

Study of Fuzzy Structures Solutions Using the Simplex Method

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ABSTRACT

In this paper, we discussed fuzzy structure solutions using the simplex method with various techniques. The modeling and solving the optimization problems that come across our real life and business is one of the most important daily problems.

Keywords: Fully Fuzzy Linear Programming (FFLP), Linear Programming Problem (LPP).

I. INTRODUCTION

Linear programming has proved to be extremely useful for solving certain types of industrial problems for it provides a precise way of using statements of limitations such as “not more than” and “not less than” in mathematical computations. When applying OR methods to industrial problems, for instance, the problems to be modelled and solved are normally quite clear cut, well described, and crisp. They can generally be modelled and solved by using classical linear programming methods. If uncertainty occurs, it can be properly modelled using fuzzy theory. Linear programming is one of the most frequently applied operation research techniques. LPP is developing new approaches to fit better with real world problems. Classical LPP have crisp or well-defined constraints but in real life problems it is not the case. Constraints are not always well defined or clear or crisp. In such cases FFLP plays a very important role. Basically,

FFLP can be divided into two categories (a) FFLP problems with inequality constraints. (b) FFLP problems with equality constraints. These are some methods in which FFLP problem is converted into crisp linear programming problem and then obtained crisp LLP is solved to find the fuzzy optimal solution of the FFLP.

II. DEFINITIONS & TYPES OF FUZZY SETS

Fuzzy Set: If X is a universal set then a fuzzy set A on X is defined with the help of membership function μ_A which is a function defined by $\mu_A: X \rightarrow [0,1]$ it assigns grades to each element of set A and then a fuzzy set is expressed by ordered pairs $\{(x, \mu_A(x)): x \in X\}$ here the function μ_A is called membership function of fuzzy set A .

Types of Fuzzy Sets

Interval Valued Fuzzy Set: A fuzzy set whose membership function has the form

$\mu_A: X \rightarrow \in([0,1])$ where $\in([0,1])$ is family of all closed intervals of real numbers in $[0, 1]$.

Type-2 Fuzzy Set: A Fuzzy Set whose membership function has the form $\mu_A: X \rightarrow F([0,1])$ where $F([0,1])$ is fuzzy power set of $[0, 1]$.

L-Fuzzy Set: A fuzzy set whose membership function has the form $\mu_A: X \rightarrow L$ where L is a lattice or Partially Ordered Set.

Level-2 Fuzzy Set: A fuzzy set whose membership function has the form $\mu_A: F(X) \rightarrow [0,1]$ where $F(X)$ fuzzy power set of X .

Fuzzy Number: A fuzzy number is a fuzzy set of real line with normal, convex and continuous membership function of bounded support.

Triangular Fuzzy Number: A fuzzy number A is called triangular number with peak point 'a' left width $\alpha > 0$ and right width $\beta > 0$ if its membership

Functions has the following for $A(x) =$

$$\begin{cases} 1 - \frac{x-a}{\alpha} ; & a - \alpha \leq x \leq a \\ 1 - \frac{x-a}{\beta} ; & a \leq x \leq a + \beta \\ 0 & ; \text{ otherwise} \end{cases}$$

We use this notation as $A = (a, \alpha, \beta)$. This fuzzy number $A = (a, \alpha, \beta)$ is called nonnegative if if and only if $a > 0, \alpha > 0, \beta > 0$

Trapezoidal Fuzzy Number: A fuzzy set A is called trapezoidal fuzzy number with tolerance interval $[a, b]$, left width α and right width β if its membership has the following form $A(x) =$

$$\begin{cases} 1 - \frac{x-a}{\alpha} ; & a - \alpha \leq x \leq a \\ 1 & ; a \leq x \leq b \\ 1 - \frac{x-b}{\beta} ; & b \leq x \leq b + \beta \\ 0 & ; \text{ otherwise} \end{cases}$$

We use this notation as $A = (a, b, \alpha, \beta)$. This fuzzy number $A = (a, b, \alpha, \beta)$ is called non-negative if if and only if $a > 0, b > 0, \alpha > 0, \beta > 0$.

