© 2017 IJSRST | Volume 3 | Issue 8 | Print ISSN: 2395-6011 | Online ISSN: 2395-602X
Themed Section: Science and Technology

Thermoelastic Problem of Thin Finite Rectangular Plate with Heat Source by Marchi-Fasulo Transform Technique

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ABSTRACT

This paper concerned with the temperature, displacement and thermal stresses at any point of a thin rectangular plate due to internal heat source with third kind boundary conditions by Marchi-Fasulo Transform Technique.

Keywords : Thin Finite Rectangular Plate, Marchi-Fasulo Transform, Heats Source.

I. INTRODUCTION

Mostly plates are used in engineering applications such as aeronautical navel and structural field. A lot of analytical approaches for finding solution of the plane problem in terms of stresses are derived from last hundred years. The present paper is to gain an effective solution and good understanding of thermal stresses in thin rectangular plate due to internal heat source. Khobragade N.W. [1] has studied the inverse steady-state thermoelastic problem and to determine the temperature, displacement function and thermal stresses. Lamba et al. [2] have studied thermoelastic problem of thin rectangular plate due to partially distributed heat supply. Nowacki [3] has determined the steady-state thermal stresses in circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively.

Tanigawa and Komatsubara [5] and Vihak et al. [6] have studied the direct thermo elastic problem in a rectangular plate.

II. Statement of the problem

Consider a thin rectangular plate occupying the space $D: -a \le x \le a, -b \le y \le b, -h \le z \le h$, with displacement components u_x, u_y, u_z in the x, y, z direction respectively as

$$u_x = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - v\frac{\partial^2 U}{\partial x^2}\right) + \lambda T\right] dx \tag{2.1}$$

$$u_y = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - v\frac{\partial^2 U}{\partial y^2}\right) + \lambda T\right] dy \tag{2.2}$$

$$u_z = \int \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - v\frac{\partial^2 U}{\partial z^2}\right) + \lambda T\right] dz \tag{2.3}$$

where E, ν and λ are the Young modulus, the poisson ratio and the linear coefficient of thermal expansion of the material of the plate respectively and $U(x, y, z, t)$ is the Airy stress function which satisfies the differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^2 U(x, y, z, t) = -\lambda E\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T(x, y, z, t) \tag{2.4}$$

here $T(x, y, z, t)$ denotes the temperature of the thin rectangular plate satisfying the following differential equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\theta(x,y,z,t)}{k'} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{2.5}$$

where k' is thermal conductivity and k is the thermal diffusivity of the material of the plate and $\theta(x, y, z, t)$ is the heat generated within the rectangular plate for t >0 subject to initial conditions.

$$T(x, y, z, 0) = F(x, y, z) \tag{2.6}$$

The boundary conditions

$$\left[\mathrm{T(x, y, z, t)} + \mathrm{k}_1 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = F_1\,(y, z, t) \tag{2.7}$$

$$\left[\mathrm{T(x, y, z, t)} + \mathrm{k}_2 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = F_2\,(y, z, t) \tag{2.8}$$

$$\left[\mathrm{T(x, y, z, t)} + \mathrm{k}_3 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b} = F_3\,(x, z, t) \tag{2.9}$$

$$\left[\mathrm{T(x, y, z, t)} + \mathrm{k}_4 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b} = F_4\,(x, z, t) \tag{2.10}$$

$$\left[\mathrm{T(x, y, z, t)} + \mathrm{k}_5 \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=-h} = \mathrm{f}_1\,(x, y, t) \tag{2.11}$$

$$\left[\mathrm{T}(x,y,z,t) + \mathrm{k}_6 \frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = \mathrm{f}_2\,(x,y,t) \quad (2.12)$$

The components in term of $U(x,y,z,t)$ are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) \quad (2.13)$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right) \quad (2.14)$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) \quad (2.15)$$

