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Some Results on Generalized Recurrent K-Contact Manifolds Admitting Quarter-Symmetric Metric Connection

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ABSTRACT

The objective of the present paper is to study some properties of generalized ϕ-recurrent K-contact manifold admitting quarter-symmetric metric connection.

Keywords: K-Contact Manifold, Generalized Recurrent.

AMS Subject Classification (2010) : 53C05, 53D10, 53C20, 53C15, 53C25

1. Introduction:

In 1924, A. Friedman and J.A. Schouten [4] introduced the notion of a semi-symmetric linear connection on a differentiable manifold. In 1932, H.A. Hayden [8] introduced the idea of metric connection with torsion on a Riemannian manifold. In 1970, K. Yano [16] studied some curvature and conditions for semi-symmetric connections in Riemannian manifolds. In 1975, S. Golab [5] defined and studied quarter-symmetric linear connection on a differentiable manifold.

The notion of local symmetry of a Riemannian manifold has been weakened by many authors in several ways to a different extent. As a weaker version of local symmetry, Takahashi [15] introduced the notion of local ϕ-symmetry on a Sasakian manifold. Generalizing the notion of ϕ-symmetry, the authors De et al. [3] introduced the notion of ϕ-recurrent Sasakian manifolds. This notion was further studied by many authors like C.S. Bagewadi and et al. ([1], [6], [7]), Nagaraja [10], A.A. Shaikh and Ananta Patra [14]. In the present paper we studied generalized ϕ-recurrent K-contact manifold admitting quarter-symmetric metric connection. The second section devoted to the basic concepts of K-contact manifold. In the third section we study the relation between the Levi-Civita connection and the quarter-symmetric metric connection in a K-contact manifold. In the fourth section we study the generalized recurrent K-contact manifold with respect to the quarter-symmetric metric connection. In the next section we study the generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection. In the section we studied C-Bochner generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection

A linear connection $\widetilde{\nabla}$ in an n-dimensional differentiable manifold is said to be a quarter-symmetric connection if its torsion tensor T is of the form

(1.1) $$T(X,Y) = \widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X - [X,Y] = \pi(Y)F\,X - \pi(X)F\,Y,$$

where π is a 1-form and F is a tensor of type (1,1) and quarter-symmetric linear connection $\widetilde{\nabla}$ satisfies the condition

$$\big(\widetilde{\nabla}_X g\big)(Y,Z) = 0$$

for all $X, Y, Z \in \varkappa(M)$, where $\varkappa(M)$ is the Lie algebra of vector fields of the manifold M, then $\widetilde{\nabla}$ is said to be a quarter-symmetric metric connection.

For contact manifold admitting quarter-symmetric connection we take $\pi = \eta$ and $F = \phi$ in (1.1) then the equation can be written as

(1.2) $$T(X,Y) = \eta(Y)\phi\,X - \eta(X)\phi\,Y.$$

2. Preliminaries:

An n-dimensional differentiable manifold M is said to have an almost contact structure $(\phi,\ \xi,\eta)$ if it carries a tensor field ϕ of type (1,1), a vector field ξ and a 1-form η on M respectively such that,

(2.1) $$\phi^2 X = -X + \eta(X)\xi,\ \ \phi\xi = 0,\ \ \eta(\xi) = 1,\quad \eta\cdot\phi = 0.$$

Thus a manifold M equipped with this structure is called an almost contact manifold and is denoted by $(M,\phi,\ \xi,\eta)$. If g is a Riemannian metric on an almost contact manifold M such that,

(2.2) $$g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y),\qquad \eta(X) = g(X,\xi),$$
(2.3) $$g(X,\phi Y) = -\,g(\phi X, Y),$$
where X, Y are vector fields defined on M, then M is said to have an almost contact metric structure $(\phi,\ \xi,\eta,g)$ and M with this structure is called an almost contact metric manifold and is denoted by $(M,\phi,\ \xi,\eta,g)$.

If on $(M,\phi,\ \xi,\eta,g)$ the exterior derivative of 1-form η satisfies,

$$d\eta(X,Y) = g(X,\phi\,Y)$$

then $(M,\phi,\ \xi,\eta,g)$ is said to be a contact metric structure and M equipped with a contact metric structure is called contact metric manifold.

