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Study of Fuzzy Stochastic Games

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ABSTRACT
In this paper, we present about the study of fuzzy stochastic games.
Keywords: Fuzzy Logic, Game Theory, Stochastic Game.

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I. INTRODUCTION

The topological games G (I,X) with a slight change as follows:

Each topological space considered in this paper is assumed to be a Hausdorff space. N denotes the set of all natural numbers and m denotes an infinite cardinal number. Also let L = {E_i | E_i are closed subsets of X}.

There are two players P and E. Player P chooses a closed set Et of X with E1 L and player E chooses an open set U_1 of X with $E_1 \subset U_1$.

Again, player P chooses a closed set E_2 of X with E_2 L and player E chooses an open set U_2 of X with $E_2 \subset U_2$ and so on.

The infinite sequence < E_1, U_1, E_2, U_2……> is play of G (L,X). Player P wins the play < E_1, U_2, E_2, U_2,…> if {Un : n N} covers X, otherwise player E wins.

A finite sequence < E_1, U_1,…., E_n, U_n > of subsets in X is said to be admissible for G(L,X) if the infinite sequence < E_1, U_1, …E_n, U_n, ϕ,ϕ,…..> is a play of G (L,X).

A function s is said to be a strategy for player P in G (L,X) if the domain of S consists of the void sequence ϕ and the finite sequence < U1, …..,Un > of open sets in X and if s (ϕ) ad s (U_1,……,U_n) are closed in X an belong to L.

A strategy s for player P in the game G (L,X) is said to be winning if he wins each play < E_1, U_1, E_2, U_2,….. in (L,X) such that E_1 = S (ϕ) and E_{n+1} = S (U_1, ….,U_n), for all $n \in N$.

We denote the following:

DL - The class of all spaces which have a discrete closed cover consisting of members of L.

FL - The class of all spaces which have a finite closed cover consisting opf members of L.

C - The class of all compact spaces.

C_m - The class of m-compact space.

I_1, I_2 - Arbitrary classes of spaces possessing hereditary property s.t.

I_1 x I_2 = {X x Y : $X \in I_1$ and Y I_2}

Firstly, we define the following two product spaces:

D- Product: A product space X x Y is said to be a D-product if for each closed set M of X x Y and each open set O of X x Y with $M \subset O$, there is a discrete collection J by closed rectangles in X x Y such that $M \subset \cup J \subset O$.

For a closed rectangle R in X x Y, R' and R" denote the projection of R into X and Y respectively. Thus, R is a closed rectangle in X x Y iff R' and R" are closed in X & Y and R is an open rectangle in X x Y iff R'R" are open in X and Y such that R = R' and R".

C-Product: A product space X x Y is said to be a C-product if for each closed set M of X x Y and each open set O of X x Y with $M \subset O$ there is a countable collection J by closed rectangles in X x Y such that $M \subset \cup J \subset O$. With the help of definition of D-product, we have,

Theorem: (1) Let X and Y be spaces such that X x Y is a D-Product. If player P has winning strategies in G (l_1, X) and (l_2,Y), then he has a winning strategy in G (D (l_1 x l_2), X x Y).

Now we prove the following

Theorem: (2) Let X be a collection wise normal space and Y a subpar compact space with χ (Y) $\leq$ m. If player P has a winning strategy in G (DC_m, X), then every open cover of X x Y with power < m has a σ-discrete refinement by closed rectangles in X x Y.

Proof: Let s be a winning strategy of player P in G (DC_m, X). Let C be an arbitrary open cover of X x Y with $|C| \leq m$.

We construct:

(i) a sequence $\{J_n : n > 0\}$ collections of closed rectangles in X x Y;

(ii) sequence $\{< \mathfrak{R}_n, < \psi_n > : n \geq 0\}$ of the pairs of collections Rn by closed rectangles in X x Y;

(iii) the function $\psi_n : \mathfrak{R}_n \rightarrow \mathfrak{R}_{n-1}$; satisfying the following five conditions:

(a) J_n is σ-discrete in X x Y.

(b) Rn is σ-discrete in X x Y.

(c) Each $F \in J_n$ is contained in some $G \in C$.

(d) If $(x,y) \in R_{n-1} \in \mathfrak{R}_{n-1}$ and $(x,y) \in \cup J_n$. Then there is $R_n \in \mathfrak{R}_n$ such that $(x,y) \in R_n$ and $\psi_n (R_n) = R_{n-1}$.

(e) for an $R \in \mathfrak{R}_n$, Let U_n = X - R and U_k = X - (ψ_{k+1} 0……0 $\psi_n(R)$)', for $1 \leq k \leq n-1$.

Then the finite sequence < E_1, U_1, ….., E_n, U_n > is admissible for G (DC_m, X).

Let $J_0 = \{\phi\}$ and $\mathfrak{R}_0$ = {X x Y}.

We suppose that the above $\{J_i : i < n\}$ and $\{< R_i, \psi_I > : I \leq n\}$ are already constructed. We pick an $R \in \mathfrak{R}_n$.

Let < E_1, U_1,……, E_n, U_n > be the admissible sequence in G (DC_m, X).

Hence there is a discrete collection $\{C\alpha : \alpha \in \Omega (R)\}$ by m-compact closed sets in R1 such that s(U_1,……., U_n) R' = $\cup \{C\alpha : \alpha \in \Omega (R)\}$.

