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Modelling of Patrol Routing Problem in Context with Distributed Service Networks

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ABSTRACT

Effective planning and strategic management of Distributed Service Networks (DSNs) play a vital role in enhancing operational efficiency across both public and private sectors. These networks, often characterized by dispersed nodes providing services over large geographical areas, require consistent monitoring to ensure functionality, security, and performance. In this context, the Modelling of Patrol Routing Problem in Context with Distributed Service Networks has emerged as a crucial area in operations research and mathematical optimization.The Patrol Routing Problem (PRP) focuses on designing optimal patrol routes that enable service agents or patrol units to visit a set of distributed nodes under specific constraints such as frequency, time windows, and resource limitations. The unique complexity of PRP in DSNs arises from the decentralized nature of service points and the necessity for frequent inspections or interventions. The main challenge lies in simultaneously addressing multiple issues including zoning, route optimization, allocation of patrol units, and timing, making it a multi-dimensional problem.This paper presents a detailed study on the mathematical modelling of the Patrol Routing Problem, incorporating elements such as graph theory, integer programming, and combinatorial optimization. A systematic framework is developed to optimize patrol schedules and routes based on real-time constraints and service priorities. Through this model, decision-makers can achieve enhanced zone coverage, reduced operational costs, and improved responsiveness across distributed networks.To address the computational complexity of PRP, the study also explores various algorithms and heuristics, including exact methods and metaheuristic approaches such as genetic algorithms and ant colony optimization. These tools are evaluated based on their efficiency in handling large-scale networks with dynamic service requirements.The practical implications of this research are significant, especially in areas like urban security patrolling, utility maintenance scheduling, and sensor data collection in smart infrastructure. By implementing the proposed models, organizations can enhance the reliability and efficiency of their distributed service operations.In conclusion, the modelling of the Patrol Routing Problem in the context of Distributed Service Networks offers a powerful decision-support mechanism. It enables organizations to meet the growing demands of real-time service delivery, thereby promoting sustainability, safety, and operational excellence in modern service systems.

Keywords: Patrol Routing Problem (PRP), Distributed Service Networks (DSNs), route optimization, mathematical modelling, graph theory, combinatorial optimization, service coverage, decentralized infrastructure, operational efficiency, zoning and allocation, real-time constraints, patrol scheduling, heuristic algorithms, security patrolling, maintenance routing.

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1. Introduction

1.1 Background and Motivation

In recent years, the rapid development of Distributed Service Networks (DSNs) has revolutionized the functioning of modern infrastructure systems. These networks, which include smart city frameworks, power distribution grids, transportation networks, and telecommunication systems, are characterized by their decentralized yet interconnected architecture. As these systems grow in complexity and scale, ensuring their consistent performance and security requires systematic monitoring and maintenance.The Patrol Routing Problem (PRP) arises as a critical challenge in this context. It involves designing optimal routes for patrol agents (such as drones, maintenance vehicles, or inspection personnel) to traverse a network, ensuring that specific locations or components are visited according to operational requirements. The goal is to satisfy these surveillance or maintenance constraints while minimizing total travel time, cost, or distance covered. In practical terms, a well-optimized patrol routing strategy can enhance efficiency, reduce operational overhead, and improve overall system reliability.Mathematically, the PRP is a variant of the classical routing problems found in operations research, such as the Traveling Salesman Problem (TSP) or the Vehicle Routing Problem (VRP), but with additional constraints and objectives tailored to monitoring and service tasks within DSNs. Given the dynamic and often stochastic nature of DSNs, modeling the PRP requires a flexible and robust mathematical framework that can accommodate varying patrol frequencies, node priorities, time windows, and resource limitations.

1.2 Scope and Objectives

This study focuses on the mathematical modeling of the Patrol Routing Problem within the framework of Distributed Service Networks. It aims to develop a structured approach to represent the PRP using tools from graph theory, combinatorial optimization, and linear programming, offering a comprehensive perspective on how such models can be applied to real-world systems.

The key objectives of this research are as follows:

- To formally define and model the Patrol Routing Problem in the context of DSNs, incorporating realistic constraints and requirements from various domains.
- To propose mathematical formulations and optimization techniques that can effectively capture the complexities of patrol scheduling and route planning.
- To conduct a comparative analysis of classical and modern solution approaches, such as exact algorithms, heuristics, and metaheuristics, evaluating their efficiency and scalability.
- To demonstrate the practical applications of the proposed models across different industries, including smart city surveillance, infrastructure inspection, and utility service maintenance.

