

Thresholds for Light Confinement : Theoretical Insights into Self-Trapping of the Entire Spatial Laser Beam in the Nonlinear Medium

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ABSTRACT

Article Info Volume 9, Issue 3	A theoretical study for forming self-trapping solitons in the nonlinear medium using the entire transverse spatial characteristics of the intense beam is analysed. Using the poplinear Schrödinger equation (NLSE) as the basic equation
Page Number : 133-139	osnig the holimical semounger equation (NLSL) as the basic equation,
Publication Issue	nonlinearity in various nonlinear mediums are derived using the nonparaxial
May-June-2022	approach in other media. The results obtained for self-guided beam formation are
Pages : 606-611	compared with well-known paraxial theory, providing insights for applications in
Article History	optical communication and photonic devices.
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1. INTRODUCTION

Self-focusing and self-trapping are significant in nonlinear optics, with applications ranging from optical communication to advanced photonic devices.

Self-trapping is a fascinating phenomenon in nonlinear media, where an intense beam maintains its shape while propagating through a medium due to a balance between diffraction and nonlinearity. This balance allows the wave to localise in space, effectively "trapping" itself. It is observed in various nonlinear mediums like optical fibres, dielectric medium and plasma.[1-7]

An electromagnetic beam can produce its own dielectric waveguide and propagate without spreading. This may occur in materials whose dielectric constant increases with field intensity but are pretty homogeneous in the absence of the electromagnetic wave. Such self-trapping in dielectric waveguide modes appears to be possible in intense laser. Such self-trapping in self-made waveguide modes, also known as solitons, are a manifestation of self-trapped light waves that maintain their shape during propagation in optical fibres[8-15]

A new approach using entire spatial characteristics, also known as the nonparaxial approach for nonlinear Schrodinger equation for intense electromagnetic beam wave propagation in plasma, is considered by dropping many approximations as used in the known paraxial approach[7,8,14].



2. DIELECTRIC CONSTANT

The concept of an effective dielectric constant in the presence of a beam, particularly a high-intensity optical beam, arises from the nonlinear optical properties of a medium. In nonlinear optics, a material's refractive index depends on the intensity of the light passing through it, leading to a modification of the material's dielectric constant. The refractive index in a nonlinear medium can be expressed as:

$$\mathbf{\mathcal{E}} = \mathbf{\mathcal{E}} \mathbf{L} + \mathbf{\mathcal{E}} \mathbf{N} \mathbf{L} < (\mathbf{E} \mathbf{E}^* >)$$
(1)

Where ϵ_L is the linear refractive index, ϵ_{NL} is the nonlinear refractive index coefficient, and <(EE'>) is the average value of the intensity of the optical beam.

Using Maxwell's equations for the electromagnetic field, the nonlinear Schrodinger equation for self-focusing using non - paraxial approximation method avoiding Taylor series expansion of dielectric constant and dropping various approximations as used in paraxial ray approximation is written as[17]

$$\frac{d^2 f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^2 f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} - \frac{f(z) \varepsilon_{NL}}{r^2 \epsilon_L} \cdot \langle \boldsymbol{E}\boldsymbol{E}^* \rangle$$
(2)

Where

$$\frac{d^2f(z)}{dz^2}$$

- represents the second derivative of the function f(z)) concerning z.
- f(z) represents the dimensionless beamwidth parameter of an optical beam along the propagation direction z.
- $\frac{2}{\kappa^2 r_0^2 r^2 f(z)} \frac{1}{\kappa^2 r_0^A f^3(z)}$
 - \circ $\;$ These terms represent diffraction term which depends on several factors:
 - \circ $\;$ Characteristic scales related to the wave number and initial beam width or curvature.
 - o r² denotes a radial coordinate or some measure related to beam radius.
 - \circ f(z) in the denominator suggests that this effect weakens as f(z) increases.
- $\frac{f(z) \varepsilon_{NL}}{r^2 \epsilon_L} \cdot \langle \boldsymbol{E} \boldsymbol{E}^* \rangle$

This represents a nonlinear effect related to a self-focusing or defocusing process, such as a pondermotive relativistic, Kerr nonlinearity,

• $\epsilon_{NL} \langle EE^* \rangle$

(EE*) is the time-averaged intensity of the electric field, E is the electric field vector and E* is its complex conjugate, which is responsible for variation in the nonlinear part of the dielectric constant.