We can a choose discrete collection $\{W\alpha : \alpha \in \Omega (R)\}$ of open sets in R' s.t. $C\alpha \subset W\alpha$, for all $\alpha \in \Omega$ (R).

Since $C\alpha$ is m-compact, $|C| < m$, $\chi(Y) \leq m$ and R" is subparacompact.

There is a collection $J_{n+1}^{\alpha} = \{Cl\, U_{\lambda}^{\alpha,i} \times H_{\lambda}: i = l, \dots\dots, K_{\lambda}\ and\ \lambda \in \Lambda(k)\}$and by closed rectangle in R, which satisfying the following four conditions:

(i) Each $U_{\lambda}^{\alpha,i}$ is open in R'.

(2) $C_{\alpha} \subset \{U_{\lambda}^{\alpha,i}: i = 1, \dots\dots, K_{\lambda}\} \subset W_{\alpha}$.

(3) Each CI $U_{\lambda}^{\alpha,i} \times H_{\lambda}$ is contained in some $G \in C$.

(4) $\{H : \lambda \in \Lambda(\alpha)\}$ is a σ-discrete closed cover of R". Then

$J_{n+1}^{\alpha}(R) = \cup \{J_{n+1}^{\alpha}: \alpha \in \Omega\}$ is a -discrete in X x Y.

Put $R_{\lambda}^{\alpha} = \{Cl\, W_{\alpha} - \cup\{U_{n+1}^{\alpha}: 1 \leq i \leq K_{\lambda}\} \times H_{\lambda}\}$, for all $\lambda \in \Lambda(k)$.

Again put $\bar{R} = (R' - \cup \{W_{\alpha} :\ \alpha \epsilon \Omega\ (R)\}) \times R''$

Moreover, we put R_{n+1} (R) = { $\bar{R} \cup \{R_{\lambda}^{\alpha}: \lambda \in \Lambda(\alpha)\}$ and $\lambda \in \Lambda(R)\}$.

Then R_{n+1} (R) is also σ-discrete collection by closed rectangles in R.

We set $J_{n+1} = \{J_{n+1} (R) : R \in \mathfrak{R}_n\}$ and $\mathfrak{R}_{n+1} = \cup \{\mathfrak{R}_{n+1} (R) : R \in \mathfrak{R}_n\}$.

The function $\psi_n: \mathfrak{R}_{n+1} \rightarrow R_n$ defined as $\psi_{n+1} : (\mathfrak{R}_{n+1})$ (R) = (R) for all $R \in \mathfrak{R}$.

From (a), J_{n+1} and $\mathfrak{R}_{n+1}$ are σ-discrete in X x Y.

The conditions (a) and (b) are satisfied.

By (3), the the condition (c) is also satisfied.

The conditions (d) and (e) are very clear.

Let $J = \cup \{J_n : n \in N\}$.

We can easily show that J is a cover of X x Y. There fore J is a σ-discrete refinement of C by closed rectangles in X x Y.

With the consequences of the above theorem and by assuming PC_m to be the class of all product spaces with the first factor being non-compact, the following results can be obtained easily.

(R_1) Let X be a collection wise normal space and Y and a sub paracompact space with $\chi(Y) \leq m$. If player P has a winning strategy in G (DCm, X), then X x Y is a

D-product.

(R_2) Let X be a paracompact space and Y be a sub paracompact space.

IF player P has a winning strategy is G (DC, X), then X x Y is sub paracompact.

(R_3) Let X be a collection wise normal space and Y be a sub paracompact space with $\chi(Y) \leq m$. If player P has a winning strategy in G (DCm, X), then he has a winning strategy in G (D(PCm_m), X x Y).

II. FUZZY STOCHASTIC GAME

A game is determined by information, decisions and golas. But human notions (ideas) and decisions are fuzzy. For, a man with immense entropy functions may err, set right and understanding a little may increase his understanding in the pursuit of some knowledge. Therefore, in a game, perfect information, decisions & goals may not be feasible. We are therefore, led to the introduction of fuzzy games.

Let G = (N,v) be a nonfuzzy game of the set N = {1,2,3,…, n} of n players in which $v : S \rightarrow R$ is a real valued function (characteristic function) from a family of coalition $S \subset N$ to the set of real numbers R. Hence v(A) means the gain which a coalition. A can acquire only through the action of A, the coalition A can be specified by the characteristic function τ^A as follows:

$$\tau^A (i) = \begin{cases} 1 & \text{if } I \in A; \\ 0 & \text{if } I \notin A. \end{cases}$$

A rate of participation τ^A (i) of a player i is defined by

τ^A (i) = 1, if a player i participates in A and

τ^A (i) = 0, if a player i does not not participate in A.

Consequently, a coalition A is represented by

= (τ^A (1), τ^A (2),….. τ^A (n)).

A fuzzy coalition τ is defined as a coalition in which a player I can participate with a rate of participation $\tau_I \in [0,1]$ instead of {0,1}. The characteristic function nor coalitional worth function of a fuzzy game is a real valued function f: $[0,1]^n \rightarrow R$ which specifies a real number f (τ) for any fuzzy coalition τ.

This fuzzy game is denoted by FG = (N. f).

III. REFERENCES

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