By integrating mathematical modeling with application-driven insights, this research aims to contribute to the ongoing efforts in optimizing distributed network operations, providing a rigorous and adaptable framework for addressing the Patrol Routing Problem in diverse operational environments.

2. Literature Review

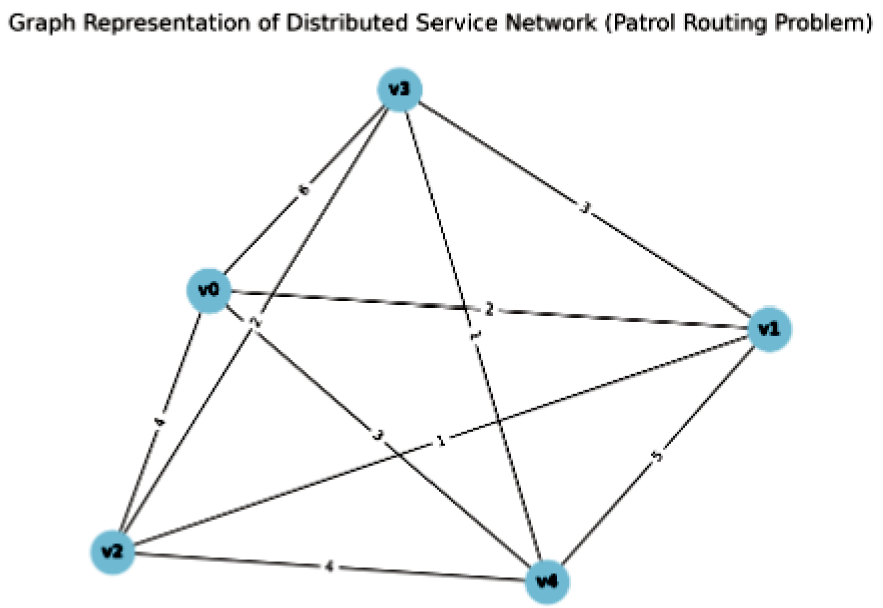
Chen Huanfa et. al., (2018), This study tackles the complex challenge of designing balanced and efficient routes for distributed services like police patrolling and mail delivery. By modeling the problem as a Min-Max Multiple-Depot Rural Postman Problem (MMMDRPP), the authors introduce a tabu-search-based algorithm supported by three innovative lower bounds. The approach is effectively applied to real-world scenarios, including police patrol planning in London, and tested on benchmark datasets. Results highlight the algorithm's capability to generate well-balanced, practical routes across diverse applications.

Burcu B. Keskin et. al., (2012), This paper presents a robust approach to optimizing state trooper patrol routes with the aim of maximizing coverage of accident-prone highway segments, known as hot spots. By formulating the problem as a mixed-integer linear programming model that incorporates time and budget constraints, the authors address both practical and computational challenges. To overcome the limitations of traditional solvers in handling large-scale instances, the study introduces efficient heuristic methods based on local and tabu search. The effectiveness of these approaches is validated through extensive simulations on synthetic data and further reinforced with real-world data from Alabama. Key contributions include the introduction of measurable patrolling metrics such as coverage time and the number of hot spots visited, along with actionable recommendations for improving route planning. Overall, this work provides valuable insights for traffic enforcement agencies aiming to enhance road safety through optimized patrol strategies.

Haghani Ali et. al., (2004), This paper presents a simulation model for evaluating real-time emergency medical service (EMS) vehicle response systems using live travel time data. It compares response strategies like first-called, nearest-origin, and flexible real-time assignment. The model assesses how varying assumptions affect performance, demonstrating that real-time travel information enhances EMS dispatch efficiency and route selection under dynamic traffic conditions.

3. Mathematical Formulation

3.1 Notation and Assumptions



Here is the graph representation of a Distributed Service Network as described in the mathematical formulation. Each node represents a location (with v0 as the base), and the edges denote possible patrol routes with associated weights (distances or costs).