III. FUZZY LINEAR PROGRAMMING

The most general type of fuzzy linear programming problem is formulated as follows:

$$\text{Max } \sum_{j=1}^n C_j X_j$$

$$\text{Subject to } \sum_{j=1}^n A_{ij} X_j \leq B_i \quad (i \in N_m), X_j \geq 0 \quad (j \in N_n),$$

Where A_{ij}, B_i, C_j are fuzzy numbers and X_j are variables whose states are fuzzy numbers ($i \in N_m, j \in N_n$); the operations of addition and multiplication are operations of fuzzy arithmetic and \leq denote the ordering of fuzzy numbers.

There are two special types of fuzzy linear programming problems.

Case-I: Fuzzy linear programming problem in which only the right-hand side numbers B_i are fuzzy numbers:

$$\text{Max } \sum_{j=1}^n C_j X_j$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} X_j \leq B_i \quad (i \in N_m), X_j \geq 0 \quad (j \in N_n),$$

Case-II

Fuzzy linear programming problems in which the right-hand-side numbers B_i and the coefficients A_{ij} of the constraint's matrix are fuzzy numbers:

$$\text{Max } \sum_{j=1}^n C_j X_j$$

$$\text{Subject to } \sum_{j=1}^n A_{ij} X_j \leq B_i \quad (i \in N_m), X_j \geq 0 \quad (j \in N_n).$$

IV. METHODS FOR SOLUTION

Real world engineering systems are too complex to be defined in precise terms; imprecision is often involved in any engineering design process. Fully fuzzy systems have an essential role in this fuzzy, modelling, which can formulate uncertainty in actual environment. Mehdi Dehghan et al. worked in LR- Fuzzy numbers which were defined and used by Dubois and Prade with some useful and easy approximation arithmetic operations on them. They employ some heuristics-based methods on Dubois and Prade's approach, finding some positive vector x which satisfies $Ax=y$, where A and b are fuzzy matrix and a fuzzy vector respectively. They proposed some new methods to solve this system that are comparable to the well-known methods such as the Cramer's rule, Gaussian elimination, LU decomposition method and its simplifications. They developed a new method to solve square and non-square fuzzy systems. [1]

Since FFLP can be divided into two groups according to fuzziness of decision parameters and decision variables, so many researchers have given solutions by

assuming that decision parameters are fuzzy numbers while the decision variables are crisp numbers. Tanaka was the first to initially propose a method for solving the FFLP with fuzzy decision variables. It was observed by T. Allahviranlov et al. that most of the methods developed for fuzzy decision variables were based on application of Zadeh's Extension Principle. In these methods a FFLP is solved by some multiplication and special type of distance. T. Allahviranlov et al. proposed a new method for solving FFLP by applying the concept of comparison. They solved the FFLP by using kind of defuzzification method. The core of the nearest symmetric triangular fuzzy number is applied for an approximation of fuzzy numbers in objective function and coefficient matrix in the constraints. [2]

Any LP model representing real world situation involves a lot of parameters whose values are assigned by experts. However, both experts and decision makers frequently do not precisely know the value of these parameters. There for it is useful to consider the knowledge of expert about the parameters as fuzzy data introduced by Zadeh. Amit Kumar et al. proposed Mehar's Method for solving the special type of fuzzy linear programming problems. They proved that it is easy to apply as compare to the other methods for solving the same type of fuzzy linear programming problems. [3]

Lotif et al. proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints. This method can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers. If this method is to be applied on a FFLP in which the elements of coefficient matrix are non-symmetric triangular fuzzy numbers then firstly it is required to approximate the non-symmetric fuzzy numbers to the nearest symmetric fuzzy number, due to this conversion there were some limitations or shortcomings of these methods as the given solution of FFLP problem but these solutions do not satisfy the constraints. That is it was not possible to obtain the fuzzy number of the right hand side of

the constraint by putting the obtained solution in the left hand side of the constraint. To overcome the shortcomings of this method, Amit Kumar et al. proposed a new method for finding the fuzzy optimal solution of FFLP with equality constraints. They claimed that new method can overcome the shortcomings of existing method and perform better as compared to existing methods. [4] In this method firstly linear ranking function is used to convert the fuzzy objective function to the crisp objective function by doing this fuzziness of the objective function is neglected. [13] However this method proposed by Amit Kumar et al. is revised by H. Saberi et al. They studied this method and presented a revised method by imposing conditions of non-negative to the variables. [10] Abbas Hatami et al. used the reference of method proposed by Amit Kumar et al. for solving fuzzy linear programming in which the elements of coefficient matrix of the constraints are represented by real numbers and rest of the parameters are represented by fuzzy numbers. [11]