The equations (2.1) to (2.15) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo integral transform stated in three times to the equations (2.5), (2.6) and using (2.7) (2.8) (2.9) (2.10) (2.11), (2.12), we obtain

$$-q^2\bar{\bar{\bar{T}}} + \bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}}{k'} = \frac{1}{k}\frac{d\bar{\bar{\bar{T}}}}{dt} \quad (3.1)$$

where $q^2 = {a_m}^2 + {a_n}^2 + {a_l}^2$,the eigen values a_m , a_n , a_l are the solution of the equations

$$[\alpha_1\, a_m \cos(a_m a) + \beta_1 \sin(a_m a)] \times [\beta_2 \cos(a_m a) + \alpha_2\, a \sin(a_m a)] = [\alpha_2\, a_m \cos(a_m a) - \beta_2 \sin(a_m a)] \times [\beta_1 \cos(a_m a) - \alpha_1\, a_m \sin(a_m a)] ,$$

$$[\alpha_1\, a_n \cos(a_n b) + \beta_1 \sin(a_n b)] \times [\beta_2 \cos(a_n b) + \alpha_2\, a \sin(a_n b)] = [\alpha_2\, a_n \cos(a_n b) - \beta_2 \sin(a_n b)] \times [\beta_1 \cos(a_n b) - \alpha_1\, a_n \sin(a_n b)] ,$$

$$[\alpha_1\, a_n \cos(a_n l) + \beta_1 \sin(a_n l)] \times [\beta_2 \cos(a_n l) + \alpha_2\, a \sin(a_n l)] = [\alpha_2\, a_n \cos(a_n l) - \beta_2 \sin(a_n l)] \times [\beta_1 \cos(a_n l) - \alpha_1\, a_n \sin(a_n l)]$$

and

$$\bar{\bar{\phi}} = P_m(a)\overline{\overline{F_2}} - P_m(-a)\overline{\overline{F_1}} + P_n(b)\overline{F_4} - P_n(-b)\overline{F_3} + P_l(h) f_2 - P_l(-h) f_1 \quad (3.2)$$

$$\bar{\bar{\bar{T}}}(m,n,l,0) = \bar{\bar{\bar{F}}}(m,n,l) \quad (3.3)$$

where $\bar{\bar{\bar{T}}}$ denotes the Marchi- Fasulo integral transform of T and m, n ,l are the parameters, $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants.

From equation (3.1), we obtain

$$\frac{d\bar{\bar{\bar{T}}}(m,n,l,t)}{dt} + kq^2\bar{\bar{\bar{T}}}(m,n,l,t) = k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}(m,n,l,t)}{k'}\right) \quad (3.4)$$

where equation (3.4) is first order differential equation has solution,

$$\bar{\bar{\bar{T}}}\,(m,n,l,t) = e^{-kq^2 t}\left[\int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}(m,n,l,t)}{k'}\right) e^{kq^2 t'}\, dt' + c\right]$$

where c is constant which to determine by using (3.3)

$$c = \bar{\bar{\bar{F}}}(m,n,l) \quad (3.5)$$

$$\bar{\bar{\bar{T}}}\,(m,n,l,t) = e^{-kq^2 t}\left[\int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}(m,n,l,t)}{k'}\right) e^{kq^2 t'}\, dt' + \bar{\bar{\bar{F}}}(m,n,l)\right] \quad (3.6)$$

Apply inverse finite Marchi-Fasulo Transform stated in (2.19) three times to the equation (3.6) and using boundary conditions, we obtain

$$\mathrm{T}(x,y,z,t) = \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right] e^{-k({a_m}^2+{a_n}^2+{a_l}^2)t}\bar{\bar{\bar{F}}}(m,n,l) + \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right]$$

$$\times \int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}(m,n,l,t)}{k'}\right) e^{-k({a_m}^2+{a_n}^2+{a_l}^2)(t-t')}\, dt' \quad (3.7)$$

substituting the value of $T(x,y,z,t)$ from equation (3.7) in the equation (2.4), we obtain

$$\mathrm{U}(x,y,z,t) = -\lambda E \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right] e^{-k({a_m}^2+{a_n}^2+{a_l}^2)t}\bar{\bar{\bar{F}}}(m,n,l) - \lambda E \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right] \times \int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\bar{\theta}}}(m,n,l,t)}{k'}\right) e^{-k({a_m}^2+{a_n}^2+{a_l}^2)(t-t')} dt' \quad (3.8)$$

A. Determination of Displacement Components

Substituting the value of $U(x,y,z,t)$ from equation (2.8) in (2.1), (2.2), (2.3), we obtain, the displacement functions u_x , u_y , u_z as

$$u_x = -\sum_{n,l=1}^{\infty}\frac{\lambda}{\lambda_m \mu_n \nu_l}\left(P''_n P_l + P_n P''_l - P_n P_l\right)\int_{-a}^{a} P_m(x)dx \times \left(\int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\theta}}}{k'}\right) e^{-k({a_m}^2+{a_n}^2+{a_l}^2)(t-t')} dt' + e^{-k({a_m}^2+{a_n}^2+{a_l}^2)t}\bar{\bar{F}}^*(m,n,l)\right)$$