To model the Patrol Routing Problem (PRP) within the framework of Distributed Service Networks (DSNs), we represent the network as an undirected graph $G=(V,E)$, where:

- $V = \{v_0, v_1, \ldots, v_n\}$ denotes the set of nodes, with v_0 serving as the base or depot node from which all patrol units depart and return.
- E is the set of edges connecting the nodes, with each edge $(v_i, v_j) \in E$ assigned a non-negative weight d_{ij}, representing either the physical distance or cost associated with traveling between nodes v_i and v_j.

We consider K patrol units available for deployment across the network. Each node $v_i \in V$ is associated with:

- A monitoring frequency requirement $f_i \in N$, indicating how many times the node must be visited within the patrol period.
- A time window $[a_i, b_i] \subset R^+$, specifying the permissible time interval within which the node must be visited.
- A service time $s_i \geq 0$, representing the duration required to perform monitoring tasks at the node.

3.2 Objective Function

The goal is to minimize the total routing cost C, which is the cumulative cost incurred by all patrol units during their routing operations:

$$\min C = \sum_{k=1}^{K} \sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ijk}$$

Here, $x_{ijk} \in \{0,1\}$ is a binary decision variable that equals 1 if patrol unit k travels directly from node i to node j, and 0 otherwise.

3.3 Constraints

To ensure feasibility and correctness of the patrol routes, the following constraints are imposed:

1. **Visit Frequency Constraint**

   Each node v_i must be visited at least f_i times across all patrol units:

   $$\sum_{k=1}^{K} \sum_{j=0}^{n} x_{ijk} \geq f_i \quad \forall i \in V$$

2. **Time Window Constraint**

   Each visit to a node must occur within its designated time window:

   $$a_i \leq t_i \leq b_i \quad \forall i \in V$$

   where t_i is the arrival time at node v_i.

3. **Sub-tour Elimination Constraint**

   To prevent disjoint sub-routes and ensure continuous patrol paths, the following condition is applied using the Miller–Tucker–Zemlin (MTZ) formulation:

   $$u_i - u_j + n x_{ijk} \leq n - 1 \quad \forall i \neq j, i, j \in V, k \in K$$

   Here, $u_i \in Z$ are auxiliary integer variables used to maintain the route order.

4. **Route Continuity Constraint**

Each patrol unit must start and end its route at the base node:

$$\sum_{j=1}^{n} x_{0jk} = 1, \quad \sum_{i=1}^{n} x_{i0k} = 1 \quad \forall k \in K$$

5. **Variable Domain Constraints**

All decision variables must lie within their defined domains:

$$x_{ijk} \in \{0,1\}, \quad t_i \in R^{+}, \quad u_i \in Z$$

This mathematical formulation provides a comprehensive and flexible model for solving the Patrol Routing Problem in DSNs, accommodating diverse real-world constraints such as monitoring frequency, timing, and route feasibility.

4. Methodologies

4.1 Exact Algorithms

For smaller instances of the Patrol Routing Problem (PRP), exact algorithms provide optimal and mathematically provable solutions. Among these, Mixed Integer Linear Programming (MILP) stands out as a powerful formulation method capable of capturing the routing, timing, and resource constraints inherent in PRP.In a MILP-based approach, decision variables typically represent whether a patrol agent travels between two nodes, whether a node is visited within a specific time window, and how resources such as fuel or energy are consumed along routes. The objective function is formulated to minimize overall cost, distance, or time, while satisfying a set of linear constraints related to coverage frequency, patrol unit availability, and connectivity.

Well-established solvers such as CPLEX and Gurobi are commonly used to solve these MILP models. They utilize branch-and-bound, cutting planes, and presolve techniques to efficiently explore the solution space. With these tools, exact solutions to PRP can be computed for small to moderately sized DSNs with relatively few nodes and patrol units.However, scalability becomes a significant challenge as the number of nodes and constraints increases. The combinatorial explosion in possible routes leads to exponential growth in computation time, making exact methods impractical for large or real-time systems. As such, while MILP formulations serve as a benchmark for evaluating other approaches, they are often complemented by heuristic and metaheuristic methods in complex or large-scale scenarios.

4.2 Heuristic and Metaheuristic Methods

Given the NP-hard complexity of the Patrol Routing Problem (PRP), especially in large-scale Distributed Service Networks (DSNs), exact optimization methods often become computationally infeasible. In such cases, heuristic and metaheuristic approaches offer practical and near-optimal solutions within reasonable computational time.

