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Some Curvature Properties of K-Contact Manifolds with Semi-Symmetric Non-Metric Connections

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ABSTRACT : In this paper we define a linear connection on a K-contact manifold which is semi-symmetric but non-metric and we study some properties of the Riemannian curvature tensor, conformal curvature tensor with respect to semi-symmetric non-metric connection.

Keywords : Semi-Symmetric Non-Metric Connection, K-Contact Manifold, Conformal Curvature Tensor.

1. Introduction:

In 1924, Friedmann and Schouten [12] introduced the notion of semi-symmetric linear connection on a differentiable manifold. In 1932, Hayden [15] introduced the idea of semi-symmetric metric connection with torsion on a Riemannian manifold. The idea of semi-symmetric metric connection on a Riemannian manifold was further developed by Yano [22]. Later on various properties of such connection have been studied by many geometers like K.S. Amur and S.S. Pujar [3], C.S. Bagewadi and et. al. [4, 5, 13, 14], M.M. Tripathi [20], U.C. De et. al. [10, 11], etc.

A semi-symmetric non-metric connection is characterized by its torsion tensor being expressible in terms of a 1-form, and it does not necessarily preserve the metric. This contrasts with the Levi-Civita connection, which is torsion-free and metric-compatible. In 1992, Agashe and Chafle [1] defined and studied a semi-symmetric non-metric connection in a Riemannian manifold. The study was further carried out by Agashe and Chafle [2], J. Sengupta, U.C. De and T.Q. Binh [18]. Later on many mathematicians like M.M. Tripathi and N. Nakkar [19], Chaubey and Ojha [8], Jaiswal and Ojha [16], Chaubey [9], studied semi-symmetric non-metric connection for different contact manifolds.

Motivated by the above work, in this paper we study semi-symmetric non-metric connection on a K-contact manifold. The paper is organized as follows: Section 2 deals with preliminaries. Section 3 concerned with the relations between the Levi-Civita connection and the semi-symmetric non-metric connection in a K-contact manifold. Finally, the paper ends with the properties of Conformal curvature tensor $\bar{N}$ of K-contact manifolds with respect to the semi-symmetric non-metric connection.

2. Preliminaries:

An n-dimensional differentiable manifold M is called an almost contact structure $(\phi,\ \xi,\ \eta)$ if it carries a tensor field ϕ of type (1,1), a vector field $\xi$and a 1-form η on M satisfying

$$(2.1)\ \phi^2 U = -U + \eta(U)\xi,\ \ \phi\xi = 0,$$
$$\eta(\xi) = 1, \quad \eta \cdot \phi = 0.$$

If g is a Riemannian metric with almost contact structure that is,

(2.2) $$g(\phi U, \phi V) = g(U, V) - \eta(U)\eta(V),$$
$$\eta(U) = g(U, \xi).$$

Then M is called an almost contact metric manifold equipped with an almost contact metric structure $(\phi,\ \xi,\ \eta, g)$ and denoted by $(M, \phi,\ \xi,\ \eta, g)$.If on $(M, \phi,\ \xi,\ \eta, g)$ the exterior derivative of 1-form η satisfies,

(2.3) $d\eta(U, V) = g(U, \phi V).$

Then $(M, \phi,\ \xi,\ \eta, g)$ is said to be a contact metric manifold.

If moreover ξ is Killing vector field, then M is called a K-contact Riemannian manifold. A K-contact Riemannian manifold is called Sasakian, if the relation

(2.4) $(\nabla_{\mathrm{U}}\phi)V = g(U, V)\xi - \eta(V)U,$

holds, where ∇ denotes the covariant differentiation with respect to g. From (2.4), we get

(2.5) $\nabla_{\mathrm{U}}\xi = -\phi U,$

(2.6) $(\nabla_U \eta)V = g(U, \phi V).$

In a K-contact manifold M the following relations holds:

$$(2.7)\ g(R(X, Y)Z, \xi) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y),$$

(2.8) $R(X, Y)\xi = \eta(Y)X - \eta(X)Y,$

(2.9) $R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X,$

(2.10) $R(\xi, X)\xi = \eta(X)\xi - X,$

(2.11) $S(X, \xi) = (n-1)\eta(X),$

(2.12) $S(\phi X, \phi Y) = S(X, Y) - (n-1)\eta(X)\eta(Y),$

for any vector fields X, Y and Z. Where R and S are the Riemannian curvature tensor and the Ricci tensor of M, respectively.

