

# High Energy Electromagnetic Beam Interaction with Plasma having Thermal Nonlinearity : A Non-Paraxial Approach

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#### ABSTRACT

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Publication Issue May-June-2023 Article History Accepted : 05 May 2023 Published : 25 May 2023 This research paper uses a non-paraxial approach to explore the intrinsic relationship between high energy electromagnetic l beams and their interactions with plasma. The thermal nonlinearity leads to a self-action generated lens effect, wherein high-power electromagnetic beams create a refractive index profile across their cross-section that mirrors their intensity profile, thus enabling the beam to self-focus within a nonlinear plasma medium. In this approach, many paraxial approximations are dropped. This study encompasses a comprehensive examination of laser-matter interactions across plasma. With a keen focus on the practical importance of the lowest order of nonlinear terms, this research sheds light on how an intense electromagnetic beam influences the effective refractive index and the dielectric constant of the medium it passes through, fundamentally altering the propagation characteristics of the beam and enabling novel applications in optical technologies. Through theoretical analysis and mathematical modelling, this paper contributes to a deeper understanding of the intricate dynamics at play in nonlinear optics and paves the way for future innovations in the field. Keywords : Laser-Matter Interaction, Thermal Nonlinearity, Self-Focusing, Self-Trapping And Critical Power

#### I. INTRODUCTION

The manipulation of light through various media has been a topic of intense study within the field of optics, leading to significant breakthroughs in laser technology and its applications. Among these, the phenomenon of self-action in laser beams, mainly through the intensity-dependent index of refraction, presents a fascinating aspect of nonlinear optics [1-5]. This phenomenon, where a laser beam modifies the optical properties of the material it propagates through, resulting in self-focusing or defocusing effects, stands at the forefront of contemporary research. This paper delves into the core principles behind the self-action generated lens effect in laser beams and its implications for laser-matter interaction. This subject spans across multiple disciplines, including plasma physics, semiconductor research, and the study of dielectrics. In nonlinear media, the refractive index change elicited by a high-power

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electromagnetic beam creates a profile that matches the beam's own intensity profile, thereby allowing the beam to self-focus. This self-action effect underscores the dynamic interchange between light and medium, a cornerstone in studying lasers' interaction with matter.

When an intense laser beam with a radial intensity distribution propagates through plasma, nonlinearity arises due to the radial gradient of dielectric constant ( $\in$ ) dependence on the wave's electric field (E). Different nonlinear mechanisms occur, such as

- (A) Pondermotive force-induced nonlinearity
- (B) The thermal nonlinearity
- (C) The relativistic nonlinearity.

In this paper, we only confine thermal nonlinearity. This leads to the focusing or defocusing of the beam depending on the nature of the nonlinear medium in which the beam is propagating. Many theories of selffocusing (focusing on the beam due to self-action) have been developed and reported in the literature.

Most popular theories are based on the paraxial ray approximation [7-12]. These approximations are known to give significant errors in the critical power for self-focusing [13]. Realising that paraxial approximation may also be quantitatively in error in the saturation.

Furthermore, the investigation extends to the mathematical modelling of the nonlinear optical phenomena, focusing on the relationship between the intensity of the electromagnetic beam and its effect on the medium's refractive index and dielectric constant. This analysis is pivotal for understanding how these changes in optical properties can lead to practical applications, such as improving optical communication systems, the development of advanced sensors, and enhancing laser machining processes. By synthesising early studies and recent advancements in the field, this introduction sets the stage for a detailed exploration of the essential physics

behind the lens effect and the broader implications of laser-matter interactions within nonlinear optics. This paper aims to enrich the academic discourse on the subject and to further the potential for technological innovations leveraging the self-action generated lens effect of laser beams.

# (i) Basic equation of high intense electromagnetic wave in nonlinear plasma

The wave equation for an electromagnetic wave with electric field vector (E) when propagates in a nonlinear medium is written as

$$\nabla^2 E - \frac{\varepsilon}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \tag{1}$$

Where the effective dielectric constant of the medium is in the presence of the incident beam, it can be written as

$$\varepsilon_{\text{eff}} = \varepsilon_{\text{L}} + \varepsilon_{\text{NL}} < EE >$$
 (2)

But the intensity distribution at Z=0 is in Gaussion form, for which the value of E is given as

$$E^{2} = E_{0}^{2} \exp(-r^{2}/r_{0}^{2})$$
(3)

Here, the incident beam is supposed to be a singlemode laser beam with a Gaussian transverse profile. Thus, the time average of the electric field amplitude  $\langle E^2 \rangle$  It is replaced by E<sub>0</sub><sup>2</sup>/2.

