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Study of Fuzzy Topological Modelling of Fuzzy Games

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ABSTRACT

In this paper, we present about the study of fuzzy topological modelling of stochastic games. By introducing the concept of topological game over an idea of Hausdorff space, a game over some special product space is played. Fuzzy set theory has been applied to fuzzify some of the results obtained.
Keywords: Fuzzy Logic, Game Theory, Topological Game.

I. INTRODUCTION

By introducing the concept of topological game over an idea of Hausdorff space, a game over some special product space is played. Fuzzy set theory has been applied to fuzzify some of the results obtained. Over an idea of a topological space, Kumar B.P[2] has played a topological game which is explained here in brief. Also, by introducing the concept of rectangle in a topological product space, some special types of products called D-Product and C-Product are studied and a game is played over such products. Lastly, it is explained how fuzzy set theory can be applied to obtained better results.

II. TOPOLOGICAL MODELLING

Let G (I, X) be an infinite positional game of pursuit and evason over I where X is a topological space and I ⊂ P (X) s.t. (i) I is closed with respect to union (ii) I possesses hereditary property. Such collection I is called an ideal over X. This game is played as follows: There are two players-P (Pursuer) and E (Evader). They choose alternately consecutive terms of a sequence < En/n∈N, Where N = {0,1,2,…n,….)}> of subsets of X s.t. each player knows I, E_0, E_1, ….,E_n when he is choosing E_{n+1}.

A sequence < En > of subset of X is said to be a play of the game if for all n N the following holds:

(i) E_0 = X (ii) E_1, E_3, E_5, ……,E_{2n+1} are the choice of P.
(iii) E_1, E_3, E_5, ……,E_{2n+1} ∈ I.
(iv) E_2, E_4, E_6, ……,E_{2n+2} are the choice of E.
(v) E_1, E_2, ⊂ E_0, E3, E_4 ⊂E_2; ……,E_{2n+1}, E_{2n+2} ⊂ E2n
(vi) E_1 ∩ E_2 = ϕ, E_3 ∩ E4 = , ……..E_{2n+1} ∩ E_{2n+2} = ϕ.

If ∩< E_{2n} > = then player P wins the play, otherwise Evader wins the play.

A finite sequence < E_m/ m $\leq$ n > is admissible for the game if the sequence < E_0, E_1, ……E_n, ϕ,ϕ,ϕ,…..ϕ > is a play of the game. For admissible sequence < E_0,…..E_n> and even n if s: < E_0,…..,E_n > P(X) and s (<E_0,…..,E_n) = E_{n+1} then s is a strategy for player P.

In case of odd n, s is said to be strategy for evader E.

A strategy s is said to be wining for player P in the game G (I, X) if P wins each play of the game with the help of this s. Similarly, s is said to be winning for E if E wins each play of the game with the help of s.

We denote by P (I, X) the set of all winning strategies of P in the game G(I,X) and by E(I,X), the set of all winning strategies of E in the game G (I,X).

A topological space X is aid to be I-like if the set of all winning strategies of player is not empty i.e. if P (I,X) ≠ ϕ.

Similarly, a space X is said to be determined, if P (I,X) ≠ϕ or E (I,X) ≠ϕ i.e. if X is I-like or X is anti I-like.

A subset A x B of a topological product space X x Y is called a rectangle. A rectangle E is said to be:

(i) Cozero if E' & E" are cozero in X x Y;
(ii) Zero if E' & E" are zero in X x Y;
(iii) Open if E' & E" are open in X x Y;
(iv) Closed if E' & E" are closed in X x Y;

where E' & E" are the projections of E into X and Y respectively so that E = E' x E".

A topological product X x Y is said to be strong rectangular if each locally finite open cover of X x Y has a locally finite refinement by cozero rectangles.

From above definitions the following conditions are seen to be equivalent:

(i) The product X x Y is strongly rectangular.

(ii) Each finite open cover of X x Y has a locally finite refinement by by cozero rectangles.

(iii) For each closed subset F and each open set U of X x Y with F U, there is a locally finite collection w by cozero rectangles s.t. F ⊂∪ W U.

(iv) X x Y is normal and for each zero-set F and each cozero-set U of X x Y with FU, there is a locally finite collection W by cozero rectangles such that F ⊂∪ W U.

(v) There exists a continuous map

f: X x Y [0,1] such that f (x,y) = $\sum_{t\in T} g_t(x)h_1(y)$ where g_t : X→ [0,1] and h_t : Y→ [0,1] are continuous.

