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Temperature-Dependent Viscosity and Prandtl Number Effects on Natural Convection Methanol Boundary Layers about a Vertical Plate with Injection

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ABSTRACT

The present study deals with the effect of temperature-dependent viscosity and Prandtl number on the steady, natural, laminar flow of methanol past a vertical porous plate with injection. The coupled nonlinear partial differential equations governing the non-similar flow have been solved numerically using an implicit finite-difference scheme along with the quasilinearization technique. Numerical results indicate that temperature-dependent viscosity and Prandtl number, both have a major role on skin friction and heat transfer parameters as well as velocity and temperature fields.

Keywords : Heat transfer, Injection, Skin friction, Temperature-dependent Viscosity, Temperature.
MSC 2010 Codes *– 76M20, 76N20, 76R10*

I. INTRODUCTION

Applications of heat transfer are generally based on the constant physical properties of the ambient fluid in fluid dynamics research. However, it is known that these properties may change with temperature, especially the fluid viscosity and hence, the Prandtl number. Numerous researchers have studied the effect of variable viscosity on different geometries under various situations [1-7].

Free convection boundary layer flows oftenly encountered in environmental and engineering devices. Abundant literature is available on the topic of the laminar boundary layer flow over a porous vertical plate with suction and injection, having wide range of engineering applications. In many practical problems like film cooling, control of boundary layers, etc. suction and injection of fluid through porous heated or cooled surface plays a major role. In fact, the case of uniform suction and blowing (injection) through an isothermal vertical wall was treated first by Sparrow and Cess [8]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [9], who obtained asymptotic solutions, valid at large distances from the leading edge, for both suction and blowing (injection).

The present study is undertaken to investigate the effect of variable viscosity and Prandtl number on the free convection boundary layer flow (of methanol) over a vertical porous plate with injection. It may be mentioned here that methanol is a liquid in room temperature, used in thousands engineering and industrial applications including plastics, paints, cosmetics and fuel industries.

II. GOVERNING EQUATIONS

We consider a semi-infinite porous plate, which is played vertical in a quiescent fluid (methanol) of infinite extent maintained at an uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge (origin) in the stream wise direction and y is measured normal to the surface of the plate. Indeed, the flow is assumed to be in the x-direction i.e., along the vertical plate in the upward direction and the y-axis is taken to be normal to the plate.

The fluid properties are assumed to be isotropic and constant except for the fluid viscosity. The temperature difference between the surface of the plate (Tw) and the ambient fluid (T) is taken to be small. In the range of temperature (T) considered (i.e. 0-600C),the variation of both density () and specific heat (cp) of methanol with temperature is small and hence they are taken as constants.[See Table I] However, the viscosity (u) and thermal conductivity (k) [and hence the Prandtl number (Pr)] are assumed to vary as an inverse linear function of temperature:

$$\mu = 1/(b_1 + b_2 T) \quad (1)$$
$$Pr = 1/(c_1 + c_2 T) \quad (2)$$

where

$$b_1 = 126.23, \quad b_2 = 2.438, \quad c_1 = 0.1109 \text{ and } c_2 = 0.0013 \quad (3)$$

The numerical values required for these correlations are taken from table I [10].

Table I. Values of thermo-physical properties of methanol at different temperature [10]

Temperature (T^0C)	Density (ρ) (gr./cm^3)	Specific heat(c_p) (J × 10^7/kg ^{0}K)	Thermal conductivity (k) (erg × 10^5/cm.s-^{0}K)	Viscosity (μ) (g. × 10^{-2}/ cm-s)	Prandtl number (Pr)
0	0.813	2.399	0.207	0.777	9.005
10	0.804	2.449	0.204	0.664	7.971
20	0.794	2.504	0.201	0.575	7.163
30	0.785	2.566	0.199	0.504	6.498
40	0.775	2.633	0.196	0.447	6.004
50	0.765	2.706	0.193	0.399	5.594
60	0.755	2.785	0.190	0.360	5.276

The relation (1) and (2) are reasonably good approximations for liquids such as methanol, particularly for small wall and ambient temperature differences. Further, the fluid added (injection) or removed (suction) is the same as that involved in flow. The Boussinesq's approximation employed for the fluid properties to relate density changes in the flow field. Under the above-mentioned assumptions, the boundary layer equations governing the steady, two-dimensional flow are [9]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left[\left(Pr^{-1}\mu\frac{\partial T}{\partial y}\right)\right] \quad (6)$$

The initial and boundary conditions are

$$\begin{aligned} &x = 0, y > 0; u = v = 0, T = T_w \\ &x > 0; y = 0; u = 0, v = -v_0 \ (for\ suction), \\ &x > 0; y = 0; u = 0, v = +v_0 \ (for\ injection), \\ &y \to \infty; x > 0; u = 0, T = T_\infty \end{aligned} \quad (7)$$

