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Λ-Ib-Sets in Topological Space

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ABSTRACT

In this paper we studied a new type of sets that called Λ-Ib-sets, for planting this sets it was necessary to know about b-opens sets were introduced by [4], b-I-opens sets were introduced by [2] and the notion of ideal in topological space was defined by [5], thus we used those concepts and created the sets which were studied, furthermore we studied relationship between Λ-I sets and Λ-Is sets, in fact some properties were planted, on the other hand, the definition of continuity from Λ-Ib-sets was added and some examples are shown.

Keywords : Λ-Ib-Sets, Function Local, B-I-Open, Topological Space.

I. INTRODUCTION

For several years many authors have been studying the relationship between any opens sets and ideal in topological space, these researches have shown how important is investigate in this topic, because it will find others types of sets like studied in this paper, in this way Sanabria et al. have found relationship with I-opens sets and ideal topological [9], besides Aysegul and Gulhan have studied b-I-opens sets with ideal topological [2], hence those researches helped to design this research, meanwhile, continuity in topological spaces has been studied by several authors, the main author is Noiri who investigated this topic and added many properties and they are useful in this investigation.

On the other hand, we introduced a new set, this set is called $\Lambda\text{-Ib}=\cap\{U: A \subset U, U \in BIO(X,\tau)\}$, in the chapter III we proof that whole b-I-opens= $Cl^*(int(A)) \cup int(Cl^*(A))$ is Λ-Ib. As well as you saw, Cl^* means Kuratowski's closure where we need to find τ^*-opens. Moreover in this paper the interior of any set is denoted by int and the closure of any set is denoted by cl.

II. PRELIMINARIES

Definition 2.1: Let A subset X, A is called b-open [4] if $A \subset Cl(int(A) \cup int(Cl(A))$.

Remark: This type of set is represented as $BO(X, \tau)$

Definition 2.2: Let $I \neq \emptyset$, I is a collection of subset of X, I is ideal in X [10] if satisfices the properties below:

1. If $A \subset B$ and $B \in I$, then $A \in I$. (Hereditary property)
2. If $A \in I$ and $B \in I$, then $A \cup B \in I$. (Additive property)

Definition 2.3: Let X a set, A mapping $\Omega: P(X) \rightarrow P(X)$ is called kuratowski's operator closure [10] if satisfices the properties below:

1. $A \subset \Omega(A)$
2. $\Omega(\Omega(A)=\Omega(A)$
3. $\Omega(A \cup B)= \Omega(A) \cup \Omega(B)$
4. $\Omega(\emptyset)= \emptyset$

For any A, B ∈ X.

Definition 2.4: Let (X, τ, I) a topological space, for any subset A in X, the local function is defined [10] by A with I and τ, it is denoted by $A^*(I, \tau)$, as:
$A^*(I, \tau)=\{x \in X: U \cap A \notin I \text{ for each } U \in \tau(x)\}$, where $\tau(x)= \{U \in \tau : x \in U\}$.

Remark: When does not exist confusion, we will write A^* instead $A^*(X, \tau)$.

Definition 2.5: Let (X, τ, I) a topological space, for any subset A in X, it defines $Cl^*(A) = A \cup A^*$ [5].

Definition 2.6: Let A subset of X, A is called I-open [3] if $A \subset int(A^*)$.

Remark: I-open set is represented by IO

Definition 2.7: Let A subset of X, A is called semi-I-open [6] if $A \subset Cl^*(int(A))$.

Remark: Semi-I-open set is represented by SIO

Definition 2.8: Let (X, τ, I) a topological space is called Hayashi Samuels (E.H.S) if $\tau \cap I \neq \{\emptyset\}$.

III. Λ-IB-SETS

Definition 3.1: A set A is called b-I-open if $A \subset Cl^*(int(A)) \cup int(Cl^*(A))$.

Remark: b-I-opens sets are represented by $BIO(X, \tau)$ or BIO, it also important to mention that the complement of a b-I-open it is b-I-closed.

Theorem 3.2: Let V open in X, then V is b-I-open.

Proof: Let V open in X, by definition 3.1 $V \subset Cl^*(int(A)) \cup int(Cl^*(A))$, as well-know, V is open, then $int(V)=V$, therefore $V \subset Cl^*(V) \cup int(Cl^*(V)) = (V \cup V^*) \cup int(Cl^*(V))$, in consequence, V is in the operation in bracket.

The following example shows the reciprocal of the last theorem it is not always true.

Example 3.3: Let $X=\{a,b,c\}$, $\tau=\{\emptyset, X, \{a,b\}\}$ and $I=\{\emptyset, \{c\}\}$. The unitary set $\{a\}$ is b-I-open, but it is not open in X.

