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Impact of Kerr Nonlinearity on Laser Beam Self-Focusing in Dielectric Materials

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Abstract

This paper presents a detailed study on the self-focusing of laser beams in dielectric materials with Kerr nonlinearity. The theoretical framework underlying this phenomenon derives the basic nonlinear Schrödinger equation (NLSE) and discusses its implications for beam propagation [1,2,3,4]. Numerical simulations using the Runga-kutta method are performed to analyse the beam dynamics under various conditions, highlighting the critical power threshold and the effects of diffraction and nonlinearity on the beam profile. A cylindrical core has been presented. The Kerr nonlinearity acts as a perturbation on the geometrically built-in radial inhomogeneity of the dielectric fiber. The beam-width parameter at different axial points is studied using varying parameters. An appreciable decrease in critical beam power for focusing has been reported. It has been found that when the beam power attains this threshold value, it propagates in a self-trapped waveguide mode. The propagation characteristics of the laser beam are compassionate concerning the delta parameter of the fiber, and the self-focusing length decreases with the increase in incident beam power.

Nonlinear guided wave optics has become a field of tremendous importance because of its applications in soliton transmission and the development of a variety of devices, such as frequency doublers for constructing blue lasers, optical power filters, etc. It is of interest, therefore, to look into the effects

Keywords : Self-Action, Nonlinearities In Dielectric Media, Nonlinear Schrödinger Equation, Critical Power Threshold.

1. Introduction

The self-actions are related to self-focusing; self-defocusing and self-trapping are nonlinear optical phenomena wherein a laser beam propagating through a medium with a refractive index that depends on the light intensity can focus and defocus itself, potentially leading to beam collapse.

The primary objective of this paper is to explore the self-focusing dynamics of laser beams in dielectric media with Kerr nonlinearity. We aim to derive and solve the governing equations, investigate the critical power threshold for self-focusing, and analyse the beam evolution through numerical simulations. A non-paraxial ray approach developed and obtained an equation for self-actions and compared results with the paraxial method used by Akhmanov and Sodha and others [1-6,18,20] using WKB approximation. Results indicate the excellent matching with available practical results, and it may open a new kind of passive nonlinear optical device for manipulating output light intensity,
depending mainly on its refractive index profile
This effect is critical in high-intensity laser applications, including laser-induced breakdown, nonlinear optics, and optical communications.

(A) Nonlinearity in the dielectric medium

In a laser beam propagating in an isotropic, transparent dielectric medium, there is a change in the refractive index, which is proportional to the laser intensity through different mechanisms like optical Kerr effect, electrostriction and thermal effect. These effects are responsible for different types of nonlinearity and are described below:

(i) Kerr nonlinearity

In liquids, an intense light beam leads to the orientation of anisotropic molecules, owing to the interaction with induced dipole moment [7]. The orientation of anisotropically polarised molecules is such that the direction of maximum polarizability of the dipole moments is along the direction of the electric vector of the laser beam; the medium becomes birefringent, i.e. the refractive index along the field is different from that perpendicular to the field[8]. This changes the dielectric constant of the medium. Hence, the nonlinear dielectric constant depends on the intensity of the incident laser beam; the nature and magnitude of the nonlinearity are determined by the structure of molecules, such as symmetrical/non-symmetrical or long chain, etc., of the electric field. The nonlinear dielectric response is characterised by:

$$\varepsilon(E) = \varepsilon_0 \left[1 + \chi^{(1)} + \chi^{(3)} \frac{|\mathbf{E}|^2}{\varepsilon_0}\right] \qquad (1)$$

Where
ε_0: Permittivity of free space.
$\chi(1)$: Linear susceptibility.
$\chi(3)$: Third-order nonlinear susceptibility (Kerr coefficient).
The effective dielectric constant ε_{eff} in the presence of Kerr nonlinearity is:

$$\varepsilon_{\text{eff}} = \varepsilon_0 \left[1 + \chi^{(1)} + \frac{\chi^{(3)}}{2} \frac{|\mathbf{E}|^2}{\varepsilon_0}\right] \qquad (2)$$