Introducing the following transformations

$$u = \frac{\partial\psi}{\partial y}; v = -\frac{\partial\psi}{\partial x};$$

$$\psi = \frac{v^2 g\beta(T_w - T_\infty)\xi^3}{V_0{}^3}\left[f(\eta,\xi) \pm \frac{\xi}{4}\right]$$

$$T = T_\infty + (T_w - T_\infty)G(\eta,\xi); \ \eta = \frac{V_o y}{v\xi}$$

$$\xi = V_0\left[\frac{4x}{v^2 g\beta(T_{wo} - T_\infty)}\right]^{1/4} \quad (8)$$

to Equations (4) – (6), the continuity Equation (4) is identically satisfied and Equations. (5)–(6) reduces, respectively, to

$$(NF')' + 3fF' - 2F^2 \pm \xi F' + G = \xi\left(FF_\xi - F'f_\xi\right) \quad (9)$$

$$(N\,Pr^{-1}\,G')' + 3fG' \pm \xi G' = \xi\left(FG_\xi - G'f_\xi\right) \quad (10)$$

where

$$u = \frac{{V_0}^2 4x}{\nu \xi^2} F;\, v = -\frac{V_0}{\xi}\left(3f + \xi f_\xi - \eta F \pm \xi\right)$$

$$f = \int_0^\eta F d\eta\,;\, N = \left(\frac{\mu}{\mu_\infty}\right) = \frac{b_1 + b_2 T_\infty}{b_1 + b_2 T} = \frac{1}{1 + a_1 G}',$$

$$Pr = \frac{1}{c_1 + c_2 T} = \frac{1}{a_2 + a_3 G}',\quad a_1 = \left(\frac{b_2}{b_1 + b_2 T_\infty}\right)\Delta T_w,$$

$$a_2 = c_1 + c_2 T_\infty,\ \ a_3 = c_2 \Delta T_w,\ \ \Delta T_w = (T_w - T_\infty) \quad (11)$$

It is noted here that the upper and lower signs in Eqns. (9) and (10) is taken for suction and injection, respectively. The present study, however, restricted case of injection only. The transformed boundary conditions are:

$$\begin{aligned} F = 0;\ G = 1\ \ at\ \ \eta = 0 \\ F = 0;\ G = 0\ \ as\ \ \eta \to \infty \end{aligned} \quad (12)$$

The local skin friction and heat transfer parameters can be expressed, respectively, as

$$\tau_w = \frac{V_0}{g\beta(T_{wo} - T_\infty)}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi(F')_{\eta=0} \quad (13)$$

$$Q = \frac{\nu}{V_0(T_{wo} - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{1}{\xi}(G')_{\eta=0}\ (\xi \neq 0) \quad (14)$$

Here, u and v are velocity components in x and y-directions respectively; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimensionless stream functions respectively Pr is the Prandtl number; □1,□2,□3,□1,□2,□1 □□□ □2 are constants; g is the gravitational acceleration; □ is the coefficient of thermal expansion; w and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively, the subscript ξ and prime (′) denote, respectively partial derivatives with respect ξ and η.

III. RESULTS AND DISCUSSION

The system of dimensionless nonlinear coupled partial differential equations (9)-(10) with boundary conditions (12) has been solved numerically employing an implicit finite-differrnece method with aquasilinearzation technique [11, 12]. In order to assess the accuracy of the numerical method which we have used, the skin friction and heat transfer parameters () for injection have been obtained by solving equations (9) and (10) for constant viscosity [N=1] case, taking Pr=1.0, and compared with those of Merkin [9]. Our results are found to be in good agreement with those of [9], as shown in Fig.1, validating the accuracy of the numerical method used in the present study. The computed results for variable viscosity as well as Prandtl number have been presented in the graphical form and analyzed.

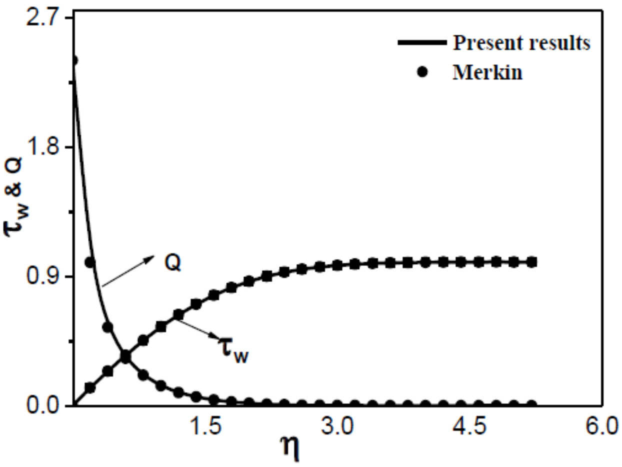

Fig. 1. Comparison of skin friction and heat transfer parameters with Merkin [9] for injection