III. MODIFIED FUZZY GAMES

We define the topological games G (I,X) with a slight change as follows:

Each topological space considered in this paper is assumed to be a Hausdorff space. N denotes the set of all natural numbers and m denotes an infinite cardinal number. Also let L = {E_i | E_i are closed subsets of X}.

There are two players P and E. Player P chooses a closed set Et of X with E1 L and player E chooses an open set U_1 of X with $E_1 \subset U_1$.

Again, player P chooses a closed set E_2 of X with E_2 L and player E chooses an open set U_2 of X with $E_2 \subset U_2$ and so on.

The infinite sequence < E_1, U_1, E_2, U_2……> is play of G (L,X). Player P wins the play < E_1, U_2, E_2, U_2,…> if {Un : n N} covers X, otherwise player E wins.

A finite sequence < E_1, U_1,…., E_n, U_n > of subsets in X is said to be admissible for G(L,X) if the infinite sequence < E_1, U_1, …E_n, U_n, ϕ,ϕ,…..> is a play of G (L,X).

A function s is said to be a strategy for player P in G (L,X) if the domain of S consists of the void sequence ϕ and the finite sequence < U1, …..,Un > of open sets in X and if s (ϕ) ad s (U_1,……,U_n) are closed in X an belong to L.

A strategy s for player P in the game G (L,X) is said to be winning if he wins each play < E_1, U_1, E_2, U_2,….. in (L,X) such that E_1 = S (ϕ) and E_{n+1} = S (U_1, ….,U_n), for all n ∈ N.

We denote the following:

DL - The class of all spaces which have a discrete closed cover consisting of members of L.

FL - The class of all spaces which have a finite closed cover consisting opf members of L.

C - The class of all compact spaces.

C_m - The class of m-compact space.

I_1, I_2 - Arbitrary classes of spaces possessing hereditary property s.t.

$I_1 \times I_2 = \{X \times Y : X \in I_1$ and Y $I_2\}$

Firstly, we define the following two product spaces:

D- Product: A product space X x Y is said to be a D-product if for each closed set M of X x Y and each open set O of X x Y with M ⊂ O, there is a discrete collection J by closed rectangles in X x Y such that M ⊂∪ J⊂ O.

For a closed rectangle R in X x Y, R' and R" denote the projection of R into X and Y respectively. Thus, R is a closed rectangle in X x Y iff R' and R" are closed in X & Y and R is an open rectangle in X x Y iff R'R" are open in X and Y such that R = R' and R".

C-Product: A product space X x Y is said to be a C-product if for each closed set M of X x Y and each open set O of X x Y with M ⊂ O there is a countable collection J by closed rectangles in X x Y such that M ⊂∪ J ⊂ O.

With the help of definition of D-product, we have,

Theorem: (1) Let X and Y be spaces such that X x Y is a D-Product. If player P has winning strategies in G (l_1, X) and (l_2,Y), then he has a winning strategy in G (D (l_1 x l_2), X x Y).

Now we prove the following

Theorem: (2) Let X be a collection wise normal space and Y a subpar compact space with χ (Y) $\leq$ m. If player P has a winning strategy in G (DC_m, X), then every open cover of X x Y with power < m has a σ-discrete refinement by closed rectangles in X x Y.

Proof: Let s be a winning strategy of player P in G (DC_m, X). Let C be an arbitrary open cover of X x Y with |C| $\leq$ m.

We construct:

(i) a sequence $\{J_n : n > 0\}$ collections of closed rectangles in X x Y;

(ii) sequence $\{<\mathfrak{R}_n, <\psi_n> : n \geq 0\}$ of the pairs of collections Rn by closed rectangles in X x Y;

(iii) the function $\psi_n : \mathfrak{R}_n \rightarrow \mathfrak{R}_{n-1}$; satisfying the following five conditions:

(a) J_n is σ-discrete in X x Y.

(b) Rn is σ-discrete in X x Y.

(c) Each F ∈ J_n is contained in some G ∈ C.

(d) If (x,y) ∈ R_{n-1} ∈ $\mathfrak{R}_{n-1}$ and (x,y) ∈ ∪J/n.

Then there is $R_n \in \mathfrak{R}_n$ such that (x,y) ∈ R_n and $\psi_n(R_n) = R_{n-1}$.