(ii) Electrostriction

In liquids that have isotropic molecules, solids, and gases, the major contribution to the intensity-dependent refractive index is caused by electrostriction. Electrostriction is the force extended on dielectric materials due to the nonuniform electric field of the incident wave. Due to this force, the material tends to be drawn into the high-field region[10]. The magnitude of the electrostrictive force is directly proportional to the gradient of the beam's intensity. This force affects the density of the material, which modifies the medium's dielectric constant. Under the influence of this force, the medium becomes nonlinear and related nonlinearity is termed electrostriction nonlinearity.

Electrostriction involves a quadratic dependence of the dielectric constant on the electric field[12]. The dielectric constant can be expressed as:

$$\varepsilon(E) == \varepsilon_0\left[1+\chi^{(1)}+\frac{1}{2}Q|\mathbf{E}|^2\right] \qquad (3)$$

χ(1): Linear susceptibility.

Q: Electrostrictive coefficient.

It is observed that:

- Nonlinearity is quadratic in electric field strength.
- Electrostriction results from the mechanical strain induced by the electric field

(iii) Thermal

When an intense laser beam with the radial distribution of the intensity is incident on an absorbing medium, a radial gradient of temperature is observed, which is responsible for the radial gradient of the dielectric constant. The nature and magnitude of variation of the dielectric constant of medium with temperature will decide the focus or defocus of the beam. The change in density and, consequently, of the refractive index connected with the heating (due to the energy dissipation) of the laser beam leads to nonstationary effects even in the field of an unmodulated wave [10].

For such cases, the time intervals of the beam must be shorter than the time of establishing a stationary temperature gradient of the nonlinear part of the dielectric constant. The time interval is directly proportional to. For most of the medium, thermal expansion reduces the refractive index, i.e. <0, and the thermal effect leads to defocusing. At the same time, for >0, heating leads to self-focusing of the wave. Self-focusing due to thermal heating of the medium by laser beam has been observed in liquids [11], crystals [12], optical glasses [13], and liquified nobel gases [14], and results were analysed accordingly.

In some cases, the change in dielectric constant with temperature may not be linear. For a more general expression that considers higher-order terms, the temperature dependence can be written as:

$$\varepsilon(T) == \varepsilon_0\left[1+\alpha(T-T_0)+\frac{\beta}{2}(T-T_0)^2+\cdots\right] \qquad (4)$$

where:

ε(T): Dielectric constant at temperature T,

ε_0: Dielectric constant at reference temperature T_0,

α: Thermal coefficient of the dielectric constant,

β: The second-order coefficient for temperature dependence, capturing more complex thermal effects.

T: Current temperature,

In the present paper, Kerr nonlinearity refers to the intensity-dependent refractive index in dielectric materials and is considered for detailed study

2. Saturation Behaviour of Dielectric Constant In Kerr Nonlinearity

The Kerr dielectric medium is based on quadratic dependence of the dielectric constant, and using (1), one gets the effective dielectric constant of the medium in the presence of the beam.

$$\varepsilon_{\mathrm{eff}} < EE* > = \varepsilon_{\mathrm{L}} + \varepsilon_2 |\mathbf{E}|^2 \tag{5}$$

where ε_2, is the Kerr coefficient of the medium.

There are many mechanisms, like the Kerr effect, where the dielectric constant of the medium attains saturation value at high intensity.

This saturation behaviour is observed in the Kerr medium because, at high intensity, all the anisotropic molecules align in the same direction. Hence, even for a moderately high intensity of the incident laser beam, the intensity near the focal point will be high enough to make equation (4) a poor approximation. Hence, a general expression is required to satisfy the dielectric constant saturation behaviour at high intensity.

$$\varepsilon_{\mathrm{eff}}(E) < EE* >= \varepsilon_{\mathrm{L}} + \varepsilon_2 < EE* > -\varepsilon_4 < EE* >^2 \tag{6}$$