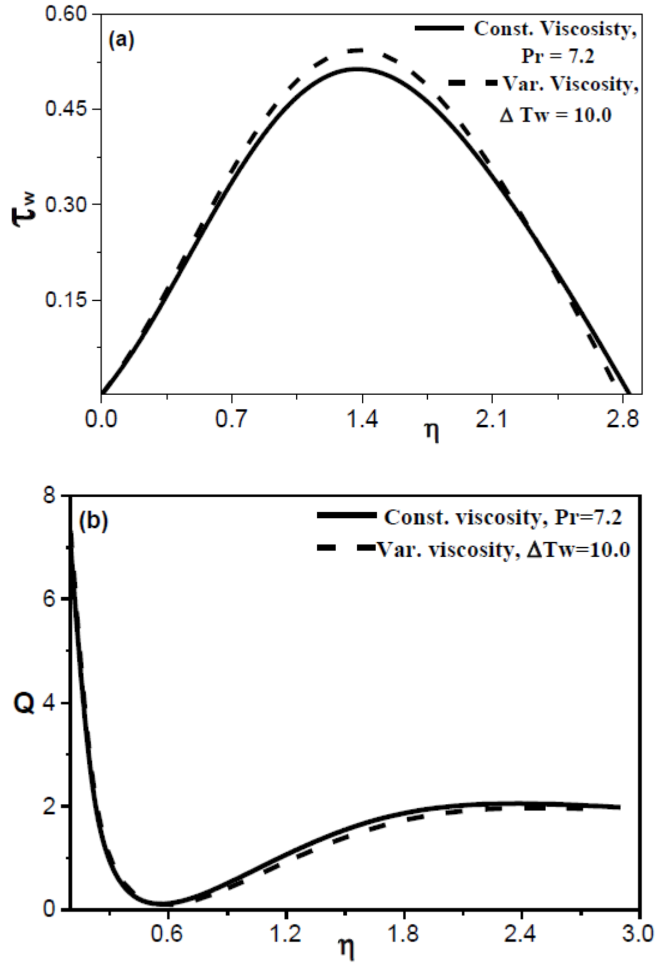

Fig.2. Variation of (a) skin friction and (b) heat transfer parameters along stream-wise directions

Figure 2 describes the variation of skin friction (□□)and heat transfer parameters (□) with the stream wise coordinate □, in the presence of both variable fluid properties [T∞ =28.0oC,

Tw= 10.0] and constant fluid properties [N =1 and Pr = 7.2 for methanol at room temperature] and injection. It is observed from Fig.2(a) that skin friction (□□) increases from zero to a maximum value in a certain range of □ =1.4, and then decreases as □ further increases. It is also observed that the effect of variable fluid properties is to increase the skin friction and to decrease the heat transfer. In fact, for variable fluid properties differs from that of constant fluid properties by about 55.14% while ,the percentage of difference in the case of (□)is about 10.69% [Fig.2(b)], at the stream-wise coordinate ξ =1.5 Further, it is observed that the zero-skin friction is moved upstream in the presence of variable fluid properties. Indeed, in the case of constant fluid properties zero skin friction occurs at the stream-wise location ξ = 2.9 whereas for variable fluid properties, the same occurs at ξ = 2.8 This justifies the advancement in the boundary layer separation under the influence of variable viscosity, Prandtl number and injection.