$$+\sum_{n,l=1}^{\infty}\frac{\lambda \nu}{\lambda_m \mu_n \nu_l}\left(P_n P_l\right)\int_{-a}^{a} P_m(x)dx \times \left(\int_0^t k\left(\bar{\bar{\phi}} + \frac{\bar{\bar{\theta}}}{k'}\right) e^{-k({a_m}^2+{a_n}^2+{a_l}^2)(t-t')} dt' + e^{-k({a_m}^2+{a_n}^2+{a_l}^2)t}\bar{\bar{F}}^*(m,n,l)\right) \quad (3.9)$$

$u_y =$

$$-\sum_{n,l=1}^{\infty}\frac{\lambda}{\lambda_m\mu_n\nu_l}(P_m P''_l + P''_m P_l - P_m P_l)\int_{-b}^{b}P_n(y)dy \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' + \\ e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

$$+\sum_{n,l=1}^{\infty}\frac{\lambda v}{\lambda_m\mu_n\nu_l}(P_m P_l)\int_{-b}^{b}P_n(y)dy \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' + \\ e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

(3.10)

$u_z =$

$$-\sum_{n,l=1}^{\infty}\frac{\lambda}{\lambda_m\mu_n\nu_l}(P''_m P_n + P_m P''_n - P_m P_n)\int_{-h}^{h}P_l(z)dz \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' \\ +e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

$$+\sum_{n,l=1}^{\infty}\frac{\lambda v}{\lambda_m\mu_n\nu_l}(P_m P_n)\int_{-h}^{h}P_l(z)dz \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' \\ +e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

(3.11)

B. Determination of the Stress Functions

Using (2.8) in the equation (2.13), (2.14) and (2.15) the stress functions are obtained as

$\sigma_{xx} =$

$$-\lambda E\left(\sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P''_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P''_l(z)}{\nu_l}\right]\right) \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' + \\ e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

(3.12)

$$\sigma_{yy} = -\lambda E\left(\sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P''_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P''_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right]\right) \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' + \\ e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

(3.13)

$\sigma_{zz} =$

$$-\lambda E\left(\sum_{m,n,l=1}^{\infty}\left[\frac{P''_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P''_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right]\right) \times \left(\begin{array}{c}\int_0^t k\left(\bar{\bar{\phi}}+\frac{\bar{\bar{\theta}}}{k'}\right)e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)(t-t')}dt' + \\ e^{-k(a_m{}^2+a_n{}^2+a_l{}^2)t}\bar{\bar{F}}^*(m,n,l)\end{array}\right)$$

(3.14)

C. Special Case and Numerical Results

Setting,

$$\theta(x,y,z,t) = e^{-t}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \quad (3.15)$$

and the initial condition is

$$F(x,y,z,) = 0, and\ \emptyset = 0 \quad (3.16)$$

Applying finite Marchi-Fasulo transform three times we obtain

$$\bar{\bar{\theta}}(m,n,l,t) = \iiint_{x=-a,y=-b,z=-h}^{x=a,y=b,z=h} e^{t}\delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \times P_m(x)P_n(y)P_l(z)dxdydz$$

$$= [8(k_1+k_2)(k_3+k_4)(k_5+k_6)e^{t}] \times \left[\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right] \times \left[\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right] \times \left[\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right] \quad (3.17)$$

Substitute the values, we obtain

$$T(x,y,z,t) = k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right] \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \quad (3.18)$$

$$U(x,y,z,t) = -\lambda Ek[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(y)}{\mu_n}\right]\left[\frac{P_l(z)}{\nu_l}\right] \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.19}$$

$$u_x = -\lambda k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P''_n P_l + P_n P''_l - P_n P_l)\int_{-a}^{a}P_m(x)dx\, e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t}$$
$$+\lambda\upsilon k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P_n P_l)\int_{-a}^{a}P_m(x)dx \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.20}$$

$$u_y = -\lambda k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P_m P''_l + P''_m P_l - P_m P_l)\int_{-b}^{b}P_n(y)dy \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t}$$
$$+\lambda\upsilon k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P_m P_l)\int_{-b}^{b}P_n(y)dy \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.21}$$

$$u_z = -\lambda k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P''_m P_n + P_m P''_n - P_m P_n)\int_{-h}^{h}P_l(z)dz \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t}$$
$$+\lambda\upsilon k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \sum_{n,l=1}^{\infty}\frac{1}{\lambda_m\mu_n\nu_l}(P_m P_n)\int_{-h}^{h}P_l(z)dz \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.22}$$