The remaining portion of the medium where the beam is not interacting will have

dielectric constant  $\varepsilon_L$ . The axial (central) part of the beam having higher intensity should experience an extensive refractive index than the edge for the medium with a positive value of  $\varepsilon_{NL}$ . Consequently, the plane wavefront of the wave is progressively more distorted and as the wave propagates in the medium, it bends towards the propagation axis. It undergoes focusing, and the medium behaves like a lens. Consider a particular incident ray of the wave that makes an angle with the axis will suffer total internal



reflection. However, rays with the beam self-focus only when the refraction effect is more effective than the diffraction effect.

As a result, the dielectric constant inside the beam is higher than outside. This effect is nonlinear and causes the plasma to act as a convex lens for the beam, focusing the beam to a smaller diameter.

#### (ii) COLLISIONAL HEATING

This type of nonlinearity is dominant in weakly ionised plasma. When the duration of the electromagnetic wave is much higher than the energy relaxation time (T>>T), the nonlinearity appearing through the heating of carriers is much more critical than the ponderomotive force effect. This is the case with many laboratory and ionosphere plasmas. This type of nonlinearity arises from the heating of plasma by wave.

When an electromagnetic wave propagates through plasma, the electron acquires momentum and energy from the wave, which is lost in collision with ions and neutral particles. The mechanism for the lens effect for heating by collision is described as follows.

In the absence of a propagating wave in the medium, the temperature of electrons is the same as that of the heavy particles. Hence, the net energy exchange between electrons and heavy particles is zero. While in the presence of the electric field of the wave, the electrons gain energy from the field and, in a steady state, attain a temperature higher than that of the heavy particles, although the rate of power absorbed by electrons from the field becomes equal to the rate of power lost by electrons in collision [14]. If the incident wave has a nonuniform intensity distribution, the carrier temperature is also nonuniform, and corresponding electron concentration variation occurs in the plasma, and the plasma becomes the inhomogeneous medium. The space charge field generated in this process also makes ions follow the electrons. Due to this redistribution of charge carriers, a transverse gradient of the dielectric constant is established, leading to the wave's lens effect. This is

responsible for thermal nonlinearity, as discussed below.

#### (iii) THERMAL NONLINEARITY

As already discussed earlier, thermal nonlinearity dominates in the plasma medium when the time period of the laser pulse is higher than the energy relaxation time (t >> Te). In collisional plasma, the primary source of effective dielectric constant is the nonuniform redistribution of charge carriers due to the energy of propagating electromagnetic waves. The carriers are redistributed in the plasma due to their inhomogeneous heating resulting from the transverse variation of the electric field along the wavefront of the propagating beam.

Gaussian form of electromagnetic wave propagating in plasma has been considered.

The axial component of the wave vector is given as

$$\Psi^{2} = \Psi_{0^{2}}(z) \exp((-r^{2})/(a^{2}(z)))$$
(4)

Where  $\Psi_0(z)$  is the axial amplitude of the wave, and an (x) is the effective size of the wave at z coordinate. According to equation (4.), it is concluded that due to high intensity near the axis, charge carriers of plasma are more heated than those away from it. Due to this, a temperature gradient occurs along the transverse direction and carriers' redistribution occurs.

#### (D) NONLINEAR DIELECTRIC CONSTANT

A detailed investigation of carriers redistribution and nonlinear dielectric constants based on kinetic theory was efficiently conducted by Sodha et al.[67].

The redistribution of carrier concentration strongly depends on the nature of the collision, as

$$\frac{N_e}{N_0} = \left(\frac{2T_0}{T_0 + T_e}\right)^{1-\frac{s}{2}}$$
(5)

Ne, N0 represents the electron and equilibrium carrier concentration respectively and Te , T0 represents the corresponding temperatures.

The drift velocity (v) acquired by the electrons of plasma due to beam-plasma collision is given as

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$$\mathbf{v} = -\frac{(\nu - i\omega)}{m\,\omega^2}\,\mathbf{e}\mathbf{E}\tag{6}$$

where  $\nu$  is the electron-heavy particle collision frequency.

