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Critical Power of Intense Laser Beams in Nonlinear Media : A Comparative Study for Paraxial and Nonparaxial Approach

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ABSTRACT

Self-focusing actions have central importance for most nonlinear optical effects. The critical power for self-focusing is commonly theoretically without considering a material's absorption. This paper aims to comprehensively review the critical power of intense laser beams in nonlinear media—numerical results based on the nonlinear wave equation. The critical power is a key parameter in nonlinear optics that determines when significant nonlinear effects occur using the nonlinear Schrodinger equation(NLSE), which is the fundamental equation of nonlinear optics, a nonparaxial approach is used for the mathematical formulation of critical power by dropping many approximations. Critical power is numerically calculated without any series expansion and compared with the result obtained by the popular paraxial approach.
Keywords: Nonlinear Optics, Nonlinear Schrödinger Equation, Self-Trapping, Solitons.

1. INTRODUCTION

Lasers have revolutionized numerous scientific and industrial fields, from telecommunications to medical imaging. One critical aspect of laser technology is the behaviour of intense laser beams when they propagate through nonlinear media like plasma, dielectric medium and fiber[1-5]. Nonlinear optics, a field that explores how light interacts with matter in ways that depend on the light intensity, is known as self-action. Among these phenomena, the concept of critical power holds particular importance. The critical power of a laser beam is the threshold at which the nonlinear effects become pronounced, leading to various optical phenomena such as self-focusing, self-phase modulation, and filamentation. Understanding and controlling this critical power is essential for optimizing the performance of laser systems in various applications.

This paper aims to comprehensively review the critical power of intense laser beams in nonlinear media. It will cover the fundamental principles of nonlinear optics and explore the mathematical formulations of critical power for paraxial and new nonparaxial method. A new approach that considers the entire spatial propagation of laser beams in a medium for calculating critical power is used.

Furthermore, in this paper, a comparative study is performed. This exploration seeks to elucidate the pivotal role of critical power in advancing laser technology and its applications.

2. FUNDAMENTAL EQUATION OF INTENSE LASER BEAM IN NONLINEAR MEDIUM

The present paper for steady-state self-action assumes that nonlinearity does not depend on time, so the moving focus phenomena [19] are excluded when the steady state self focusing equation is used.

(i) The nonlinear Schrodinger equation

Maxwell's equations are basic equations for electromagnetic fields. Using these, the wave equation for the electric field (E) is written as a beam width parameter differential equation for self-focusing:

Wave equation governing the electric vector (E) of the propagating beam in the nonlinear medium can easily be obtained by solving Maxwell's equations and may be written [1-4]

$$\nabla^2 E - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \tag{1}$$

Where ε is the effective dielectric constant of a nonlinear medium.

Which can be written as

$$\varepsilon = \varepsilon_L + \varepsilon_{NL} \langle(EE^*\rangle) \tag{2}$$

Where ε_L is the linear refractive index, ε_{NL} is the nonlinear refractive index coefficient, and $\langle(EE^*\rangle)$ is the average value of the intensity of the optical beam. For an azimuthally symmetric beam, the wave equation is written as

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial \mathrm{r}} + \frac{\partial^2 E}{\partial z^2} = (\epsilon_{\mathrm{L}} + \epsilon_{NL})\frac{\partial^2 E}{\partial t^2} \tag{3}$$

The general solution of the above equation is given as

$$[\mathrm{E} = \mathrm{A(r,z)}\exp\{\mathrm{i}[\kappa \mathrm{z} - \omega \mathrm{t}]\}] \tag{4}$$

Where k is the wave propagation constant of the beam

$$\mathrm{k} = \frac{\omega\sqrt{\epsilon_{\mathrm{L}}}}{c} \tag{5}$$

and complex amplitude

$$\mathrm{A(r,z)} = \mathrm{A_0(r,z)}\exp(-\mathrm{ikS(r,z)}) \tag{6}$$

the equation of propagating in the medium is written as

$$2\mathrm{ik}\frac{\partial A}{\partial \mathrm{z}} + \left(\frac{\partial^2 \mathrm{A}}{\partial \mathrm{r}^2} + \frac{1}{\mathrm{r}}\frac{\partial \mathrm{A}}{\partial \mathrm{r}}\right) + \frac{\partial^2 E}{\partial z^2} = \mathrm{k}^2 \mathrm{A} - \frac{\epsilon_{\mathrm{L}}\omega^2}{c^2}A - \frac{\epsilon_{\mathrm{NL}}\omega^2}{c^2}A \tag{7}$$