If ξ is a killing vector field, then M is called a K-contact Riemannian manifold. A K-contact Riemannian manifold is called Sasakian, if the relation

(2.4) $$(\nabla_X\phi)Y = g(X,Y)\xi - \eta(Y)X$$
holds, where ∇ denotes the operator of covariant differentiation with respect to g.

In a K-contact manifold M, the following relations holds:

(2.5) $\nabla_X \xi = -\phi X,$

(2.7) $g(R(X,Y)Z,\xi) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y),$

(2.8) $R(\xi,X)\xi = -X + \eta(X)\xi,$

(2.8) $S(X,\xi) = (n-1)\,\eta(X),$

where R and S are the Riemannian curvature tensor and the Ricci tensor of M, respectively.

Definition 2.1: A K-contact manifold is called generalized ϕ-recurrent if its curvature tensor R satisfies the condition

$$\phi^2((\nabla_W R)(X,Y)Z) = \alpha(W)R(X,Y)Z + \beta(W)\{g(Y,Z)X - g(X,Z)Y\} \tag{2.9}$$

for arbitrary vector fields X, Y, Z and W. Where \$R\$ is the Riemannian curvature tensor, α and β are two 1-forms defined by

$$\alpha(W) = g(W,\rho_1), \quad \beta(W) = g(W,\rho_2) \tag{2.10}$$

where ρ_1 and ρ_2 are vector fields associated with 1-forms α and β respectively.

3. Relation between the curvature tensors of Levi-Civita connection and the quarter-symmetric metric connection in a K-Contact manifold:

A quarter-symmetric metric connection $\widetilde{\nabla}$ in a K-Contact manifold is given by

$$\widetilde{\nabla}_X Y = \nabla_X Y - \eta(X)\phi Y. \tag{3.1}$$

Therefore equation (3.1) is the relation between the Levi-Civita connection and the quarter-symmetric metric connection on a K-Contact manifold.

A relation between the curvature tensor of M with respect to the quarter-symmetric metric connection $\widetilde{\nabla}$ and the Levi-Civita connection ∇ is given by

$$\tilde{R}(X,Y)Z = R(X,Y)Z + 2g(\phi X,Y)\phi Z + [\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi + [\eta(Y)X - \eta(X)Y]\eta(Z) \tag{3.2}$$

where $\tilde{R}$ and R are the Riemannian curvature of the connection $\widetilde{\nabla}$ and ∇, respectively.
From (3.2), it follows that

$$\tilde{S}(Y,Z) = S(Y,Z) - g(Y,Z) + n\,\eta(Y)\eta(Z) \tag{3.3}$$

where $\tilde{S}$ and S are the Ricci tensors of the connection $\widetilde{\nabla}$ and ∇, respectively. Contracting (3.3), we get

(3.4) $\tilde{r} = r$

where $\tilde{r}$ and r are the scalar curvatures of the connection $\widetilde{\nabla}$ and ∇, respectively.

4. Generalized recurrent K-contact manifold with respect to the quarter-symmetric metric connection:

We define a generalized recurrent K-contact manifold with respect to the quarter-symmetric metric connection if its curvature tensor $\tilde{R}$ satisfies the condition

(4.1) $(\widetilde{\nabla}_W \tilde{R})(U,V)Z = \alpha(W)\tilde{R}(U,V)Z + \beta(W)\{g(V,Z)U - g(U,Z)V\}$

where α and β are 1-form and it is defined by

(4.2) $\alpha(W) = g(W,\rho_1),\quad \beta(W) = g(W,\rho_2)$

where ρ_1 and ρ_2 are vector fields associated with 1-forms α and β respectively. Now by substituting the value of $U = Z = \xi$ in (4.1) then we have

(4.3) $(\widetilde{\nabla}_W \tilde{R})(\xi,V)\xi = \alpha(W)\tilde{R}(\xi,V)\xi + \beta(W)\{\eta(V)\xi - V\}$

It is also known that

(4.4) $\left(\widetilde{\nabla}_W \tilde{R}\right)(\xi,V)\xi = \widetilde{\nabla}_W \tilde{R}(\xi,V)\xi - \tilde{R}(\widetilde{\nabla}_W \xi,V)\xi - \tilde{R}(\xi,\widetilde{\nabla}_W V)\xi - \tilde{R}(\xi,V)\widetilde{\nabla}_W \xi$

then by virtue of (3.2), (3.1) and (2.7) in (4.4) then the equation (4.4) reduces in the form

