International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
doi : https://doi.org/10.32628/IJSRST

A Study on Conservative Pseudo Quasi-Conformal Curvature Tensor in K-Contact Manifold

S. N. Manjunath[a1] *, K. J. Jayashree[b] and P. Rashmi[c]
[a*]Lecturer, Department of Science, Govt. VISSJ Polytechnic, Bhadravathi, Karnataka, India
[b]Senior Scale Lecturer, Department of Science, Govt. Polytechnic, Hiriyur, Karnataka, India
[c]Lecturer, Department of Science, Govt. Polytechnic, Tumkur, Karnataka, India
* Corresponding author. E-mail : mathsmanju@gmail.com

ARTICLEINFO

Article History:
Accepted : 10 Dec 2021
Published: 30 Dec 2021

Publication Issue :
Volume 8, Issue 6
November-December-2021

Page Number :
1176-1179

ABSTRACT

The paper deals with the study on Pseudo quasi-conformal curvature tensor in K-contact manifolds and it is shown that the manifold is Einstein.

Keywords : Conservative, Conformal Curvature Tensor, Einstein, K-Contact Manifold.

1. INTRODUCTION

Geometry of contact metric manifolds is important because of their applications, for instance in the theory of Einstein metrics K-contact manifolds have nice topological properties. Two important classes of contact manifolds are K-contact manifolds and Sasakian manifolds ([3], [4], [12]). K-contact and Sasakian manifolds have been studied by several authors such as ([1], [7], [9], [10]). It is known that if the characteristic vector field of a contact metric manifold is Killing vector field then the manifold is called a K-contact manifold. It is well known that every Sasakian manifold is K-contact, but the converse is not true, in general. However a three-dimensional K-contact manifold is Sasakian.

In [14], Yano and Sawaki defined and studied a tensor field W called quasi-conformal curvature tensor on a Riemannian manifold which includes both the conformal curvature tensor and concircular curvature tensor as special cases. In [13], A.A. Shaikh and Sanjib Kumar Jana introduced and studied the pseudo quasi-conformal curvature tensor and it is a generalization of conformal, quasi-conformal [1], concircular and projective curvature tensor as special cases. Recently, D.G. Prakasha et.al [8] and Satyabrota Kundu [11] studied different properties of pseudo quasi-conformal curvature tensor in P-Sasakian manifolds.

Motivated by the above work in this research article we studied pseudo quasi-conformal curvature tensor in K-contact manifolds:

The paper is organized as follows: Section 2 deals with preliminaries of K-contact manifold. Section 3 deals with the study pseudo quasi-conformally conservative K-contact manifold.

2. PRELIMINARIES

An n-dimensional differentiable manifold M is said to have an almost contact structure (ϕ, ξ, η) if it carries a tensor field ϕ of type(1,1), a vector field $\xi$and a 1-form η on Msatisfying

$$(2.1) \qquad \phi^2 X = -X + \eta(X)\xi, \ \phi\xi = 0, \eta(\xi) = 1, \quad \eta \cdot \phi = 0.$$

If g is a Riemannian metric with almost contact structure that is,

$$(2.2) \qquad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \qquad \eta(X) = g(X, \xi).$$

Then M is called an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g) and denoted by (M, ϕ, ξ, η, g).
If on (M, ϕ, ξ, η, g) the exterior derivative of 1-form η satisfies,

$$d\eta(X, Y) = g(X, \phi Y).$$

Then(M, ϕ, ξ, η, g) is said to be a contact metric manifold.
If moreover ξ is Killing vector field, then M is called a K-contact manifold. A K-contact manifold is called Sasakian, if the relation

$$(2.3) \qquad (\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X,$$

holds, where ∇ denotes the covariant differentiation with respect to g. From (2.3), we get

$$(2.4) \qquad \nabla_X \xi = -\phi X, (\nabla_X \eta)Y = gX, \phi Y.$$

In a K-contact manifold M the following relations holds:

$$(2.5) \qquad g(R(\xi, X)Y, \xi) = g(X, Y) - \eta(X)\eta(Y),$$

$$(2.6) \qquad R(X, Y)\xi = \eta(Y)X - \eta(X)Y,$$

$$(2.7) \qquad R(\xi, X)\xi = \eta(X)\xi - X,$$

$$(2.8) \qquad S(X, \xi) = (n-1)\eta(X),$$

for any vector fields X, Y. Where R is the Riemannian curvature tensor and S is the Ricci tensor of the manifold M.
The pseudo quasi-conformal curvature tensor [13] $\widetilde{W}$ of type $(1, 3)$ on a K-contact manifold is defined by

$$(2.9) \qquad \widetilde{W}(X, Y)Z = (p + d)R(X, Y)Z \quad + \left[q - \frac{d}{n-1}\right][S(Y, Z)X - S(X, Z)Y]$$
$$+q[g(Y, Z)QX - g(X, Z)QY] - \frac{r(p + 2(n-1)q)}{n(n-1)}[g(Y, Z)X - g(X, Z)Y],$$

for all vector fields X, Y, Z, where p, q, d are arbitrary constants not simultaneously zero, R is the Riemannian curvature tensor, S is the Ricci tensor, Q is the Ricci operator and r is the scalar curvature tensor of the manifold M.
In particular, if

- $p = q = 0, d = 1$ in (2.9) then $\widetilde{W}$ reduces to the projective curvature tensor.
- $p \neq 0, q \neq 0, d = 0$ in (2.9) then $\widetilde{W}$ reduces to the quasi-conformal curvature tensor.
- $p = 1, q = -\frac{1}{n-2}, d = 0$ in (2.9) then $\widetilde{W}$ reduces to the conformal curvature tensor.
- $p = 1, d = q = 0$ in (2.9) then $\widetilde{W}$ reduces to the concircular curvature tensor.

3. PSEUDO QUASI-CONFORMALLY CONSERVATIVE K-CONTACT MANIFOLD

In this section, we study the conservative Pseudo Quasi-Conformal curvature tensor in K-contact manifold. Then we have

(3.1) $div\, \widetilde{W}(U,V)Z = 0.$

Differentiating (2.9) covariantly along the vector field U_i, we get

(3.2) $(\nabla_{U_i}\widetilde{W})(U,V)Z = (p+d)(\nabla_{U_i}R)(U,V)Z + \left[q - \frac{d}{(n-1)}\right][(\nabla_{U_i}S)(V,Z)U - (\nabla_{U_i}S)(U,Z)V]$
$+q[g(V,Z)(\nabla_{U_i}Q)U - g(U,Z)(\nabla_{U_i}Q)V] - \frac{dr(U_i)}{n(n-1)}[p+2(n-1)q][g(V,Z)U - g(U,Z)V].$

On contracting (3.2), we obtain

(3.3) $$div\, \widetilde{W}(U,V)Z = (p+d)\ div\, R(U,V)Z + \left[q - \frac{d}{(n-1)}\right][(\nabla_U S)(V,Z) - (\nabla_V S)(U,Z)]$$
$$-\left[\frac{p}{n(n-1)} - \frac{(n-4)q}{2n}\right][g(V,Z)dr(U) - g(U,Z)dr(V)].$$

With the help of (3.1) in (3.3), we have

(3.4) $$\left[p+q+\frac{d(n-2)}{(n-1)}\right][(\nabla_U S)(V,Z) - (\nabla_V S)(U,Z)]$$
$$-\left[\frac{p}{n(n-1)} - \frac{(n-4)q}{2n}\right][g(V,Z)dr(U) - g(U,Z)dr(V)] = 0.$$

By substituting $U = \xi$ in (3.4), we get

(3.5) $$\left[p+q+\frac{d(n-2)}{(n-1)}\right][(\nabla_\xi S)(V,Z) - (\nabla_V S)(\xi,Z)]$$
$$-\left[\frac{p}{n(n-1)} - \frac{(n-4)q}{2n}\right][g(V,Z)dr(\xi) - g(\xi,Z)dr(V)] = 0.$$