The difference between the temperature of the electron and equilibrium is given as

$$T_{e} - T_{0} = \frac{e^{2}MEE^{*}}{6m^{2}k_{0}\omega^{2}}$$
(6)

and effective electric displacement vector for such a case is given as

$$D_{\rm eff} = \epsilon_{\rm eff} E \tag{7}$$

Using equations (1-4), the effective dielectric constant for such non-absorbing collisional plasma can be written as [15]

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^2}{\omega^2} \left[ 1 - \left\{ \frac{1}{1 + \alpha \frac{LEE^*}{2}} \right\}^{1 - \frac{5}{2}} \right]$$
(8)

In plasma, various collision processes can occur, depending on the nature of the medium. These can be classified as

(a) For weakly ionised laboratory plasmas, where electron-neutral particle collision takes place, the collision parameter (s) value is found to be one, i.e. s = 1

(b) In the case of electron-diatomic molecule collision, the value of s = 2

(c) For the electron-ion collision process, s = -3

Using equation (8) the nonlinear part of the dielectric constant is given as

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2} \left[ 1 - \left\{ \frac{1}{1 + \alpha \frac{EE^*}{2}} \right\}^{1 - \frac{5}{2}} \right]$$
(9)

The nonlinear part of dielectric constant is calculated for different types of plasma (s =-1 and s = -3) at different values of incident intensity parameter ( $\alpha E_0^2$ ) and plotted in Figure (1).



Fig. 1 graph between nonlinear part of dielectric constant and intensity parameter

Variation of the nonlinear part of dielectric constant (8) of the collisional plasma with intensity parameter of the incident beam (E). Curve A for the case of electron- ion collision mechanism (S-3) and Curve B for the electronneutral particle collision (S-1).

It is observed that the nonlinear part of the dielectric constant attains saturated value for both types of collision in plasma. At the same time, it is also concluded that for large value of intensity parameter ( $\alpha E_0^2$ ),  $\varepsilon_{NL}$  attains a higher saturated value in case of electron- ion collision mechanism (s - 3) as compared to that for electron- neutral particle collision s = 1)

## (5) EQUATION FOR LENS EFFECT USING NON-PARAXIAL TECHNIQUE IN COLLISIONAL PLASMA

to discuss the self-focusing parameter of beam in collisional plasma for entire intense electromagnetic beam following main features are used

- (1) Paraxial approximation are dropped
- (2) To analyse dimensionless beam width parameter (f) is used for effective focusing and defocusing of beam
- (3) The focusing parameter (f) depends on transverse radial distance r and axial coordinate z

The intensity distribution in medium at any distance z may be given as

$$I = A_0^2(r,z) = (E_0^2/f^2(z)) \exp(-r^2/r_0^2f^2(z))$$

Where  $r_0 f(z)$  represents the spot size of beam at any axial distance z.

If  $\omega_p$  is the plasma frequency in the absence of laser beam and  $\alpha$  represents the charestric parameter depends on frequency of incident beam and  $k = \omega \sqrt{\acute{\epsilon}/c}$  is propagation constant of beam.

Considering the present non paraxial technique developed by the author, the equation for self generated lens effect of the beam in collisional homogeneous plasma has been obtained by substituting the value of nonlinear part of dielectric constant from equation (9) into equation (1) and given as

$$\frac{d^2 f(z)}{dz^2} = \frac{2}{k^2 r_0^2 r^2 f(z)} - \frac{1}{k^2 r_0^4 f^3(z)} - \frac{f(z)}{r^2} \frac{\omega_p^2}{\omega^2} \frac{1}{\epsilon_L} \left[ 1 - \left(\frac{1}{1 + \alpha E E^*}\right)^{1 - \frac{S}{2}} \right] 10$$

Using Runge-Kutta method above equation has been solved with the help of computer programming and the numerical results are presented and studied. These results are compared with those of paraxial ray approximation method [15].



# Fig. 2: The variation of normalised self-focusing parameter (f) with propagation distance (z) in collisional plasma. Curve A: present nonparaxial analysis and Curve B: paraxial ray approach.

The variation of the focusing parameter in plasma due to lens effect is plotted in Figure (2) with axial distance z and a comparative study has been done with paraxial ray approximation obtained by using same parameters as used in the present analysis. It is predicted from the Figure (2) that the focusing parameter f (z) shows oscillatory behaviour during propagation of the beam in collisional plasma. The axial distance which corresponds to minimum value of the focusing parameter (f) is large in comparison to the paraxial results and the minimum value of the focusing parameter (f) is smaller in comparison to the value corresponding to paraxial results. This discrepancy in the results is observed because entire transverse portion of the beam has been considered in present approach in place of only small portion of beam used in paraxial theory.

# CONCLUSION

The present analysis for the self-action of the laser beam in thermal nonlinear plasma using nonparaxial approach considering entire spatial part of the laser beam yields interesting results. The nonlinearity of the plasma depends on the characteristic length as well as In the electron charge density profile function. The results prove that the effective nonlinear- density involved in plasma depends on inhomogeneity of the plasma as well as on the nonuniform distribution of the intensity of the incident beam [15]. The results for normalised self-focusing and tensity show irregular no oscillatory behaviour for inhomogeneous plasma. The results of the present analysis are supposed to be accurate and near to al life situation.

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