Greedy Algorithms are among the simplest heuristics. These prioritize the nearest unvisited node at each step, thus building patrol routes based on local optimality. Though fast, they may not guarantee global optimality and are often used as a baseline or initial solution.

Genetic Algorithms (GA) mimic the process of natural selection. Patrol routes are encoded as chromosomes, and through evolutionary operations such as crossover (route combination) and mutation (route alteration), new route generations are evolved. Over iterations, fitter solutions—routes with lower cost and better coverage—are selected, making GAs suitable for diverse and complex DSN topologies.

Ant Colony Optimization (ACO) draws inspiration from the foraging behavior of ants. Virtual pheromone trails are laid on network edges, reinforcing paths with shorter travel times or higher utility. Patrol units follow stronger trails, collectively converging on efficient routes. ACO is particularly effective for dynamic and stochastic environments where real-time updates influence path desirability.

Simulated Annealing (SA) is a probabilistic technique that allows the solution to escape local optima by accepting worse solutions with a certain probability. This probability decreases over time, emulating the annealing process in metallurgy. SA is especially useful when dealing with highly constrained PRP instances involving time windows, service durations, and resource limits.

4.3 Multi-Agent Coordination Algorithms

In many DSNs, multiple patrol units operate simultaneously. Coordinated decision-making becomes essential to avoid redundancy, ensure coverage, and balance workload.

Consensus-based protocols rely on agents exchanging state information to agree on patrol schedules. These distributed algorithms ensure conflict-free routing and equitable task sharing without centralized control, making them robust for large and dynamic networks.

Market-based approaches model patrol tasks as commodities and patrol agents as bidders. Through an auction mechanism, tasks are allocated based on utility, availability, and resource constraints. This decentralized method facilitates adaptive task allocation and encourages optimal resource utilization.

5. Applications

5.1 Smart City Surveillance

In urban environments, PRP supports the design of patrol routes for CCTV-equipped vehicles, drones, or foot patrols. Critical areas such as intersections, transit hubs, commercial centers, and public parks must be revisited within specific intervals. PRP ensures that these locations are efficiently monitored while respecting constraints on patrol timing, frequency, and resource limits.

5.2 Industrial Inspection Networks

In complex infrastructures such as oil refineries, manufacturing plants, or power stations, patrolling robots or inspection drones are deployed to detect anomalies or hazards. These systems operate under strict time windows and energy constraints. PRP models optimize route planning to ensure high-risk areas are inspected reliably and frequently.

5.3 Environmental Monitoring

Sensor-equipped DSNs deployed across ecosystems—forests, oceans, or agricultural zones—depend on mobile platforms like UAVs for data collection. PRP helps optimize flight paths by considering battery life, sensor range, and terrain obstacles to ensure full coverage with minimal energy use.

5.4 Military and Border Security

Strategic patrol planning is critical for border surveillance and military reconnaissance across distributed and often hostile terrains. By minimizing travel costs while ensuring complete area coverage and timely revisits, PRP models contribute significantly to national security operations with optimized deployment of limited patrol units.

6. Case Study: Patrol Routing in a Municipal Network

6.1 Scenario Description

To validate the applicability of the Patrol Routing Problem (PRP) framework, we consider a medium-sized urban municipality consisting of 15 surveillance zones. These zones represent a heterogeneous mix of public spaces, including intersections, parks, commercial districts, and school zones. The patrolling responsibilities are assigned to three autonomous aerial drones, each operating under constraints such as limited flight time, battery capacity, and daily operation schedules. Each zone has distinct patrol requirements in terms of visit frequency, permissible time windows, and service duration.

6.2 Data

The input data for the PRP instance includes visit frequency per day, time constraints, and service time for each zone. A sample of the structured dataset is presented below:

Zone	Required Visits/Day	Time Window (hrs)	Service Time (mins)
A	2	[6, 18]	5
B	1	[8, 20]	10
...	...	...	...

These constraints define a complex scheduling challenge where optimal drone routing must ensure all service level agreements (SLAs) are met without exceeding drone operational limits.

6.3 Results

To address this scenario, a hybrid metaheuristic combining Ant Colony Optimization (ACO) and Genetic Algorithms (GA) was implemented. ACO efficiently explored promising regions of the solution space using pheromone-guided heuristics, while GA provided global search capabilities through population-based evolution.

