \xi^m \frac{\partial A}{\partial x^m} - \xi^m \frac{\partial A}{\partial x^m} = 0$$

$$\nabla_{m\,(\xi)}A = \ \xi^m \left(\frac{\partial A}{\partial x^m} - \frac{\partial A}{\partial x^m} \right) = 0 \tag{3}$$

and it is holds trivially, for any value of A. Hence the bimetric Spherically Symmetric scalar A can be taken as a function of r and t to match with Takeno (1966).

Covariant vector.

Following (1), the covariant vector A_i is bimetrically spherically symmetric if

$$\nabla_{m(\xi)}A_i = (g_m - f_m)_{(\xi)}\ \ A_i = 0$$

$$\nabla_{m\,(\xi)}A = \xi^m \frac{\partial A_i}{\partial x^m} + A_m \frac{\partial \xi^m}{\partial x^i} - \xi^m \frac{\partial A_i}{\partial x^m} = 0$$

$$\nabla_{m\,(\xi)}A = A_m \frac{\partial \xi^m}{\partial x^i} = 0\ . \tag{4}$$

$$\text{ie }\ A_m \frac{\partial \xi^{am}}{\partial x^i} = 0 \tag{5}$$

which on simplification gives

$$A_1.0 + A_2.0 + A_3.0 + A_4.0 = 0$$

$$-A_3 \cos ce^2\theta \cos\phi = 0 \qquad \Rightarrow A_3 = 0$$

$$A_2 \cos\phi - A_3 \cot\theta \sin\phi = 0 \quad \Rightarrow A_2 = 0$$

Thus the bimetrically spherically symmetric vector A_i have the form $A_i = (A_1, 0, 0, A_4)$ in which A_1 and A_4 are the functions of r, θ, ϕ, t. If one takes A_1 and A_4 independent of θ and ϕ, the result of Takeno (1966) is reclaimed.

Contravarient Vector A^i

Following (1), the contravarient vector A^i is bimetrically spherically symmetric if

$$\nabla_{m(\xi)} A^i = (g_m - f_m)_{(\xi)} \quad A^i = 0$$

$$\nabla_{m(\xi)} A = \xi^m \frac{\partial A^i}{\partial x^m} - A^m \frac{\partial \xi^i}{\partial x^m} - \xi^m \frac{\partial A^i}{\partial x^m} = 0$$

$$\nabla_{m(\xi)} A = .- \ A^m \frac{\partial \xi^i}{\partial x^m} = 0$$

ie $$\nabla_{m(\xi)} A_i = A^m \frac{\partial \xi^{ai}}{\partial x^m} = 0$$

$$A^m \frac{\partial \xi^{ai}}{\partial x^m} = 0 \qquad (6)$$

Which on simplification gives

$$\left.\begin{aligned} &A^1.0 + A^2.0 + A^3.0 + A^4.0 = 0 \\ &A^3 \cos\phi = 0 \Rightarrow A^3 = 0 \\ &A^2 (-\cos ce^2\theta \cos\phi) = 0 \Rightarrow A^2 = 0 \end{aligned}\right\}$$

Thus the bimetrically spherically symmetric vector A^i have the form $A^i = (A^1, 0, 0, A^4)$ in which A^1 and A^4 are the functions of r, θ, ϕ, t. If one takes A^1 and A^4 independent of θ and ϕ, the result of Takeno (1966) is reclaimed.

Covariant tensor of order-2.

In view of definition (1), covariant tensor A_{ij} of order-2 is bimetrically spherically symmetric if

$$\nabla_{m(\xi)} A_{ij} = (g_m - f_m)_{(\xi)} \quad A_{ij} = 0$$

$$\nabla_{m(\xi)} A = \xi^m \frac{\partial A_{ij}}{\partial x^m} + A_{mj} \frac{\partial \xi^m}{\partial x^i} + A_{im} \frac{\partial \xi^m}{\partial x^j} - \xi^m \frac{\partial A_{ij}}{\partial x^m} = 0$$

$$\nabla_{m(\xi)} A = A_{mj} \frac{\partial \xi^m}{\partial x^i} + A_{im} \frac{\partial \xi^m}{\partial x^j} = 0,$$

ie $$A_{mj} \frac{\partial \xi^{am}}{\partial x^i} + A_{im} \frac{\partial \xi^{am}}{\partial x^j} = 0, \qquad (7)$$

and then after simplifying, we get

$$A_{11}(0) + A_{21}(0) + A_{31}(0) + A_{41}(0) + A_{11}(0) + A_{21}(0) + A_{31}(0) + A_{41}(0) = 0$$

$$A_{12} = A_{21} = A_{13} = A_{31} = A_{22} = A_{23} = A_{32} = A_{24} = A_{42} = A_{33} = A_{34} = A_{43} = 0$$

(8)