3. Expression of $\widetilde{R}(U, V)Z$ in terms of $R(U, V)Z$:

Let M be an n-dimensional K-contact manifold with Riemannian metric g. If ∇ is the Levi-Civita connection of a K-contact manifold M. A semi-symmetric non-metric connection $\tilde{\nabla}$ in a K-contact manifold is given by

(3.1) $\tilde{\nabla}_U V = \nabla_{\mathrm{U}} \mathrm{V} + \eta(V)U,$

where η is a 1-form associated with the vector field ξ on M. By virtue of (3.1), the torsion tensor $\tilde{T}$ of the connection $\tilde{\nabla}$ and is given by

(3.2) $\tilde{T}(U,V) = \tilde{\nabla}_U V - \tilde{\nabla}_V \mathrm{U} - [U,V]$.

A linear connection $\tilde{\nabla}$ on M is said to be a semi-symmetric connection if its torsion tensor $\tilde{T}$ of the connection $\tilde{\nabla}$ satisfies

(3.3) $\tilde{T}(U,V) = \eta(V)U - \eta(U)V$.

If moreover $\tilde{\nabla}g = 0$ then the connection is called a semi-symmetric metric connection. If $\tilde{\nabla}g \neq 0$ then the connection $\tilde{\nabla}$ is called a semi-symmetric non-metric connection.

From (3.1), we get

(3.4) $(\tilde{\nabla}_X g)(Y,Z) = -\eta(Y)g(X,Z) - \eta(Z)g(X,Y)$,

for all vector fields X, Y, Z on M.

A relation between Riemannian curvature tensors R and $\tilde{R}$ with respect to Riemannian connection ∇ and semi-symmetric non-metric connection $\tilde{\nabla}$ of a K-contact manifold M is given by

(3.5) $\tilde{R}(U,V)Z = R(U,V)Z - \alpha(V,Z)U + \alpha(U,Z)V$,

for all vector fields U, V, Z on M where α is a tensor field of (0,2) type defined by

(3.6) $\alpha(U,V) = (\nabla_{\mathrm{U}}\eta)\mathrm{V} - \eta(\mathrm{U})\eta(\mathrm{V}) = (\tilde{\nabla}_{\mathrm{U}}\eta)\mathrm{V}$.

By using (2.6) in (3.6), we obtain

(3.7) $\alpha(U,V) = g(U,\phi V) - \eta(\mathrm{U})\eta(\mathrm{V})$.

By virtue of (3.7) in equation (3.5), we get

(3.8) $\tilde{R}(U,V)Z = R(U,V)Z - g(V,\phi Z)\mathrm{U}$
$+\eta(\mathrm{Z})\eta(\mathrm{V})U + g(U,\phi Z)\mathrm{V} - \eta(\mathrm{U})\eta(\mathrm{Z})\mathrm{V}$.

A relation between Ricci tensors $\tilde{S}$ and S with respect to semi-symmetric non-metric connection $\tilde{\nabla}$ and the Riemannian connection ∇ of a K-contact manifold M is given by

(3.9) $\tilde{S}(U,Z) = S(U,Z) - (n-1)\alpha(U,Z)$.

On contracting (3.9), we obtain

(3.10) $\tilde{r} = r - (n-1)trace(\alpha)$.

Lemma 3.1: Let M be an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$. Then

(3.11) $(\tilde{\nabla}_X \phi)Y = (\nabla_X \phi)Y - \eta(Y)\phi X$,

(3.12) $\tilde{\nabla}_X \xi = X - \phi X$,

(3.13) $(\tilde{\nabla}_X \eta)Y = (\nabla_X \eta)Y - \eta(X)\eta(Y) = \alpha(X,Y)$.

Proof: By using (3.1) and (2.1), we obtain (3.11). From (3.1) and (2.5), we get (3.12). Finally, by virtue of (3.1), (2.4) and (2.6) we get (3.13).

From (3.13), we can easily state the following corollary:

Corollary 3.1: In a K-contact manifold, the tensor field α satisfies

(3.14) $\tilde{\alpha}(X,\xi) = -\eta(X)$.

Theorem 3.1: In a K-contact manifold with semi-symmetric non-metric connection $\tilde{\nabla}$, we have

(3.15) $\tilde{R}(U,V)Z + \tilde{R}(V,Z)U + \tilde{R}(Z,U)V$
$= [\alpha(U,Z) - \alpha(Z,U)]V + [\alpha(Z,V) - \alpha(V,Z)]U$
$+[\alpha(V,U) - \alpha(U,V)]Z$.