(4.5) $\left(\widetilde{\nabla}_W \tilde{R}\right)(\xi,V)\xi = 0.$

By substituting the above value of equation (4.5) in (4.3) then we have

(4.6) $\alpha(W)\tilde{R}(\xi,V)\xi + \beta(W)\{\eta(V)\xi - V\} = 0.$

So by using (3.2) and (2.7) in (4.6), we get

(4.7) $[2\alpha(W) + \beta(W)]\{\eta(V)\xi - V\} = 0.$

Remarking that the equation $\{\eta(V)\xi - V\} = 0$ does not hold for K-contact manifold with respect to quarter-symmetric metric connection. So we obtain

(4.8) $[2\alpha(W) + \beta(W)] = 0,$

which implies that $2\alpha(W) = -\beta(W)$ got any vector field W. Then we state the following:

Theorem 4.1. Let M be a generalized recurrent K-contact manifold with respect to quarter-symmetric metric connection, then $2\alpha(W) = -\beta(W)$.

5. Generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection:

We define a Generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection if its curvature tensor $\tilde{R}$ satisfies the condition

$$(5.1)\quad (\tilde{\nabla}_W \tilde{R})(U,V)Z = \alpha(W)\tilde{R}(U,V)Z + \beta(W)\{g(V,Z)U - g(U,Z)V\}$$

where α and β are 1-form and it is defined by

$$(5.2)\qquad \alpha(W) = g(W,\rho_1),\quad \beta(W) = g(W,\rho_2)$$

where ρ_1 and ρ_2 are vector fields associated with 1-forms α and β respectively.

Then by virtue of (2.1) and (5.1) can be written in the form

$$(5.3)\quad -\left(\tilde{\nabla}_W \tilde{R}\right)(U,V)Z + \eta\left(\left(\tilde{\nabla}_W \tilde{R}\right)(U,V)Z\right)\xi = \alpha(W)\tilde{R}(U,V)Z + \beta(W)\{g(V,Z)U - g(U,Z)V\}$$

from which it follows that

$$(5.4)\qquad \begin{aligned} &-g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(U,V)Z, U_i\right) + \eta\left(\left(\tilde{\nabla}_W \tilde{R}\right)(U,V)Z\right) g(\xi, U_i) \\ &= \alpha(W) g\left(\tilde{R}(U,V)Z, U_i\right) + \beta(W)\{g(V,Z)g(U,U_i) - g(U,Z)g(V,U_i)\}. \end{aligned}$$

Let $e_i,\ i = 1,2,\dots,n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $U = U_i = e_i$ in (5.4) and taking summation over i, then we get

$$(5.5)\qquad -(\tilde{\nabla}_W \tilde{S})(V,Z) + \sum_{I=1}^{n} \eta((\tilde{\nabla}_W \tilde{R})(e_i,V)Z)\eta(e_i) == \alpha(W)\tilde{S}(V,Z) + (n-1)\beta(W)g(V,Z).$$

The second term of (5.5) it is reduced to

$$(5.6)\qquad \begin{aligned} g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(e_i,V)Z,\xi\right) &= g\left(\tilde{\nabla}_W \tilde{R}\,(e_i,V)Z,\xi\right) - g\left(\tilde{R}\left(\tilde{\nabla}_W e_i,V\right)Z,\xi\right) \\ &\quad - g(\tilde{R}\,(e_i,\tilde{\nabla}_W V)Z,\xi) - g(\tilde{R}\,(e_i,V)\tilde{\nabla}_W Z,\xi). \end{aligned}$$

On simplification we obtain, $g\left(\left(\tilde{\nabla}_W \tilde{R}\right)(e_i,V)Z,\xi\right)$. So then the equation (5.5) can be written in the form

$$(5.7)\qquad -(\tilde{\nabla}_W \tilde{S})(V,Z) = \alpha(W)\tilde{S}(V,Z) + (n-1)\beta(W)g(V,Z).$$

putting $Z = \xi$ in the above equation then we get

$$(5.8)\qquad -(\tilde{\nabla}_W \tilde{S})(V,\xi) = \alpha(W)\tilde{S}(V,\xi) + (n-1)\beta(W)g(V,\xi).$$

and on simplification we have

(5.9) $(2n-1)g(V,\phi W) - S(V,\phi W) = 2(n-1)\alpha(W)\eta(V) + (n-1)\beta(W)\eta(V)$

if we put $V = \phi V$ in the above equation (5.9) then we have

(5.10) $S(\phi V,\phi W) = (2n-1)g(\phi V,\phi W)$

and on simplification we have

(5.11) $S(V,W) = (2n-1)g(V,W) - n\,\eta(W)\eta(V)$.