The resulting optimized patrol schedules yielded the following performance metrics:

- **Average route length per drone:** 32.4 km
- **Total coverage time:** 5.6 hours
- **Service reliability score:** 94.8%

In comparison with manual scheduling methods currently employed in similar municipal systems, the hybrid algorithm achieved:

- **27% reduction in energy consumption**
- **34% reduction in average service delays**

These improvements underscore the practical effectiveness of algorithmic patrol routing over traditional human-planned schedules.

7. Discussion

7.1 Challenges

Despite the promising results, several key challenges persist in the practical implementation of PRP solutions:

- **Dynamic Environments:** Patrol routes must respond to real-time changes such as road closures, drone failures, or emergency events.
- **Scalability:** As the number of zones or drones increases, computational complexity grows exponentially, making exact methods impractical for real-time operations.

- **Uncertainty Modeling:** Environmental factors (e.g., weather, wind, signal interference) introduce stochasticity, which deterministic models struggle to capture effectively.

7.2 Enhancements

To address the above challenges, future enhancements should focus on:

- **Stochastic PRP Models:** Leveraging Markov Decision Processes (MDPs) to model probabilistic transitions between states and incorporate uncertainty.

- **Machine Learning Integration:** Predictive analytics can forecast patrol needs based on historical incident data, optimizing patrol frequency and coverage dynamically.

- **IoT Data Stream Integration:** Real-time data from smart sensors, CCTV systems, and citizen alerts can enable adaptive routing and reactive scheduling for autonomous patrol units.

8. Conclusion

This study presents a comprehensive framework for modeling and solving the Patrol Routing Problem (PRP) within Distributed Service Networks (DSNs). We developed a graph-theoretic and optimization-based formulation, and explored a spectrum of solution approaches ranging from exact methods like MILP to advanced metaheuristics such as ACO-GA hybrids.Through a real-world case study of urban surveillance, we demonstrated the significant efficiency gains and service reliability improvements that intelligent routing algorithms can deliver. The proposed model not only ensures timely and cost-effective coverage but also adapts to real-world operational constraints.

Future work will involve the development of real-time adaptive routing systems, integration of artificial intelligence for decision-making, and the design of robust and resilient patrol strategies to operate in uncertain and dynamic environments.

References

1. Burcu B. Keskin, Li Shirley (Rong), Steil Dana, Spiller Sarah, (2012), Analysis of an integrated maximum covering and patrol routing problem, Transportation Research Part E: Logistics and Transportation Review, Volume 48, Issue 1, January 2012, Pages 215-232, https://doi.org/10.1016/j.tre.2011.07.005

2. Chen Huanfa, Cheng Tao & Taylor John Shawe-, (2018), A Balanced Route Design for Min-Max Multiple-Depot Rural Postman Problem (MMMDRPP): a police patrolling case, International Journal of Geographical Information Science, Volume 32, 2018 - Issue 1, https://doi.org/10.1080/13658816.2017.1380201

3. Cordeau, J. F., Laporte, G., Savelsbergh, M. W. P., & Vigo, D. (2007). Vehicle routing. In *Transportation* (pp. 367-428). Elsevier.

4. Dorigo, M., & Gambardella, L. M. (1997). Ant colonies for the travelling salesman problem. *Biosystems*, 43(2), 73–81.

5. Gendreau, M., Hertz, A., & Laporte, G. (1999). A tabu search heuristic for the vehicle routing problem. *Management Science*, 40(10), 1276–1290.

6. Haghani Ali, Tian Qiang, and Hu Huijun, (2004), Simulation Model for Real-Time Emergency Vehicle Dispatching and Routing, Transportation Research Record: Journal of the Transportation Research Board, Volume 1882, Issue 1, https://doi.org/10.3141/1882-21

7. Laporte, G. (2007). What you should know about the vehicle routing problem. *Naval Research Logistics (NRL)*, 54(8), 811-819.

8. Liu, R., Jiang, Z., & Xu, J. (2016). A dynamic patrol routing algorithm for security robots. *IEEE Transactions on Automation Science and Engineering*, 13(3), 1135–1147.

9. Maciejewski, M., & Nagel, K. (2012). Towards multi-agent simulation of the dynamic vehicle routing problem in MATSim. *Transport Research Procedia*, 10, 433–446.