(3.16) $\tilde{R}(U,V,Z,W) + \tilde{R}(V,U,Z,W) = 0$.

(3.17) $\tilde{R}(U,V,Z,W) - \tilde{R}(Z,W,U,V)$
$$= [\alpha(U,Z) - \alpha(Z,U)]g(V,W) + \alpha(W,U)g(V,Z) - \alpha(V,Z)g(U,W).$$

Proof: By using (3.5), we obtain

(3.18) $\tilde{R}(U,V)Z + \tilde{R}(V,Z)U + \tilde{R}(Z,U)V$
$$= R(U,V)Z + R(V,Z)U + R(Z,U)V + [\alpha(U,Z) - \alpha(Z,U)]V + [\alpha(Z,V) - \alpha(V,Z)]U + [\alpha(V,U) - \alpha(U,V)]Z.$$

By using first Bianchi identity $R(U,V)Z + R(V,Z)U + R(Z,U)V = 0$ in (3.18) we obtain (3.15).

Again by using (3.5), we get

(3.19) $\tilde{R}(U,V,Z,W) = R(U,V,Z,W)$
$$-\alpha(V,Z)g(U,W) + \alpha(U,Z)g(V,W).$$

If we change the role of U and V in (3.19), we have

(3.20) $\tilde{R}(V,U,Z,W) = R(V,U,Z,W)$
$$+\alpha(V,Z)g(U,W) - \alpha(U,Z)g(V,W).$$

By virtue of (3.19) and (3.20), we obtain

(3.21) $\tilde{R}(U,V,Z,W) + \tilde{R}(V,U,Z,W)$
$$= R(U,V,Z,W) + R(V,U,Z,W).$$

Since $R(U,V,Z,W) + R(V,U,Z,W) = 0$ and then we get (3.16).

Now by using (3.19), we have

(3.22) $\tilde{R}(U,V,Z,W) - \tilde{R}(Z,W,U,V)$
$$= R(U,V,Z,W) + R(Z,W,U,V) + [\alpha(U,Z) - \alpha(Z,U)]g(V,W) + \alpha(W,U)g(V,Z) - \alpha(V,Z)g(U,W).$$

We know that $R(U,V,Z,W) = R(Z,W,U,V)$, then (3.22) reduces as (3.17).

Lemma 3.2: Let M be an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection $\tilde{\nabla}$. Then

(3.23) $\tilde{R}(X,Y)\xi = 2[\eta(Y)X - \eta(X)Y],$

(3.24) $\tilde{R}(\xi,X)\xi = 2[\eta(X)\xi - X],$

(3.25) $\tilde{R}(\xi,X)Y = g(X,Y)\xi - 2\eta(Y)X - \alpha(X,Y)\xi,$

Proof: By using (2.8) in (3.5), we get (3.23). By using (2.10) and (3.5), we have (3.24). From (2.9) and (3.5), we obtain (3.25).

Lemma 3.3: In an n-dimensional K-contact manifold with respect to the semi-symmetric non-metric connection, we have

(3.26) $\tilde{S}(X,\xi) = 2(n-1)\eta(X),$

(3.27) $\tilde{S}(\phi X,\phi Y) = \tilde{S}(X,Y).$

Proof: By using (2.11) and (3.9), we obtain (3.26). From equation (2.12) and (3.9), we get (3.27).

4. Conformal curvature tensor of K-contact manifold admitting semi-symmetric non-metric connection:

Let M be an n −dimensional K-contact manifold, then the Conformal curvature tensor $\bar{N}$ of M with respect to the Levi-Civita connection is defined by

$$(4.1) \quad \bar{N}(U,V)Z = R(U,V)Z - \frac{1}{(n-2)}[S(V,Z)U - S(U,Z)V + g(V,Z)QU - g(U,Z)QV] + \frac{r}{(n-1)(n-2)}[g(V,Z)U - g(U,Z)V].$$

By taking an inner product with W in (4.1), we get

$$(4.2) \quad \bar{N}(U,V,Z,W) = R(U,V,Z,W) - \frac{1}{(n-2)}[S(V,Z)g(U,W) - S(U,Z)g(V,W) + g(V,Z)g(QU,W) - g(U,Z)g(QV,W)] + \frac{r}{(n-1)(n-2)}[g(V,Z)g(U,W) - g(U,Z)g(V,W)],$$

where R, S and r are the Riemannian curvature tensor, Ricci tensor and the scalar curvature of the K-contact manifold M.