There are some methods in which FFLP is decomposed into three crisp linear programming problems (CLPP) with bounded variable constraints, then three CLPP are solved separately and using its solution the optimal solution of FFLP is obtained. In this method fuzzy ranking function and addition of non negative variables is not used. The solution obtained by this method exactly satisfies the constraints. There for bound and decomposition method can serve managers by providing an approximate best solution to a variety of linear programming models with fuzzy numbers in simple and effective way. [5]

Amit. K. et al. proposed a general method to solve FFLP problems in which all the parameters are flat fuzzy numbers positive or negative both signs. Earlier such type of problems has solutions only if parameters have positive sign. Their method worked better than other methods. [6] In 2013 Amit Kumar et al. proposed Mehar's method to solve FFLP in which

some or all the parameters are represented by unrestricted L-R flat fuzzy numbers and they proved that this method can provide solution. [7] Amit Kumar et al. also proposed Mehar's method to deal with sensitivity analysis of such fuzzy linear programming problems in which all the parameters are represented by LR flat fuzzy number. [8] Izaz Ullah Khan et al. used a modified version of well-known simplex method for solving FFLP problems. They used ranking method together with Gaussian elimination method. The algorithm proposed by them is flexible, easy and reasonable. They solved FFLP problem directly without converting it into crisp LPP problem. [9] The proposed scheme presented promising results from the aspect of computing efficiency and performance. [10-11]

V. REFERENCES

- [1] Mehdi Dehghan, Behnam Hashemi, Mehdi Ghate, Computational Methods for Solving Fully Fuzzy Linear Systems, Applied mathematics and Computational 179(2006)328-343.
- [2] T. Allahviranlov, F. Hosseinzadeh Lotif, M. Kh-Kiasary, N. A. Kiani and L. Alizadeh, Solving Fully Fuzzy Linear Programming problem by Ranking Function, Applied Mathematical Sciences, vol.2, 2008(19-32).
- [3] Amit Kumar, Jagdeep Kaur, A new method for solving fuzzy linear programming with trapezoidal fuzzy numbers, Journal of Fuzzy Set Valued Analysis, vol.2011, doi:10.5899/201/jfsva-00102.
- [4] Amit Kumar, Jagdeep Kaur and Pushpinder Singh, New method for solving Fully fuzzy Linear Programming Problems, Applied Mathematical Modelling, 35(2011)817-823.
- [5] M. Jayalakshmi and P. Pandian, A new method for finding an optimal fuzzy solution for fully fuzzy linear programming problems, vol. 2, issue-4, July-Aug 2012(247-254).
- [6] Amit Kumar & Jagdeep Kaur, General form of LPP with fuzzy parameters, Journal of Applied Research and Technology, 629-635, vol. 11, oct-2013.
- [7] Jagdeep Kaur, Amit Kumar, Mehar's Method for solving Fully Fuzzy Linear Programming Problems, Applied Mathematical Modelling 37(2013)7142-7153.
- [8] Neha Bhatia, Amit Kumar, Mehar's Method for solving fuzzy sensitivity analysis problems with LR flat fuzzy Numbers, Applied Mathematical Modelling, 36(2012)4087-4095.
- [9] Izaz Ullah Khan, Tahir Ahmed, Narmah Maan, A simplified novel technique for solving fully fuzzy linear programming problems, J option Theory Appl(2013)159:536-546.
- [10] H. Saberi Najafi, S. A. Edalatpanah, A note on "A new method for solving fully fuzzy linear programming problems", Applied Mathematical Modelling 37(2013)7865-7867.
- [11] Abbas Hatami, Hamed Kazemipoor, Solving fully fuzzy linear programming with symmetric trapezoidal fuzzy numbers using Mehar's Method, J. Math.Comput. Sci. 4(2014) No.2, 463-470, <http://scik.org>.