(e) for an R ∈ $\mathfrak{R}_n$, Let U_n = X - R and U_k = X - $(\psi_{k+1}$ 0.......0 $\psi_n(R))'$, for $1 \leq k \leq n-1$.

Then the finite sequence < E_1, U_1, ..., E_n, U_n > is admissible for G (DC_m, X).

Let $J_0 = \{\phi\}$ and $\mathfrak{R}_0 = \{X \times Y\}$.

We suppose that the above $\{J_i : i < n\}$ and $\{< R_i, \psi_I > : I \leq n\}$ are already constructed. We pick an R ∈ $\mathfrak{R}_n$.

Let < E_1, U_1,......, E_n, U_n > be the admissible sequence in G (DC_m, X).

Hence there is a discrete collection {Cα : α ∈ Ω (R)} by m-compact closed sets in R1 such that s(U_1,......., U_n) R' = ∪ {Cα : α ∈ Ω (R)}.

We can a choose discrete collection {Wα : α ∈ Ω (R)} of open sets in R' s.t. Cα ⊂ Wα, for all α ∈Ω (R).

Since Cα is m-compact, |C| < m, χ(Y) $\leq$ m and R" is subparacompact.

There is a collection $J_{n+1}^{\alpha} = \{Cl\, U_{\lambda}^{\alpha,i} \times H_{\lambda}: i = l, \dots\dots, K_{\lambda}\ and\ \lambda \in \Lambda(k)\}$and by closed rectangle in R, which satisfying the following four conditions:

(i) Each $U_{\lambda}^{\alpha,i}$ is open in R'.

(2) $C_{\alpha} \subset \{U_{\lambda}^{\alpha,i}: i = 1, \dots\dots, K_{\lambda}\} \subset W_{\alpha}$.

(3) Each CI $U_{\lambda}^{\alpha,i} \times H_{\lambda}$ is contained in some G ∈ C.

(4) $\{H : \lambda \in \Lambda(\alpha)\}$ is a σ-discrete closed cover of R". Then

$J_{n+1}^{\alpha}(R) = \cup \{J_{n+1}^{\alpha}: \alpha \in \Omega\}$ is a -discrete in X x Y.

Put $R_{\lambda}^{\alpha} = \{CI\, W_{\alpha} - \cup\{U_{n+1}^{\alpha}: 1 \leq i \leq K_{\lambda}\} \times H_{\lambda}\}$, for all $\lambda \in \Lambda(k)$.

Again put $\bar{R} = (R' - \cup \{W_{\alpha} : \alpha \in \Omega\,(R)\}) \times R''$

Moreover, we put R_{n+1} (R) = { $\bar{R}$ ∪ $\{R_{\lambda}^{\alpha}: \lambda \in \Lambda(\alpha)\}$ and $\lambda \in \Lambda(R)\}$.

Then R_{n+1} (R) is also σ-discrete collection by closed rectangles in R.

We set $J_{n+1} = \{J_{n+1} (R) : R \in \Re_n\}$ and $\Re_{n+1} = \cup \{\Re_{n+1} (R) : R \in \Re_n\}$.

The function $\psi_n: \Re_{n+1} \rightarrow R_n$ defined as $\psi_{n+1} : (\Re_{n+1}) (R) = (R)$ for all $R \in \Re$.

From (a), J_{n+1} and $\Re_{n+1}$ are σ-discrete in X x Y.

The conditions (a) and (b) are satisfied.

By (3), the the condition (c) is also satisfied.

The conditions (d) and (e) are very clear.

Let $J = \cup \{J_n : n \in N\}$.

We can easily show that J is a cover of X x Y. There fore J is a σ-discrete refinement of C by closed rectangles in X x Y.

With the consequences of the above theorem and by assuming PC_m to be the class of all product spaces with the first factor being non-compact, the following results can be obtained easily.

(R_1) Let X be a collection wise normal space and Y and a sub paracompact space with $\chi(Y) \leq m$. If player P has a winning strategy in G (DCm, X), then X x Y is a

D-product.

(R_2) Let X be a paracompact space and Y be a sub paracompact space.

IF player P has a winning strategy is G (DC, X), then X x Y is sub paracompact.

(R_3) Let X be a collection wise normal space and Y be a sub paracompact space with $\chi(Y) \leq m$. If player P has a winning strategy in G (DCm, X), then he has a winning strategy in G (D(PCm$_m$), X x Y).

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