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Solution for "Good" Boussinesq Equation by Applying Differential Method

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ABSTRACT

In the present problem, we study travelling wave solution for 'good' Boussinesq equation by differential transform method [Lou (1999), Erturk (2007), Arikoglu (2006)], also applied Pade and convergence has been discussed. This method reduces the size of computational work compared to Taylor series method which requires computationally long time for large orders.

Keywords : Differential Method, Good BE, Pseudo Spectral Method

1. Introduction

The 'good' Boussinesq equation describes shallow water waves propagating in both directions. Kamel Al Khaled and Ameina S. Nusier [2008] have derived wave solutions for 'good' Boussinesq equation using Adomian decomposition method and Galerkin method. Marjin Uddin, Sirajul Haq, Muhammad Ishaq [1990] have solved 'good' Boussinesq equation using radial basic function pseudospectral method,

The Boussinesq equation [1872] is an example of soliton producing equation which is given by

$$u_{tt} = u_{xx} + u_{xxxx} + \left(u^2\right)_{xx}. \tag{1}$$

2. Differential method

The "good" Boussinesq equation is given by,

$$u_{tt} = u_{xx} - u_{xxxx} + \left(u^2\right)_{xx}, \tag{2}$$

with initial conditions

$$u(x,0) = u_0(x)$$
$$u_t(x,0) = v_0(x). \tag{3}$$

Let us consider traveling wave solution to "good" Boussinesq equation (2). Taking $z = x - ct$ in equation (2) and integrating twice we get

$$u_{zz} = u^2 - \left(c^2 - 1\right)u \quad (4)$$

Let us consider "good" Boussinesq equation (4) for $c = 0.5$, which will be as follows

$$u_{zz} = u^2 + 0.75u, \quad (5)$$

with initial conditions

$$u(0) = -\frac{9}{8}, \quad u'(0) = 0 \quad (6)$$

Applying differential transformation to both sides of equation (5) and (6)

we get the following solution,

$$u(z) = -1.125 + 0.210938z^2 - 0.026367z^4 + 0.002801z^6 - 0.0002736z^8 + 0.0000254z^{10}$$
$$-2.2827\times10^{-6}z^{12} + 2.00154\times10^{-7}z^{14} - 1.7237\times10^{-8}z^{16} + 1.46403\times10^{-9}z^{18}$$
$$-1.2296\times10^{-10}z^{20} + 1.0234\times10^{-11}z^{22} - 8.4532\times10^{-13}z^{24}. \quad (7)$$

4. Phase – Plane Analysis

We consider the reduced differential equation

(8)

This differential equation may be studied in phase plane by writing following two first order equations

$$\frac{du}{dz} = v \quad (9)$$

,

$$\frac{dv}{dz} = u^2 - \left(c^2 - 1\right)u. \quad (10)$$

Equation (10) and (9) have two singularities, which are

$(0,0)$, a saddle point, and $(-0.75,0)$ a centre

Conclusion

The "good" Boussinesq equation is similar to the well known Kdv and nonlinear Schrodinger equations, in the sense that it also produces soliton solutions. But it also possesses some other properties, for instance, solitary wave only exists for a finite range of velocity, also it can merge into a single solitary wave and it can interact and give antisoliton solution (Manoranjan et al., 1988).

From the graphs obtained here, we can conclude that using DTM–Pade technique in solving solitary wave equations is very efficient than using DTM alone. The DTM-Pade technique can be used in solving large scale of nonlinear differential equations.

6.6 Graphs

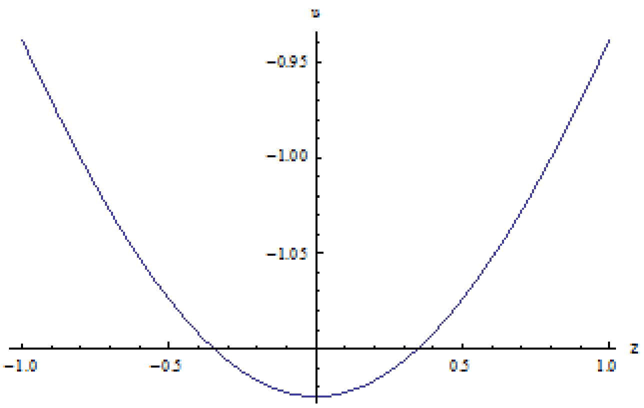
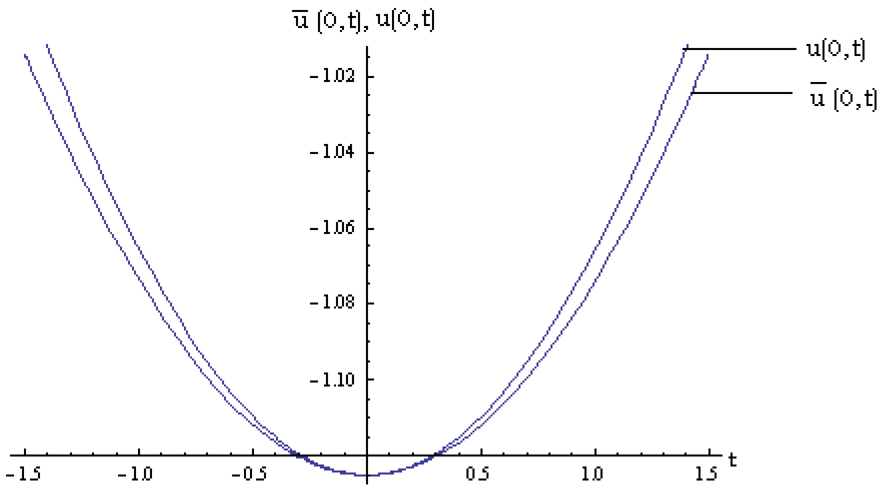


Fig 6.1 graph plotted for series obtained by DTM for u upto 25 terms



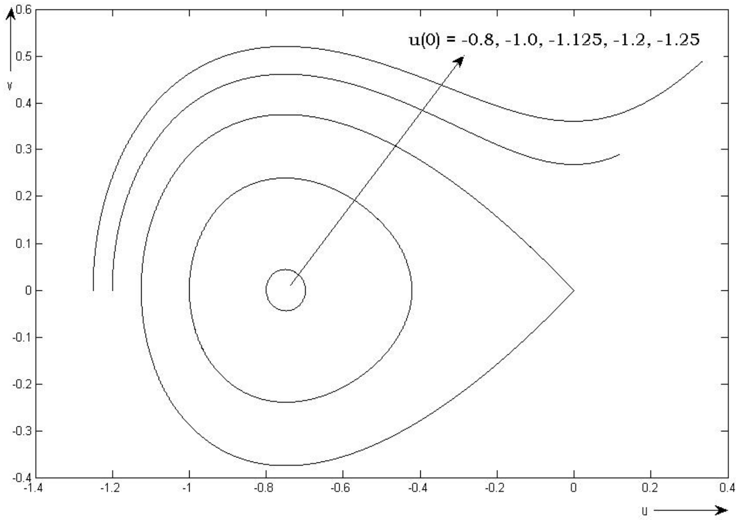


Fig 6.11 Phase plane of (u,v) showing $(0,0)$ as centre and $(-0.68,0)$ is saddle point (R-K-Merson)

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