We know that

(3.6) $$(\nabla_V S)(\xi,Z) = VS(\xi,Z) - S(\nabla_V \xi,Z) - S(\xi,\nabla_V Z)$$
$$= S(\phi V,Z) - (n-1)g(\phi V,Z).$$

And

(3.7) $$(\nabla_V S)(\xi,Z) = 0.$$

By using (3.6), (3.7) and $dr(\xi) = 0$ in (3.5), we get

(3.8) $$\left[p+q-\frac{d(n-2)}{(n-1)}\right][S(\phi V,Z) - (n-1)g(\phi V,Z)] = \left[\frac{p}{n(n-1)} - \frac{(n-4)q}{2n}\right]\eta(Z)dr(V).$$

Replacing $Z = \phi Z$ in (3.8), we obtain

(3.9) $$S(\phi V,\phi Z) = (n-1)g(\phi V,\phi Z),$$

provided $\left[p+q+\frac{d(n-2)}{(n-1)}\right] \neq 0$. On simplifying (3.9), we get

(3.10) $S(V,Z) = (n-1)g(V,Z),$

On contracting (3.10), we have

(3.11) $r = n(n-1).$

Hence, we state the following:

Theorem 3.1. If in a K-contact manifold the Pseudo quasi-conformal curvature tensor is conservative, then the manifold is Einstein and scalar curvature $r = n(n-1)$.

REFERENCES

[1]. K. S. Amur and Y. B. Maralabhavi, On quasi-conformally flat spaces, Tensor, N.S., 31, (1977), 194-198.

[2]. C. Baikoussis and T. Koufiorgos, On a type of contact manifolds, J. of Geometry, 46, (1993), 1-9.

[3]. D.E. Blair, "Contact manifolds in Riemannian geometry, Lecture Notes in Mathematics," Vol.509. Springer-Verlag, berlin-New-York, (1976).

[4]. D.E. Blair, "Riemannian geometry of contact and symplectic manifolds," Progress in Mathematics, 203. Birkhauser Bosten, Inc., Boston, MA, 2002.

[5]. Bhagwat Prasad, A pseudo projective curvature tensor on a Riemannian manifold, Bull. Cal. Math. Soc., 94 (3), (2002), 163-166.

[6]. D.E. Blair and J.A. Oubina, Conformal and related changes of metric on the product of two almost contact metric manifolds, Publ. Mat., 34 (1), (1990), 199-207.

[7]. U.C. De and A.A. Shaikh, K-contact and Sasakian manifolds with conservative quasi-conformal curvature tensor, Bull.Cal. Math. Soc., 89, (1997), 349-354.

[8]. D.G. Prakasha, T.R. Shivamurthy and Kakasab Mirji, On the pseudo quasi conformal curvature tensor of P-Sasakian manifolds, Electronic Journal of Mathematical Analysis and Applications, 5 (2), (2017), 147-155.

[9]. Mukut Mani Tripathi and Mohit Kumar Dwivedi, The structure of some classes of K-contact manifolds, Proc. Indian Acad.Sci. (Math. Sci.), 118 (3), (2008), 371-379.

[10]. H.G. Nagaraja and G.Somashekhara, On Pseudo projective curvature tensor in Sasakian manifolds, Int. J. Contemp. Math.Sciences, 6 (27), (2011), 1319-1328.

[11]. Satyabrota Kundu, On P-Sasakian manifolds, Math. Reports, 15 (65), (2013), 221-232.

[12]. S. Sasaki, "Lecture note on almost contact manifolds, Part-I," Tohoku University, (1965).

[13]. A.A. Shaikh and Sanjib Kumar Jana, A Pseudo Quasi-Conformal curvature tensor on a Riemannian manifold, South East Asian J. Math. & Math. Sc., 4(1), (2005), 15-20.

[14]. K. Yano and S. Sawaki, Riemannian manifolds admitting a conformal transformation group, J. Diff. Geom., 2, (1968), 161-184.