In this connection, an expression for the dielectric constant of the medium in the presence of the intense electromagnetic wave, suggested by Piekara [8], is given as Sodha et al. [24] suggested a similar expression for the effective value of the dielectric constant of the dielectric medium having Kerr nonlinearity and is presented as

$$\varepsilon_{\mathrm{eff}} < EE >= \varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{S}} \left[1 - \exp\{-\frac{\varepsilon_2}{\varepsilon_{\mathrm{S}}} < EE* >\}\right] \tag{7}$$

It is found that the expression for dielectric constant represents a better approach for saturating Kerr nonlinearity. For the low values of the intensity $\frac{\varepsilon_2}{\varepsilon_{\mathrm{S}}} < EE^* > \ll 1$ Both the equations (6) and (7) are reduced as

$$\varepsilon_{\mathrm{eff}}(E) < EE* >= \varepsilon_{\mathrm{L}} + \varepsilon_2 < EE* > \tag{8}$$

In the present paper, considering Kerr nonlinearity in dielectric materials, the nonlinear part of the dielectric constant can be written as [10]

$$\varepsilon_{\mathrm{NL}} < EE >= \varepsilon_{\mathrm{S}} \left[1 - exp\{-\frac{\varepsilon_2}{\varepsilon_{\mathrm{S}}} < EE* >\}\right] \tag{9}$$

where is ε_2 Kerr nonlinear constant and is the saturated value of the nonlinear part of the dielectric constant. The effective dielectric constant of the Kerr dielectric medium in the presence of an intense laser beam using the present non-paraxial approach has been calculated using equation (9), and its variation with the intensity of the incident beam has been plotted in Figure 1. For comparison, using paraxial ray approximation and considering the same parameters, the effective dielectric constant is also calculated and plotted in the same Figure 1.

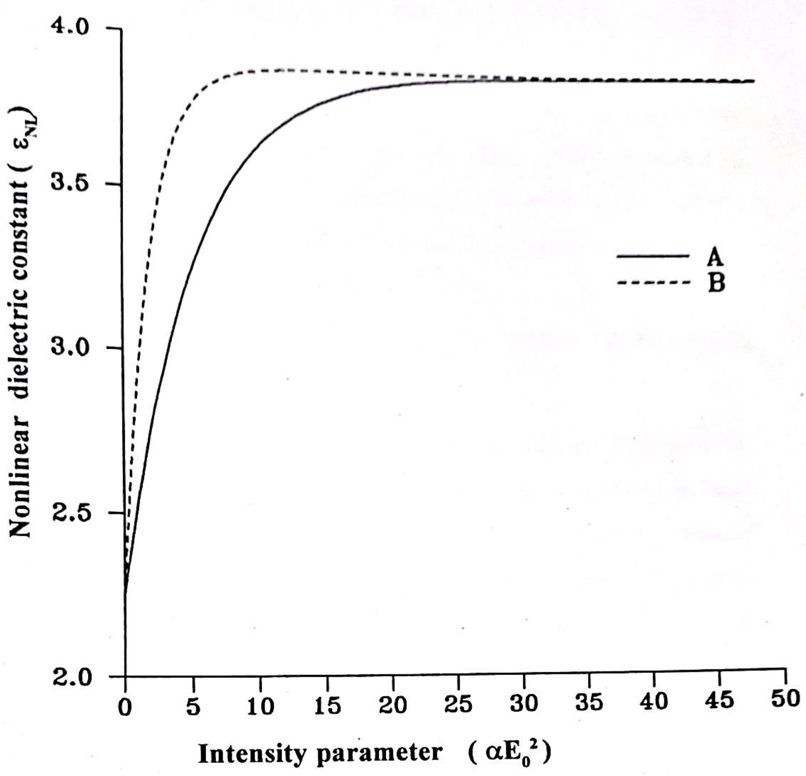

Figure 1

Variation of the effective dielectric constant (ε_{NL}) with the electric field intensity parameter (αE_0^2) for Kerr nonlinearity. Curve- A for the present analysis and Curve B for the paraxial ray approximation method. Here ε_{CO} =2.25, ε_S = 0.73

r_0 = 25µm and a_0= 25µm. (Ref. Curve B). The results of the present analysis are compared with paraxial theory (Figure 1)

Observations

(i) The result indicates that the effective dielectric constant shows saturation behaviour for higher values of the incident beam intensity, i.e., at about four times higher than the paraxial theory result. In the present case, the entire laser beam cross-section is considered instead of the near-axis part used in the paraxial ray approximation.