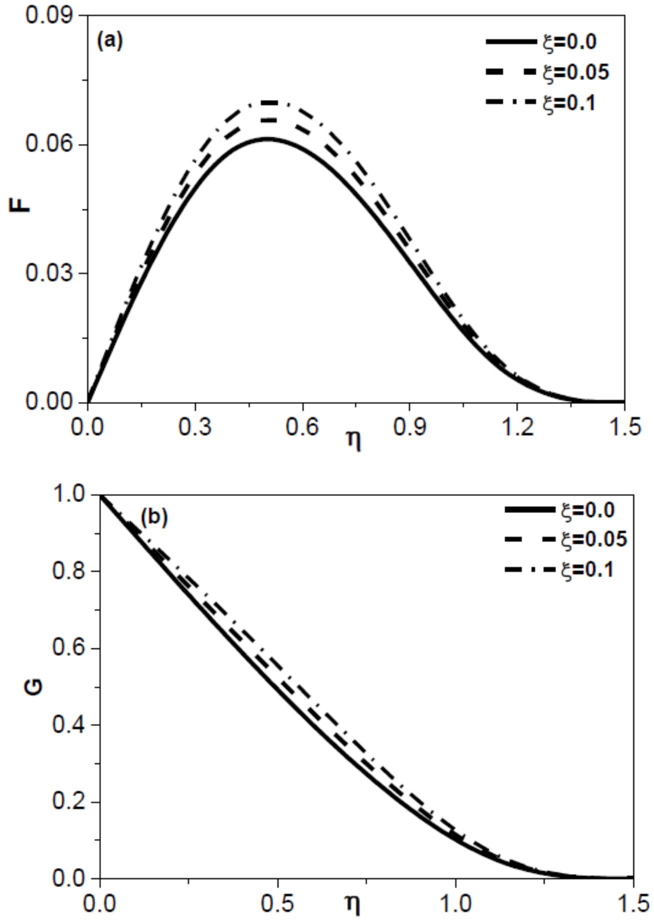

Fig. 3. Behavior of (a) velocity and (b) temperature profiles at different stream-wise locations

The relevant velocity (F) and temperature(G) profiles are shown in Fig.3, for the case of variable fluid properties. The velocity profile increases in magnitude with the increase of stream wise coordinate (□) [Fig.3(a)], as a result there is an enhancement in the velocity of the fluid and hence thickness of momentum boundary layer decreases. On the other hand, the thermal boundary layer thickness increases as increases, enhancing the temperature inside the boundary layer [Fig.3(b)].

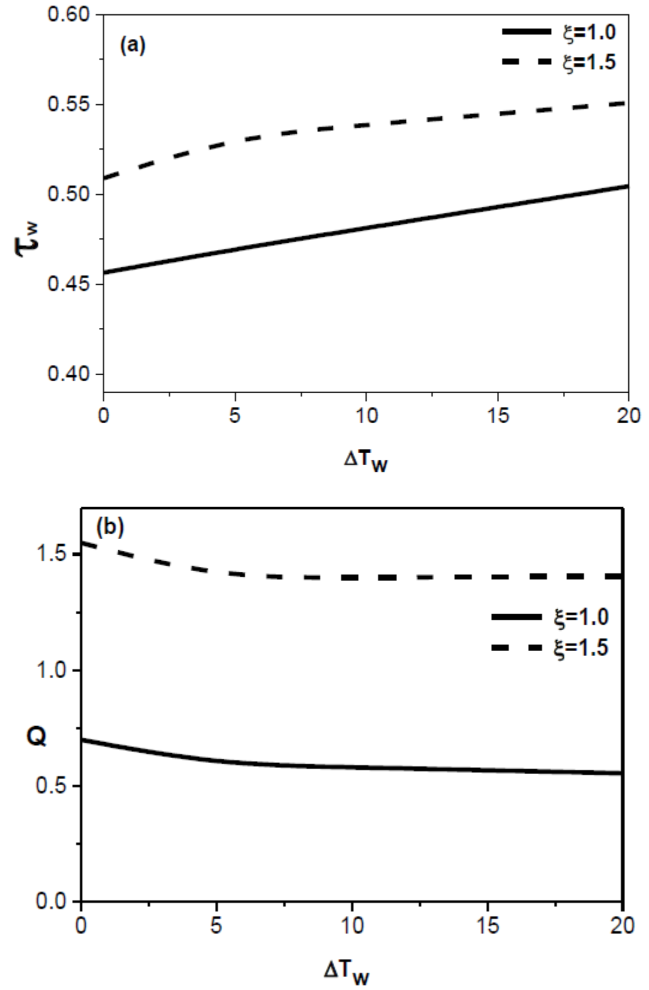

Fig. 4. Effect of ΔTw on (a) skin friction and (b) heat transfer parameters at stream-wise locations

The variation of viscosity and Prandtl number with temperature can be introduced in terms of the difference (ΔTw) in the temperature of the wall and ambient fluid [Fig.4]. Since T∞ =28.00C, the maximum value of ΔTw is taken as 200C so that numerical computations are done with in the permissible temperature. In Fig 4(a)–(b) for different stream wise locations it is observed that as ΔTw increases τ w also increases, however Q decreases as ΔTw increases. Further, it is observed that as ξ increases both skin friction and heat transfer decrease, the rate of decrease of skin friction is 11.74% and 9.50% respectively at ΔTw = 100C and 150C, while the rate of decrease in heat transfer is 58.52% and 59.54% for the same values of ΔTw.

IV. CONCLUSION

The steady, laminar methanol boundary layer flow (of methanol) past a vertical porous plate is numerically investigated assuming both viscosity and Prandtl number as linear inverse functions of temperature. The computed results show that the flow/temperature fields, skin friction and heat transfer characteristics are significantly affected by the temperature-dependent viscosity and Prandtl number in the presence of injection.

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