Hence we can state the following:

Theorem 5.2. A generalized ϕ-recurrent K-contact manifold with respect to quarter-symmetric metric connection is an η-Einstein manifold.

6. C-Bochner Generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection

Let us consider a C-Bochner Generalized ϕ-recurrent K-contact manifold with respect to the quarter-symmetric metric connection is defined by

(6.1) $$\phi^2(\tilde{\nabla}_W \tilde{B})(X,Y)Z = \alpha(W)\tilde{B}(X,Y)Z + \beta(W)[g(Y,Z)X - g(X,Z)Y],$$

where $\tilde{B}$ is a C-Bochner curvature tensor with respect to the quarter-symmetric metric connection i.e.,

(6.2) $$\begin{aligned}\tilde{B}(X,Y)Z = {} & \tilde{R}(X,Y)Z + \tfrac{1}{n+3}[g(X,Z)\tilde{Q}Y - \tilde{S}(Y,Z)X - g(Y,Z)\tilde{Q}X + \tilde{S}(X,Z)Y \\ & + g(\phi X,Z)\tilde{Q}\phi Y - \tilde{S}(\phi Y,Z)\phi X - g(\phi Y,Z)\tilde{Q}\phi X + \tilde{S}(\phi X,Z)\phi Y + 2\tilde{S}(\phi X,Y)\phi Z \\ & + 2g(\phi X,Y)\tilde{Q}\phi Z + \eta(Y)\eta(Z)\tilde{Q}X - \eta(Y)\tilde{S}(X,Z)\xi + \eta(X)\tilde{S}(Y,Z)\xi - \eta(X)\eta(Z)\tilde{Q}Y] \\ & - \frac{\tilde{D}+n-1}{n+3}[g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X + 2g(\phi X,Y)\phi Z] + \frac{\tilde{D}}{n+3}[\eta(Y)g(X,Z)\xi \\ & - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y - \eta(X)g(Y,Z)\xi] - \frac{\tilde{D}-4}{n+3}[g(X,Z)Y - g(Y,Z)X].\end{aligned}$$

Where $\tilde{D} = \frac{\tilde{r}+n-1}{n+1}$.

Then by virtue of (2.1) and (6.1) then we have

(6.3) $-\big(\tilde{\nabla}_W\tilde{B}\big)(X,Y)Z + \eta\left(\big(\tilde{\nabla}_W\tilde{B}\big)(X,Y)Z\right)\xi = \alpha(W)\tilde{B}(X,Y)Z + \beta(W)[g(Y,Z)X - g(X,Z)Y]$.

Let $\{e_i, i = 1,2,\ldots,n\}$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in the above equation and taking summation over i, and then by substituting $U = \xi$ and then by $X = \xi$ we obtain

$$\beta(W) = -\left[\frac{4}{(n+3)}\right]\alpha(W).$$

Now, we state the following:

Lemma 6.1. Let M be a C-Bochner generalized ϕ-recurrent K-contact manifold with respect to quarter-symmetric metric connection then we have

$$\beta(W) = -\left[\frac{4}{(n+3)}\right]\alpha(W).$$

Now, putting $X = U = e_i$ in (6.3) and taking summation over i, and substituting $Z = \xi$ and on simplification, we get

$$S(Y,W) = Ag(Y,W) + B\eta(Y)\eta(W)$$

where

$A = \left[\frac{\{(n-1)(2n+5)-r\}}{n+1}\right]$ and $B = \left[\frac{\{r-(n-1)(n+4)\}}{n+1}\right]$. We state the following:

Theorem 6.2. A C-Bochner generalized ϕ-recurrent K-contact manifold with respect to quarter-symmetric metric connection is an η-Einstein manifold.

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