(ii) The dielectric medium, which follows Kerr's nonlinear behaviour, has been considered in the present study. In the high-intensity region, a strong electric field associated with a propagating beam tends to orient the anisotropic molecules of Kerr nonlinear medium, owing to interaction with induced dipoles and the nonlinear part of a dielectric constant follows saturating behaviour, as suggested by Konar et al [15].

3. Equation for Focusing For Kerr Nonlinear Medium

To investigate the self-generated lens effect (i.e. self-focusing) of the laser beam in a dielectric medium having Kerr nonlinearity, the non-paraxial technique developed by the author [16] has been used. Many approximations, like paraxial approximation in beam, medium, Taylor series expansion, etc., are dropped. Hence, considering the entire spatial characteristics of the laser beam, the equation for the self-generated lens effect in dielectric medium has been obtained by using equations (9) and is given as

$$\frac{d^2f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^{2c} f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} \quad - \frac{\varepsilon_2}{\varepsilon_{CO}}\left[\frac{E_0^2}{2fr^2}\exp\left[exp\left\{-\frac{r^2}{f^2 r_0^2}\right\} - \frac{2\Delta f \varepsilon_{CO}}{\varepsilon_s}\right]\right] \tag{10}$$

Where $\frac{2}{\kappa^2 r_0^2 r^2 f(z)} - \frac{1}{\kappa^2 r_0^A f^3(z)}$ The first two terms represent diffraction, which depends on several factors related to the wave number and initial beam width or curvature.r^2 denotes a radial coordinate or some measure related to beam radius. f(z) in the denominator suggests that this effect weakens as f(z) increases.
$\frac{\varepsilon_2}{\varepsilon_{CO}}\left[\frac{E_0^2}{2fr^2}\exp\left[exp\left\{-\frac{r^2}{f^2 r_0^2}\right\} - \frac{2\Delta f \varepsilon_{CO}}{\varepsilon_s}\right]\right]$ This represents a nonlinear effect related to a self-focusing or defocusing process, such as a Kerr nonlinearity.

The nonlinear term plays an essential role in the self-focusing of the beam, so it is discussed in two different cases.

(A) *Saturating Kerr nonlinearity*

When $\frac{\varepsilon_2}{\varepsilon_S} < EE*> \ggg 1$

The nonlinear part of the dielectric constant often shows a saturating tendency for intense laser beams. The dielectric constant attains a saturating value for the high intensity of the beam. The self-focusing equation for Kerr nonlinearity in dielectrics at saturating nonlinearity is written as:

$$\frac{d^2f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^2 f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} \quad - \frac{\varepsilon_S}{r^2 \varepsilon_L}[f(z)] \quad (11)$$

In the present analysis, the numerical solution of the second-order differential equation for the beam-width parameter has been obtained without using many approximations, which are usually adopted in the popular paraxial ray theories [17,18,19,20]. Due to this, the present analysis is expected to provide better results for beam and medium characteristics. For different radial coordinates (p).

In the present case, for saturating type nonlinearity using equation (11), the beam-width focusing parameter f (z) has been calculated at different values of the axial distance of propagation z, and results are plotted in Figure 2