$$\sigma_{xx} = -\lambda Ek[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right) \times \left(\sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P''_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P''_l(z)}{\nu_l}\right]\right) \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.23}$$

$$\sigma_{yy} = -\lambda Ek[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right)$$

$$\times \left(\sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P''_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P''_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right]\right) \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.24}$$

$$\sigma_{zz} = -\lambda Ek[8(k_1+k_2)(k_3+k_4)(k_5+k_6)] \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(a_n b)cos^2(a_n b)-\cos(a_n b)sin(a_n b)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(a_l h)cos^2(a_l h)-\cos(a_l h)sin(a_l h)}{a_l{}^2}\right)$$

$$\times \left(\sum_{m,n,l=1}^{\infty}\left[\frac{P''_m(x)}{\lambda_m}\frac{P_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right] + \sum_{m,n,l=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\frac{P''_n(y)}{\mu_n}\frac{P_l(z)}{\nu_l}\right]\right) \times e^{-k(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.25}$$

D. Numerical Results

The numerical calculation has been carried out for copper thin rectangular plate. set $B = \propto k[8(k_1+k_2)(k_3+k_4)(k_5+k_6)]$, a=1 cm, b=2 cm, h=3 cm, thermal diffusivity $k = 4.42 ft^2/hr$, thermal conductivity k'=224 Btu/hr ft ^{0}F, t=1sec., $k_1 = k_2 = k_3 = k_4 = k_5 = 1$, modulus elasticity $E = 6.9 \times 10^{11}$, Poisson ratio $\upsilon = 0.48$, $\lambda = 12.84 \times 10^{-6}$,

We obtain, the temperature function

$$\frac{T(x,y,z,t)}{B} = \times \sum_{m=1}^{\infty}\left(\frac{(a_m a)cos^2(a_m a)-\cos(a_m a)sin(a_m a)}{a_m{}^2}\right) \times \sum_{n=1}^{\infty}\left(\frac{(2a_n)cos^2(2a_n)-\cos(2a_n)sin(2a_n)}{a_n{}^2}\right) \times \sum_{l=1}^{\infty}\left(\frac{(3a_l)cos^2(3a_l)-\cos(3a_l)sin(3a_l)}{a_l{}^2}\right)$$

$$\times \left[\frac{P_m(x-x_0)}{\lambda_m}\right]\left[\frac{P_n(y_0)}{\mu_n}\right]\left[\frac{P_l(z_0)}{\nu_l}\right] \times e^{-\propto(a_m{}^2+a_n{}^2+a_l{}^2+1)t} \tag{3.26}$$

E. Graphical Analysis

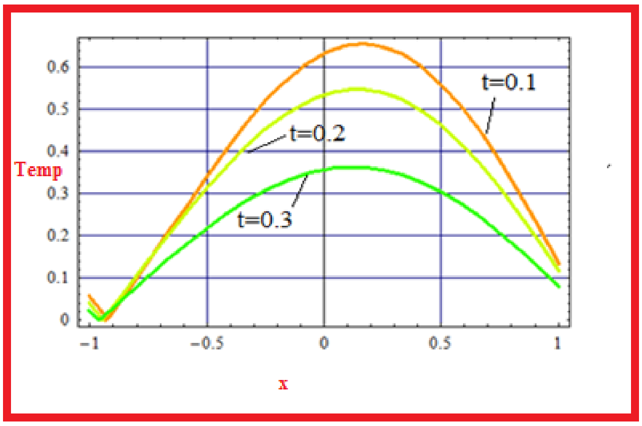

Fig:1 shows that variation of temperature T(x, y, z) Vs x. It is clear that temperature slightly increases at time t=0.1 sec, t=0.2 sec, t=0.3 sec in positive part of x and then suddenly decreases. .

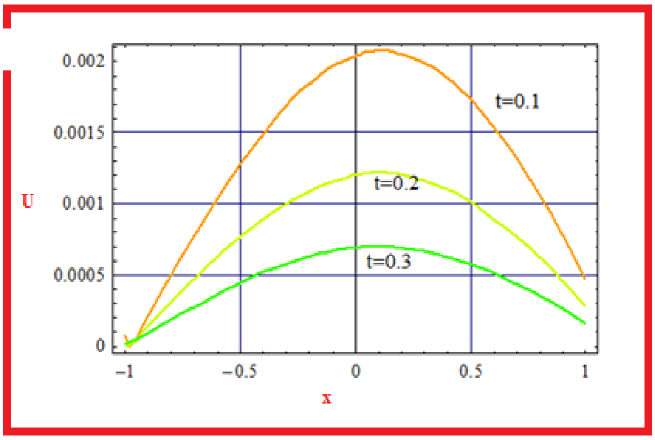

Fig:2 shows that variation of stress function U(x, y, z) Vs x. It is clear that stress slightly increases at time t=0.1 sec, t=0.2 sec, t=0.3 sec in positive part of x and then suddenly decreases.