3. SELF TRAPPING

During the propagation of an intense laser beam in a nonlinear medium, the diffraction effect continuously competes with the focusing effect, governing the propagation characteristics of the beam and its dynamics.[1-8] Here, the condition under which an electromagnetic beam can produce its own dielectric waveguide and propagate without spreading is known as uniform waveguide propagation has been discussed[15]

. Such self-trapping or diffraction-less propagation in dielectric waveguide mode is possible for intense laser beams where the dielectric constant depends on the beam's intensity.



Figure 1. Self-trapping of the beam when convergence due to the dielectric effect is equal to the divergence due to the diffraction effect. The parallel lines show the self-generated uniform waveguide in a nonlinear medium.

It can be explained by the following equation (2). One can conclude that when the diffraction divergence of the laser beam is precisely balanced by the focusing effect in the medium due to different types of nonlinear effects, the beam propagates in a self-trapped waveguide mode without convergence or divergence.

Thus, for an initial plane wavefront of the beam, at z = 0, f = 1 $r = \rho$ This leads to a condition where the terms in the right-hand side of equation (2) cancel each other

In this situation, the beam width of the laser does not change during propagation in the nonlinear medium. In other words, the beam propagates without convergence or divergence or in the self-trapped mode.

$$[\rho^{2} = k^{2} r_{0}^{4} \left(\frac{2}{k^{2} r_{0}^{2}} - \varepsilon_{NL} \langle EE^{*} \rangle \right)]$$
(3)

On applying these conditions in equation (2), one

$$\frac{\omega_p \rho}{c} = (2r_0^2 - \rho^2)^{1/2} \frac{\rho}{r_0^2} [\varepsilon_{NL} \langle \boldsymbol{E}\boldsymbol{E}^* \rangle]^{-1/2}$$
(4)

From the above equation for self-trapping, one can conclude that the normalised radius of the self-trapped beam($\frac{\omega_p \rho}{c}$) depends on the nonlinear part of the dielectric constant of the medium, which is $\varepsilon_{NL} \langle EE^* \rangle$ developed due to intensity EE^* of the beam

Some fundamental nonlinearities in plasma are as follows

(i) Pondermotive nonlinearity in plasma

The nonlinear part of the dielectric constant for pondermotive nonlinearity

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2} \left[1 - \exp\left[-\frac{3m}{4M} \alpha E_0^2 \exp\left\{ -\frac{r^2}{r_0^2} \right\} \right] \right]$$
(5)

(ii) Collisional nonlinearity in plasma

The nonlinear part of the dielectric constant for collisional nonlinearity is given as

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2} \left[1 - \left\{ \frac{1}{1 + \alpha \frac{EE^*}{2}} \right\}^{1 - \frac{S}{2}} \right]$$
(6)

Where (a) For weakly ionised laboratory plasmas, where electron-neutral particle collision takes place, the collision parameter (s) value is found to be one, i.e. s = 1

(b) In the case of electron-diatomic molecule collision, the value of s = 2

(c) For the electron-ion collision process, s = -3

(iii) relativistic nonlinearity in plasma

The nonlinear part of the dielectric constant for pondermotive nonlinearities is given as

$$\varepsilon_{NL} = \frac{\omega_p^2}{\omega^2} \left[1 - \left\{ 1 + \alpha \, \frac{EE^*}{2} \right\}^{-\frac{1}{2}} \right] \tag{7}$$