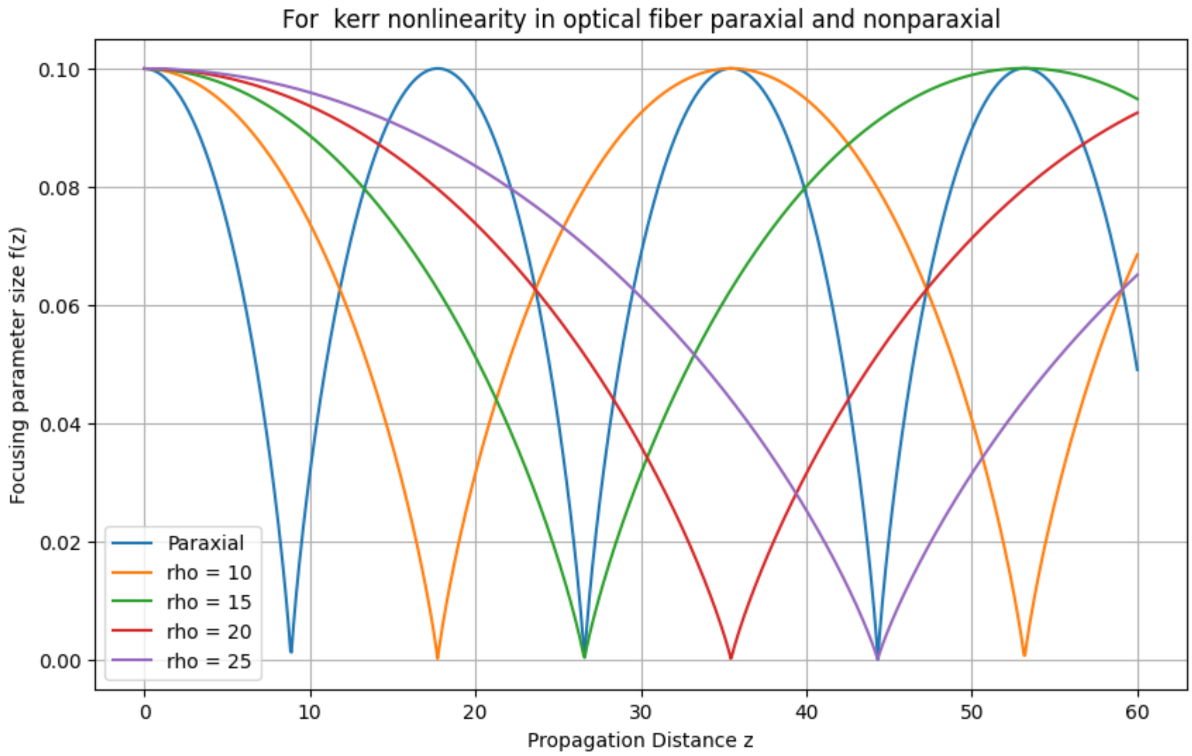

Figure 2

Observations

- In the present case, for saturating type nonlinearity using equation (11), the beam-width focusing parameter f (z) has been calculated at different values of the axial distance of propagation z, and results are plotted in Figure-2.
- These curves demonstrate that the beam-width parameter (f) oscillates between well-defined values as it propagates in a Kerr dielectric medium.
- Results also indicate that emerging beams from different spatial positions (p) focus at various points on the axis, i.e., focal length, and are found to be different for different values of p.

(B) Non-saturating Nonlinearity

If the intensity of the incident beam is small, then

When $\frac{\varepsilon_2}{\varepsilon_S} < EE > <<< 1$

Under this condition, the self-focusing equation for Kerr nonlinearity for non-saturation condition of dielectric constant using equation can be written as

$$\frac{d^2 f(z)}{dz^2} = \frac{2}{\kappa^2 r_0^2 r^{2c} f(z)} - \frac{1}{\kappa^2 r_0^4 f^3(z)} - \frac{\varepsilon_2}{\varepsilon_{CO}}\left[\frac{E_0^2}{2fr^2}\left[exp\left\{-\frac{r^2}{f^2 r_0^2}\right\}\right]\right] \tag{12}$$

It is challenging to solve this nonlinear second-order differential equation analytically, so it is solved numerically using given initial parameters, and it is observed that

(i) Beam-width focusing parameter (f) shows non-oscillatory behaviour with axial distance (z) in the case of non -saturating type nonlinearity. For incident beam power higher than the critical value, initially, the beam starts converging, but after appreciable propagation in the fiber, the influence of the diffraction divergence becomes dominant over the converging effect.