The above equation is parabolic and primarily used in open optical resonators[21]. Using equation (5), we can rewrite

$$-2\left(\frac{\partial S}{\partial z}\right)+\left(\frac{\partial S}{\partial r}\right)^2=\frac{1}{k^2A_0}\left(\frac{\partial^2 A_0}{\partial z^2}+\frac{1}{r}\left(\frac{\partial A_0}{\partial r}\right)\right)+\left(\frac{\epsilon_{NL}}{\epsilon_L}\right) \quad (8)$$

The wave eikonal S is given as

$$S=\frac{r^2}{2}\beta(z)+\emptyset(z) \quad (9)$$

And

$$\beta(z)=\frac{1}{f}\frac{df}{dz} \quad (10)$$

where f(z) is known as the dimensionless beam width parameter, which is associated with the beam's self-focusing behaviour within the medium during propagation in the z-direction.

Let us consider the initial intensity distribution of the laser beam at the vacuum-nonlinear medium interface (i.e. z =0) along the radial direction is of Gaussian form, which is in a practical situation and given as

$$A_0^2(r,z)=E_0^2 exp-\left(\frac{r^2}{r_0^2}\right) \quad (11)$$

The laser beam is propagating along the z-direction in the nonlinear medium, and its intensity distribution in the medium at any axial distance, z, may be given by

$$A_0^2\,(r,\,z)\;=\frac{E_0^2}{(f^2\,(z))}\;exp\;-\;\left(\frac{r^2}{r_0^2}\frac{1}{f^2\,(z)}\right) \quad (12)$$

where $r_0f(z)$ represents the beam's spot size at any axial distance z.

Using equations (8) and (11), the diffraction term is calculated

$$\frac{r^2}{k^2r_0^2}\frac{1}{f^4\,(z)}\;-\;\frac{1}{k^2r_0^2}\frac{1}{f^2\,(z)} \quad (13)$$

The equation simplifies

$$\frac{d^2f(z)}{dz^2}=\frac{2}{\kappa^2r_0^2r^2f(z)}-\frac{1}{\kappa^2r_0^4f^3(z)}-\frac{f(z)\,\varepsilon_{NL}}{r^2\epsilon_L}\cdot\langle \boldsymbol{EE}^*\rangle \quad (14)$$

This equation (14)represents the self-generated lens effect equation for the nonparaxial approach. The entire spatial beam, including the paraxial and peripheral portions of the axially peaked laser beam, has been considered for the study of the self-generated lens effect.

In this theory, the nonlinear part of the dielectric constant is not expanded in series form, and its termination up to a specific limit is avoided because these approximations are somewhat restrictive and limit the theory's applicability to many real-life situations.

The main features of the above equation are given as

(1) The equation is a second-order differential equation for the dimensionless beam width parameter f

(2). During the derivation of the above equation, paraxial approximations are dropped to be used for the spatial characteristics of the propagating beam in a nonlinear medium.

(3) It defines the beam dynamics and can be used to study the self-focusing of the beam with arbitrary cross-section in all types of nonlinear medium.

(4) The First two terms of the right hand side (RHS) are responsible for the diffraction divergence effect while the third term corresponds to the convergence effect due to the beam's nonlinear refraction.

(5) The self-trapping condition can be obtained by Combining the refraction and diffraction terms.

3. SELF TRAPPING CONDITION

During the propagation of an intense laser beam in a nonlinear medium, the diffraction effect continuously competes with the focusing effect, governing the propagation characteristics of the beam and its dynamics.

A condition under which an electromagnetic beam can produce its dielectric waveguide and propagate without spreading, known as uniform waveguide propagation, has been discussed. This phenomenon is known as self-trapping or diffractionless propagation.

In dielectric waveguide mode, it appears possible for an intense laser beam where the dielectric constant depends on the beam's intensity.

Following equation (14), one can conclude that when the diffraction divergence of the laser beam is precisely balanced by the focusing effect in the medium due to different types of nonlinear effects, the beam propagates in a self-trapped waveguide mode without convergence or divergence.

Thus, for an initial plane wavefront of the beam, and hence

$\frac{d^2 f(z)}{dz^2} = 0$ at z = 0 f = 1 r = ρ as well as $\frac{df(z)}{dz} = 0$ This leads to a condition where the terms on the right-hand side of equation (14) cancel each other.