(iv) Kerr nonlinearity in a dielectric medium

The nonlinear part of the dielectric constant for Kerr nonlinearity is given as

$$\varepsilon_{NL} = \varepsilon_s \left[1 - exp \left\{ -\frac{\varepsilon_2}{\varepsilon_s} \right\} < EE > \right]$$
(8)

4. WAVEGUIDE MODE PROPAGATION OF LASER BEAM: SOLITON FORMATION

Using the self-trapping equation for a nonlinear medium (4) and different types of nonlinearity, the nonlinear part of dielectric constant, as mentioned in equations (5 to 8), different equations for self-trapped radius for rays incoming into a nonlinear medium.



$$\frac{\omega_p \rho}{c} = (2r_0^2 - \rho^2)^{1/2} \frac{\rho}{r_0^2} \left[1 - \exp\left[-\frac{3m}{4M} \alpha E_0^2 \exp\left\{ -\frac{r^2}{r_0^2} \right\} \right] \right]^{-1/2}$$
(9)

Where $\frac{\omega_p \rho}{c}$, is a normalised self-trapped radius of an intense beam in pondermotive nonlinear plasma. and $\frac{3m}{4M} \alpha E_0^2$, is the intensity parameter, which is the function of the intensity of the incident beam.

For different initial sizes (ρ), the trapped radius has been calculated numerically using the nonparaxial approach for different values of intensity parameter and plotted as given below



Variation of Self trapped radius v/s intensity parameter

Figure(2)

Normalised self - trapped radius of beam ($\omega_p r_0/C$) dependence on beam intensity parameter (βE_0^2) for different initial transverse radial distance (ρ) using new nonparaxial analysis. Here $\omega_p = 2.5 \times 10^{13}$ rad / sec., $\omega = 1 \times 10^{14}$ rad / sec., and $r_0 = 30 \mu m$.

Many other methods based on different approaches, like the moment method based on the invariants of nonlinear Schrödinger equation (NLSE) such as moment method developed initially by Vlasov et al. [10] and later generalised by Lam as well as the variation method of Anderson[16] has proved to be quite helpful in estimating the self-focusing effect. These methods are supposed to be equivalent to a corrected paraxial theory because they intrinsically take care of the approximations.

The equation for self-trapped radius calculated by variational method[16] is given as

$$\frac{\omega_p \rho}{c} = \left[\left\{ -\frac{\beta E_0^2}{E(\beta E_0^2) + \exp(-\beta E_0^2) - 1} \right\} \right]^{1/2}$$
(10)

From the paraxial approximation method, the self-trapped radius (21) is written as

$$\frac{\omega_p \rho}{c} = \left\{ \frac{\exp(0.5\beta E_0^2)}{E(0.5\beta E_0^2)} \right\} \tag{11}$$

The variation of the dimensionless normalised self-trapped radius with intensity parameter is plotted for present analysis and for variational method with paraxial ray method as shown in figure(3)



Figure (3)

The decadence of normalised self-trapped radius for pondermotive nonlinearity on intensity parameter. Curve (A) for the present nonparaxial approach, Curve(B) for the variational method And Curve(C) for the paraxial approximation method

5. CONCLUSION

Considering the entire spatial part of the incoming laser beam for self-actions, such as self-focusing self-trapping in a nonlinear medium, the present nonaxial approach provides exciting results. Other methods are supposed to be equivalent to a corrected paraxial theory because they intrinsically take care of the approximations. The fact that the results for self-trapping, like the present analysis, predict a self-made waveguide is reasonably consistent with the values estimated by moments, variational theory and numerical calculations, makes the present study also interesting. Higher intensity values show that the medium behaves like a self-guided wave guide known as soliton formation in a dielectric medium mainly used in optical communication. Optical fibres are used to transmit information over long distances with minimal signal degradation. The stable, localised nature of solitons helps maintain the data's integrity. Self-trapping of plasma waves can lead to the formation of stable structures in plasmas, which are essential for understanding wave propagation and energy transfer in space and laboratory plasmas.

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