(ii) Due to nonlinearity, the beam starts diverging much before the sharp focus is reached

4. Uniform Waveguide Propagation

The condition under which the beam can produce its own waveguide and propagate without spreading is known as uniform waveguide propagation mode, also known as solitons [16]. The self-trapping in fiber (waveguide mode) is possible only when the refraction term is precisely balanced with diffraction terms in the self-focusing equation (12).

To obtain the self-trapping condition, one can use a general expression (12) for nonlinearity [14].

Consider a Gaussian beam with a plane wavefront at z = 0. Hence, at z = 0 for incident beam f = 1 and df /dz = 0, two terms on the RHS of equation (12) cancel under this condition. Thus df/dz will remains zero for all values of z. This leads to uniform d2f / dz2 = 0 waveguide motion. We get an equation for uniform waveguide propagation by substituting r = ρf for different transverse distances (p) in the incident beam in equation (12). For the present method of non-paraxial approach, one gets

$$\frac{\omega r_0}{c} = \left[\frac{\varepsilon_s r_0^2\{1-\exp\left[-\alpha E_0^2 exp\left\{-\frac{\rho^2}{r_0^2}\right\}\right]}{(2r_0^2-\rho^2)}\right]^{-\frac{1}{2}} \tag{13}$$

The expression for well-known paraxial ray approximation is also solved numerically using the following equation

$$\frac{\omega r_0}{c} = \left[\frac{\varepsilon_2 E_0^2}{(2(1+\frac{\varepsilon_2 E_0^2}{2\varepsilon_S})}\right]^{-\frac{1}{2}} \tag{14}$$

The normalised self-trapped radius is calculated for the different values of the intensity parameter for the present non-paraxial approach using equation(13) and the paraxial approximation method using equation(14) and plotted in figure(3) as given follows

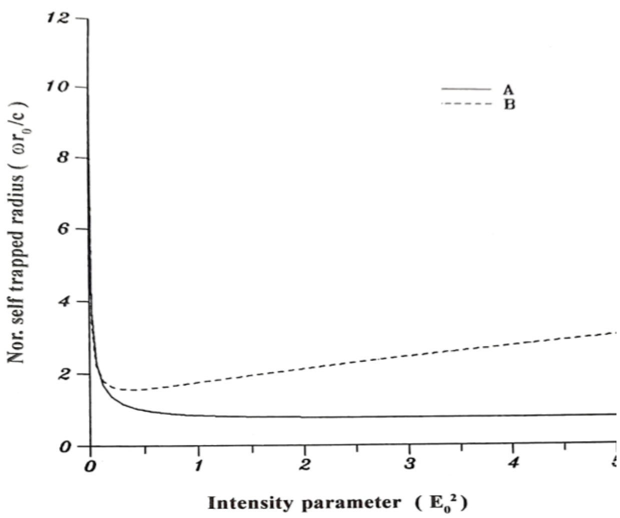

Figure 3

For dielectrics, the normalised self-trapped beam radius ($\frac{\omega r_0}{c}$) has been plotted for different intensity values and shown in Figure (3).

1) Observations

Results demonstrate that the present analysis gives a much flatter curve in the saturation region compared to the paraxial method. Such results have also been observed for detailed non-paraxial analysis by various authors in the case of plasma [16,25].

The observed self-trapping behaviour of the beam in fiber medium can easily be related to the saturating behaviour of orientation of anisotropic molecules of Kerr nonlinear medium at high intensity and associated Kerr nonlinearity saturation effect.

5. Conclusion

This paper presents a comprehensive analysis of the self-focusing of laser beams in dielectric media with Kerr nonlinearity. The theoretical derivations, supported by numerical simulations, provide a deep understanding of the factors influencing self-focusing. It is observed that the present method developed provides better results related with beam propagation in nonlinear Kerr medium and the self trapping and using this relation the calculated value of the critical power obtained near the practical results. Future work could explore the effects of higher-order nonlinearities and the influence of beam shape on the self-focusing dynamics.

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