In this situation, the beam width of the laser does not change during propagation in the nonlinear medium. In other words, the beam propagates without convergence or divergence or in the self-trapped mode.

On applying these conditions in equation (14), one obtains

$$\frac{2}{\kappa^2 r_0^2 \rho^2} - \frac{1}{\kappa^2 r_0^4} - \frac{\varepsilon_{NL}}{\rho^2 \epsilon_L} \cdot \langle \boldsymbol{E}\boldsymbol{E}^* \rangle = 0 \quad (15)$$

Finally, one gets a self-trapped equation.

$$\frac{\Omega_p \rho}{c} = (2r_0^2 - \rho^2)^{1/2} \frac{\rho}{r_0^2} [\varepsilon_{NL} \langle \boldsymbol{E}\boldsymbol{E}^* \rangle]^{-1/2} \quad (16)$$

This equation is valid for all types of plasma and dielectric medium nonlinearities. Compared to many popular paraxial ray methods

4. CRITICAL POWER

The critical power of beam P_{cr} is a vital parameter in self-focusing problems. It is defined as the minimum power of the incident beam for which it propagates in a nonlinear medium without converging or diverging, i.e. in uniform waveguide mode. In general, the minimum power of the incident beam is required to create a self-focused channel.[1-10]

We have considered a Gaussian beam propagating in the medium. The axial (central) part of the beam having a higher intensity, should The axial (central) part of the beam having a higher intensity, should experience an extensive refractive index (ε_{NL}) than the edge for the medium with a positive value. Consequently, the plane wavefront of the wave is progressively more distorted and as the wave propagates in the medium, it bends towards the propagation axis. It undergoes focusing, and the medium behaves like a lens. The remaining portion of the medium where the beam is not interacting will have a dielectric constant approach. The critical angle related to total internal reflection (θc) then

$$\theta_c^2 = \frac{n_{NL}}{nL} E_0^2 \quad (17)$$

If θc is a critical angle and related with intensity E_0^2 if

Rays with $\theta > \theta c$ emerge to the outside, and rays with theta $\theta < \theta c$ theta return to the axis. However, the beam self-focuses only when the refraction effect is more effective than the diffraction.

Similarly, because of Fraunhoffer diffraction, all the beam's power will not be carried by rays parallel to the propagation axis, but there will be a directional distribution of intensity in the medium.

For a beam whose phase front at the entrance to the medium is plane, the angle of diffraction is given by

$$\theta d = \frac{1.22\lambda}{r_0 n_L} \quad (18)$$

Where λ is the wavelength of the wave in vacuum and r_0 represents the initial size of the beam.

In all nonlinear mediums, the diffraction effect and refraction effect also play an essential role in wave propagation and produce many interesting situations.

In a particular situation where $\theta d = \theta c$, i.e. convergence due to the refraction effect is equal to the divergence of the beam due to the diffraction effect, the beam propagates any change in size. The dimensions and form of the beam remain unchanged during propagation in a nonlinear medium. This beam produces an optical waveguide in the medium by itself. The propagation mode of the beam is known as uniform waveguide mode [11-19]

Using equations (17) and (18), one can write.

$$\frac{n_{NL}}{nL} E_0^2 = (\frac{1.22\lambda}{4r_0 n_L})^2 \quad (19)$$

For the Gaussian beam at z=0, the intensity distribution is written as

$$E^2 = E_0^2 \exp(-r^2/r_0^2) \quad (20)$$

By using the above two equations, one can obtain the critical value of the electric field we

$$E_{0cr}^2 = \frac{(1.22\,\lambda)^2}{16\, r_0^2\, n_L\, n_{NL}} exp\left(-\frac{r_2^2}{r_0^2}\right) \quad (21)$$

where E_{0cr}^2 square of the critical value of electric field amplitude and can be calculated using the self-trapping condition). Above the critical power level, the incident beam may be trapped at any arbitrary diameter and not spread unless some instability and related phenomena force it to do so.