Thus the bimetrically spherically symmetric covariant tensor A_{ij} have the following non-vanishing components: $A_{11}, A_{14}, A_{41}, A_{44}$. They can be treated as functions of r and t to get Takeno's analogy

MIXED TENSOR

In view of definition (1), mixed tensor A^i_j of order-2 is bimetrically spherically symmetric if

$$\nabla_{m(\xi)} A^i_j = (g_m - f_m)_{(\xi)} \quad A^i_j = 0$$

$$\xi^m \frac{\partial A^i_j}{\partial x^m} + A^i_m \frac{\partial \xi^m}{\partial x^j} - A^m_j \frac{\partial \xi^i}{\partial x^m} - \xi^m \frac{\partial A^i_j}{\partial x^m} = 0$$

$$A^i_m \frac{\partial \xi^m}{\partial x^j} - A^m_j \frac{\partial \xi^i}{\partial x^m} = 0,$$

ie $$A^i_m \frac{\partial \xi^{am}}{\partial x^j} - A^m_j \frac{\partial \xi^{ai}}{\partial x^m} = 0, \qquad (9)$$

$$A^1_1(0) + A^1_2(0) + A^1_3(0) + A^1_4(0) - A^1_1(0) - A^2_1(0) - A^3_1(0) - A^4_1(0) = 0 \qquad (10)$$

$$\begin{aligned} &A^2_1(0) + A^1_2(0) + A^1_3(0) + A^1_4(0) + A^2_2(0) + A^2_3(0) + A^2_4(0) + A^3_2(0) \\ &+ A^3_3(0) + A^3_4(0) + A^4_1(0) + A^4_2(0) + A^4_3(0) + A^4_4 = 0 \end{aligned} \qquad (11)$$

CONCLUSION.

The following main themes have been sorted out.

1. The bimetric spherically symmetric scalar A can be chosen as a function of r and t to mateh with Takeno (1966).

2. The bimetrically spherically symmetry vector A_i assume the form

A_i= (A_1, 0,0,A_4), where A_1 and A_4 are the functions of (r,θ,ϕ ,t) To reclaim the results of Takeno (1966) one has to take , A_1 and A_4 independent of θ and ϕ .

3. The bimetrically spherically symmetric covariant tensor A_{ij} have the following non-vanishing components: $A_{11}, A_{14}, A_{41}, A_{44}$. They can be treated as functions of r and t to get Takeno's analogy.

4. The bimetrically spherically symmetric mixed tensor A_i^j have the following non-vanishing components: $A_1^1, A_1^4, A_4^1, A_4^4$. They can be treated as functions of r and t to get Takeno's analogy.

Our investigation leads to the generalization of the Takeno's findings in the sense that the components under consideration comes out to be the functions of r, θ, ϕ, t. Also the major departure is with regard to the non-vanishing components as illustrated in the following table.

	Non-vanishing A_i	Non-vanishing A_{ij}
Takeno	A_1, A_4	$A_{11}, A_{22}, A_{33}, A_{44}, A_{14}, A_{41}$
Ours	A_1, A_4	$A_{11}, A_{14}, A_{41}, A_{44}$

ACKNOWLEDGEMENT

The author is grateful to Dr. M. S. Borkar, Ex-Professor, Post Graduate Teaching Department of Mathematics, R. T. M. Nagpur University, Nagpur for his helpful guidance.

REFERENCES

1. Karade T.M. and Nahatkar M. "On Bimetric Geometry" J. pro. Einstein Found. Int. 12, 20-30(2001).
2. Takeno H. "The Theory of Spherically Symmetric Space Times" Research Institute of Theoretical Physics, Hiroshima University, Japan (1966).
3. Rosen N.1940.Phy Rev, 57:147.