Theorem 4.4: Let M be a K-contact manifold. Then the Conformal curvature tensors $\bar{N}$ and $\tilde{\bar{N}}$ of the K-contact manifolds with respect to the Levi-Civita connection and semi-symmetric non-metric connection is related as

$$(4.3) \quad \tilde{\bar{N}}(U,V,Z,W) = \bar{N}(U,V,Z,W) + \alpha(U,Z)g(V,W) - \alpha(V,Z)g(U,W) - \frac{n-1}{n-2}[\alpha(U,Z)\, g(V,W) - \alpha(V,Z)g(U,W) - \alpha(U,W)\, g(V,Z) + \alpha(V,W)\, g(U,Z)] - \frac{trace(\alpha)}{n-2}[g(V,Z)g(U,W) - g(U,Z)\, g(V,W)].$$

Proof: Let $\tilde{\bar{N}}$ and $\bar{N}$ denote the Conformal curvature tensor of M with respect to the semi-symmetric non-metric connection and the Levi-Civita connection, respectively. Conformal curvature tensor $\tilde{\bar{N}}$ with respect to the semi-symmetric non-metric connection is defined by

$$(4.4) \quad \tilde{\bar{N}}(U,V,Z,W) = \tilde{R}(U,V,Z,W) - \frac{1}{(n-2)}[\tilde{S}(V,Z)g(U,W) - \tilde{S}(U,Z)g(V,W) + g(V,Z)g(\tilde{Q}U,W) - g(U,Z)g(\tilde{Q}V,W)] + \frac{\tilde{r}}{(n-1)(n-2)}[g(V,Z)g(U,W) - g(U,Z)g(V,W)],$$

where $\tilde{R}$, $\tilde{S}$ and $\tilde{r}$ are the Riemannian curvature tensor, Ricci tensor and scalar curvature of the K-contact manifold M with respect to the semi-symmetric non-metric connection.

Then by using (3.5), (3.9) and (3.10) in (4.4), we have

$$(4.5) \quad \tilde{\bar{N}}(U,V,Z,W) = \mathrm{R(U,V,Z,W)} + \alpha(U,Z)g(V,W) - \alpha(V,Z)g(U,W) - \frac{1}{n-2}[g(U,W)\{S(V,Z) - (n-1)\alpha(V,Z)\} - g(V,W)\{S(U,Z) - (n-1)\alpha(U,Z)\} + g(V,Z)\{S(U,W) - (n-1)\alpha(U,W)\} - g(U,Z)\{S(V,W) - (n-1)\alpha(V,W)\}]$$

$$+\frac{r-(n-1)trace(\alpha)}{n-2}[g(V,Z)g(U,W) \ - g(U,Z)\ g(V,W)].$$

By virtue of (4.2) in (4.5), we obtain (4.3).

Theorem 4.5: In an n-dimensional K-contact manifold M, the Conformal curvature tensor $\tilde{\tilde{C}}$ of the manifold with respect to the semi-symmetric non-metric connection doesn't satisfy first Bianchi identity, that is,

(4.6) $\tilde{\tilde{N}}(U,V,Z,W) + \tilde{\tilde{N}}(V,Z,U,W)$

$$+\tilde{\tilde{N}}(Z,U,V,W) \neq 0.$$

Proof: First Bianchi identity for Conformal curvature tensor $\tilde{\tilde{N}}$ of K-contact manifold is given by

(4.7) $\tilde{\tilde{N}}(U,V,Z,W) + \tilde{\tilde{N}}(V,Z,U,W) + \tilde{\tilde{N}}(Z,U,V,W)$

$$= R(U,V,Z,W) + R(V,Z,U,W)$$

$$+R(Z,U,V,W)$$

$$-\frac{1}{n-2}[\{\alpha(U,Z) - \alpha(Z,U)\}g(V,W)$$

$$+\{\alpha(V,U) - \alpha(U,V)\}g(Z,W) + \{\alpha(Z,V) - \alpha(V,Z)\}g(U,W).$$

Sinc$R(U,V,Z,W) + R(V,Z,U,W) + R(Z,U,V,W) = 0$ and in view of (4.7), we obtain (4.6).

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