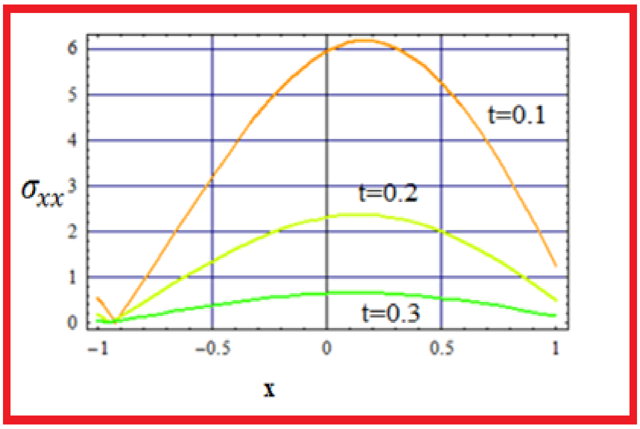

Fig:3 shows that variation of stress function σ_{xx} Vs x. It is clear that stress slightly increases at time t=0.1 sec, t=0.2 sec, t=0.3 sec in positive part of x and then suddenly decreases.

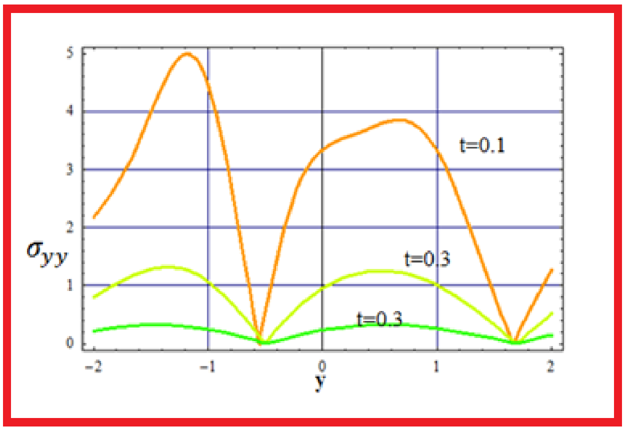

Fig:4 shows that variation of stress function σ_{yy} Vs y. It is clear that stress slightly increases at time t=0.1 sec, t=0.2 sec, t=0.3 sec in positive part of y and then suddenly decreases at y=1.6 up to zero.

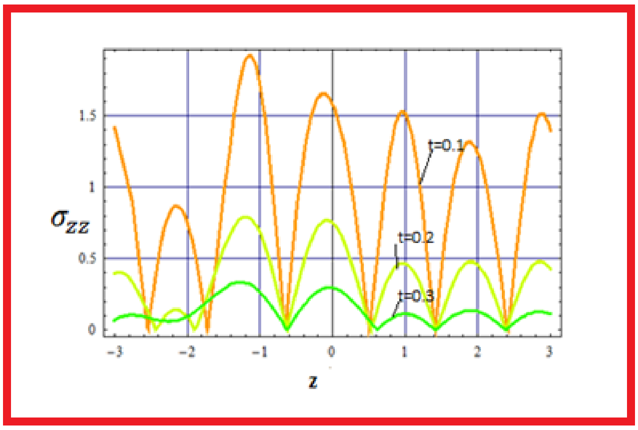

Fig.5 shows that variation of stress function σ_{zz} Vs z. It is clear that stress suddenly decreases at time t=0.1 sec, t=0.2 sec, t=0.3 sec in positive part of z up to zero at z=0.6, z=1.4, z=2.4 and it attains peak value at z=1 and z=2 .

IV. CONCLUSION

In this paper, I have discussed the thermo elastic problem of thin rectangular plate, where the non-homogeneous condition of the third kind on the edges x = -a, a , y = -b, b and z = -h, h, for t >0 heat is generated within the rectangular plate. The finite Marchi-Fasulo integral transform is used and obtained the numerical result. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable value of the parameters and function in the expression.

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