Thus, the critical power of the beam for electric field E_{0CR} is given as P_{CR} (for new nonparaxial method)

$$P_{CR} = \frac{c}{128} \cdot \frac{(1.22\,\lambda)^2}{n_{NL}} \exp\left(-\frac{r^2}{r_0^2}\right) \quad (22)$$

For the paraxial approximation approach, the critical power can be written as

$$P_{CR} = \frac{c}{128} \cdot \frac{(1.22\,\lambda)^2}{n_{NL}} \quad (23)$$

Equation (22) shows that the critical power depends on the size of the incident beam and the critical value of the electric field amplitude

There may be three possibilities for wave propagation in the medium

(i) if the power of beam **P > P$_{CR}$**

1) The self-focusing effect is stronger than the diffraction, due to which the beam tends to focus at the axial point

(ii) When the Power of the Beam is Less Than P < P$_{CR}$

2) If the power of the laser beam P is less than the critical power Pcr, the self-focusing effect will be weaker than the diffraction effect. Here's what happens in this regime:

1. **Predominant Diffraction**: The natural tendency of the beam to spread due to diffraction will dominate over the nonlinear self-focusing effect. This means the beam will expand as it propagates through the medium.

2. **Reduced Nonlinear Effects**: The intensity-dependent nonlinear refractive index change will decrease, leading to a weaker modification of the beam's phase front.

(iii) When the Power of the Beam is Less Than $P = P_{CR}$

it is the most useful condition, which is known as the self-trapped condition for this case[15-18]

1. **Balanced Self-Focusing and Diffraction**: The nonlinear self-focusing effect exactly compensates for the natural diffraction of the beam. This creates a stable propagation regime known as the nonlinear focus.
2. **Formation of a Self-Trapped Beam (Soliton)**: In certain conditions, this balance can lead to the formation of a soliton, a self-trapped beam that maintains its shape and size over a long distance. Solitons occur when the nonlinear change in the refractive index precisely balances the spreading effect of diffraction.

5. NUMERICAL CALCULATION

For numerical calculation of critical power for the paraxial approach by equation(23) and nonparaxial approach for different types of nonlinearity by equation(24)with the help of the Runga-kutta analytical method and the following parameters are used[23,24]

ω_p=2.5*10^{13} rad/sec, ω=1.0*10^{14} rad/sec and r_0=30 μm and tabulated

For pondermotive nonlinearity is as follows

Table 1

Critical power(P_{CR})corresponding to different spatial positions of the incident beam cross-section considering pondermotive nonlinearity in plasma and its ratio to paraxial method value

Spatial distance in um	Pcr in kw	Critical power ratio Pcrn/pcr*(paraxial)
10	279.27	1.00
15	321.13	1.15
20	390.53	1.40
25	502.38	1.80
30	683.77	2.46

For relativistic nonlinearity in plasma

Table 2

Critical power(P_{CR})corresponding to different spatial positions of the incident beam cross-section considering relativistic nonlinearity in plasma and its ratio to paraxial method value

Spatial distance in um	Pcr in kw	Critical power ratio Pcrn/pcr*(paraxial)
10	344.5	1.09
15	379.8	1.20

20	456.6	1.44
25	577.5	1.83
30	775.3	2.57

For Kerr nonlinearity in optical fiber

Table 3

Calculated values of Critical power(P_{CR}) for step-index optical fiber filled with Kerr nonlinear medium at a different radial distance(r_0) by using the entire spatial part of the beam method and its ratio to the value obtained by paraxial ray approximation .here P_{cr}*(paraxial) =9.34 kw [20]

Radial distance in um	Pcr in kw(Non paraxial)	Critical power ratio Pcrn/pcr*(paraxial)
.01	9.34	1
5	9.72	1.04
10	10.96	1.17
15	13.39	1.43
20	17.71	1.89
25	25.39	2.71

6. DISCUSSION

The analysis results in Tables 1 to 03 show fascinating behaviours for critical power. For the near axis region where spatial distance value of critical power for the plasma medium as well optical fibre of specific parameters (used in the present numerical analysis and it compares nearly well with the paraxial approximation method[23,24], which indicates that the approach used in the present analysis is quite satisfactory. For higher axial distance values, the critical power increases for all types of medium, as shown in the tables. For the entire beam, the critical power obtained is nearly three times that of the paraxial approach, and it matches with experimental results [22]

7. CONCLUSION

The present hod for formulation, calculation and comparison for critical power of intense laser beam in all types of nonlinear medium, especially plasma and optical fibre, using entire spatial characteristics of incoming laser beam provides exciting results. The calculated value for this approach is much closer to the experimental results. Understanding this balance is crucial for effectively utilizing high-power lasers in various scientific and industrial applications.

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