Theorem 3.4: Let A subset from (X, τ, I), then the following properties are satisficed:

1. A is SIO-open, then A is BIO-open
2. A is IO-open and X is E.H.S, then A is BIO-open
3. A is BIO-open with $I=\{\emptyset\}$, then A is BO-open

Proof:

1. Let A SIO-open, by definition 2.7, $A \subset Cl^*(int(A))$, using addition's rule, we have, $A \subset Cl^*(int(A)) \cup int(Cl^*(A))$, this shows that A is BIO-open.
2. Let A IO-open, by definition 2.6, $A \subset int(A^*)$, as well-know X is E.H.S. then, $A \subset int(A \cup A^*) = int(Cl^*(A)) \subset int(Cl^*(A)) \cup int(Cl^*(A))$, this shows that A in BIO-open.
3. Let A BIO-open, by definition 3.1, $A \subset Cl^*(int(A)) \cup int(Cl^*(A)) = int(A) \cup (int(A))^* \cup int(Cl^*(A))$, as $(int(A))^* \subset Cl(int(A))$ and as well-known $I=\{\emptyset\}$, then $Cl^*(A)=Cl(A)$, therefore $A \subset Cl(int(A)) \cup int(Cl(A))$.

Remark: In the last theorem point 2, if X does not E.H.S., $A \subset A^*$ probably will not be true and point 3, if $I \neq \{\emptyset\}$, $Cl^*(A)=Cl(A)$ probably will not be true.

Definition 3.5: Let (X, τ) a topological space and A subset of X, it defines kernel of BIO-open as $bKer(A) = \cap\{U: A \subset U, U \in BIO(X,\tau)\}$.

Definition 3.6: Let (X, τ, I) a topological space and A subset of X, it defines Λ-Ib(A) set as whole intersection of b-I-opens that contain A set, this mean $\Lambda\text{-}Ib(A)=\{\cap\{U: A \subset U, U \in BIO(X,\tau)\}$.

Definition 3.7: A subset A of (X, τ), it is Λ-Ib if $A=bKer(A)$.

Lemma 3.8: Let (X, τ, I) a topological space and A,B subset of X, then:

1. If $U_{\alpha} \in$ BIO-open, for each $\alpha \in \Psi$, then $\cup\{A_{\alpha}: \alpha \in \Psi\} \in$ BIO-open.
2. If $A \in$ BIO-open and $B \in \tau$, then $A \cap B \in$ BIO-open.

Proof:

1. Let $U_{\alpha} \in$ BIO-open for each $\alpha \in \Psi$, we have $U_{\alpha} \subset int(U_{\alpha}^{*})$ for each $\alpha \in \Psi$, then $\cup_{\alpha \in \Psi}(U_{\alpha}) \subset int((\cup_{\alpha \in \Psi}(U_{\alpha}))^*)$, therefore $U_{\alpha} \in$ BIO-open.
2. Let $A \in$ BIO-open and $B \in \tau$, then $A \cap B \subset int(A^*) \cap B \subset int((A \cap B)^*)$, this implies $A \cap B \in$ BIO-open.

The following example shows the intersection of any BIO-open is not always BIO-open.

Example3.9: Let X={ a,b,c,d}, τ={ ø, X,{a},{b},{a,b},{a,b,d}, {a,b,c}} and I={ ø,{d}}. We choose random A={b,c,d} and B={a,c,d}, where A,B ⊂ X and A,B ∈ BIO-open, we find A∩B={c,d}, but Cl*(int({c,d}))= ø and int(Cl*({c.d}))= ø, therefore {c,d}∉ ø, in addition {c,d}∉BIO-open.

Theorem 3.10: Let A, B and $\{A_{\alpha}:\alpha \in \Psi\}$ subset of (X, τ,I), then:

1. A ⊂ Λ-Ib(A)
2. A ⊂ B, then Λ-Ib(A) ⊂ Λ-Ib(B)
3. Λ-Ib(Λ-Ib(A)) = Λ-Ib(A)
4. If A ∈ BIO-open, then A= Λ-Ib(A)
5. Λ-Ib($\cup_{\alpha \in \Psi} A_{\alpha}$)= $\cup_{\alpha \in \Psi}$Λ-Ib(A_{α})
6. Λ-Ib($\cap_{\alpha \in \Psi} A_{\alpha}$) ⊂ $\cap_{\alpha \in \Psi}$Λ-Ib(A_{α})

Proof:

1. Let x ∈ A, then x ∈ U ∈ BIO-open, such that A ⊂ U, thus x ∈ ∩{U: A ⊂ U, U ∈ BIO-open} = Λ-Ib(A).
2. Let A x ∈ Λ-Ib(B), then U ∈ BIO-open, such that B ⊂ U and x ∈ U, as well-known A ⊂ B, this implies x ∈ Λ-Ib(A), in consequence Λ-Ib(A) ⊂ Λ-Ib(B).
3. By part (1) we have Λ-Ib(A) ⊂ Λ-Ib(Λ-Ib(A)), now suppose that x ∉ Λ-Ib(A), then this shows Λ-Ib(Λ-Ib(A)) ⊂ Λ-Ib(A)
4. One part is proved using part (1), the another part we choose A ∈ BIO-open and A ⊂ A, then Λ-Ib(A) ⊂ A.
5. As well-known $A_{\alpha} \subset \cup_{\alpha \in \Psi} A_{\alpha}$ and using part (2) implies ($\cup_{\alpha \in \Psi}$ Λ-Ib (A_{α}) ⊂ Λ-Ib($\cup_{\alpha \in \Psi} A_{\alpha}$), for proving the another part we will use contradiction, then we obtain Λ-Ib($\cup_{\alpha \in \Psi} A_{\alpha}$) ⊂ ($\cup_{\alpha \in \Psi}$ Λ-Ib (A_{α}).
6. Proof is similar at the point above number (5).

Definition 3.11: Let A subset of (X, τ,I) is called Λ-Ib-set if A= Λ-Ib(A).

Lemma 3.12: Let (X, τ,I) a topological space, then:

1. For each subset A of X, for all Λ-Ib(A) is a Λ-Ib-set.
2. If A is b-I-open, then A is Λ-Ib-set.
3. Any union of Λ-Ib-set is a Λ-Ib-set.
4. Any intersection of Λ-Ib-set is a Λ-Ib-set.

Proof:

1. Using part (3) of theorem 3.10, we have Λ-Ib(Λ-Ib(A)) = Λ-Ib(A), thus this proved it is Λ-Ib(A)-set.
2. As well-known A is b-I-open, then using part (4) of theorem 3.10, it has A= Λ-Ib(A), thus this proved it is Λ-Ib(A)-set.
3. The proof is similar part (5) of theorem 3.10.
4. The proof is similar part (6) of theorem 3.10.

Theorem 3.13: Any Λ-Is(A)-set is Λ-Ib(A)-set.

Proof: As well-known Λ-Is(A)= ∩{U: A ⊂ U, U ∈ SIO-open}, by theorem 3.6 any SIO-open is BIO-open, therefore A ⊂ U ∈ SIO-open ∈ BIO-open.

Theorem 3.14: Let (X, τ,I) a topological space, then any Λ-Ib(A)-set is Λ-Ib(A)-closed.

Proof: If A is Λ-Ib(A)-set, then A = A ∩ X, A is Λ-Ib(A)-set and X is τ*-closed, this proved A is Λ-Ib(A)-closed.

Theorem 3.15: Let $\{A_{\alpha}: \alpha \in \Psi\}$ a collection of subset (X, τ,I) a topological space. If A_{α} is Λ-Ib(A)-closed for each α ∈ Ψ, then $\cap\{A_{\alpha}:\alpha \in \Psi\}$ is Λ-Ib(A)-closed.

Proof: Suppose A_{α} is Λ- Ib(A)-closed for each α ∈ Ψ, exists Λ- Ib(A)-set L_{α} and τ*-closed F_{α} such that $A_{\alpha}= L_{\alpha} \cap F_{\alpha}$, therefore $\cap_{\alpha \in \Psi}A_{\alpha}= \cap_{\alpha \in \Psi}(L_{\alpha} \cap F_{\alpha})$, then $\cap_{\alpha \in \Psi}A_{\alpha}$ is Λ-Ib(A)-closed.

Corollary 3.16: Let $\{B_{\alpha}: \alpha \in \Psi\}$ a collection of subset (X, τ,I) a topological space. If B_{α} is Λ-Ib(A)-open for each α ∈ Ψ, then $\cap\{B_{\alpha}:\alpha \in \Psi\}$ is Λ-Ib(A)-closed.

Proof: Consequence D'morgan's rules and theorem 3.15.

IV. FUNCTION Λ-IB-CONTINUOUS

Definition 4.1: Let f: (X, τ,I) ⟶ (Y,μK), where τ, μ are topological space and I,K are ideal, f is called Λ-Ib-continuous if f-1(V) is Λ-Ib-open in X for each V-open in Y.

Theorem 4.2: Let f a function Λ-Is-continuous, then f is Λ-Ib-continuous.

Proof: Using theorem 3.13 any Λ-Is-open is Λ-Ib-open, in consequence f is Λ-Ib-contiuous.

Theorem 4.3: Let f a function IO-continuous and X E.H.S., then f is Λ-Ib-continuous.

Proof: Using Theorem 3.4 any IO-open is BIO-open, but any BIO-open is Λ-Ib-open, in consequence f is Λ-Ib-continuous.

V. CONCLUSION

The main aim of this paper was shown the principal results obtained trough of this investigation, besides it was looked how is the behavior of b-open if we add an ideal I, it was also shown the relationship between others types of sets, in the last section is shown a little introduction about continuity with Λ-Ib-sets and a question for this is, what will happen if we add either axioms of separation or connected?, that will be a good research to explain more about this topic.

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