# Blast Wave with Variable Energy Through A Rotating Gas 



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#### Abstract

Shock phenomenon is one of the most interesting topics in the age of Science and technology. It has many applications not only in different branches of modern science, such as astrophysics, supersonic flights, explosions and plasma physics etc. but also in the field of medical Sciences. In the study of the propagation of these waves, some assumptions are generally made, for example. (i) The energy release is instantaneous. (ii) The ambient gas pressure is negligible in comparision with the pressure immediately behind the shock. (iii) The fluid is in viscid and non-heat conducting (iv) Radiation losses are negligible.

However, in practical problems, these assumptions are only approximately valid and one has often to drop one or more of them depending on the physical situations under study. Sakurai's analysis as modified by Freeman to study the propagation of variable energy blast waves through a uniform medium has been extended to include the propagation through a non-uniform medium with particular reference to cylindrical blast wave in a rotating gas. For an energy input $E_{\alpha}=E_{0} T^{\beta}\left(1+e_{1} w+e_{2} w^{2}+\ldots\right)$. Where $E_{\alpha}$ is the energy released upto time $t, E_{0}$ is a functional contant, $\beta$ and $e_{1} e_{2}$ etc. are constants and $w$ is non-dimensiona time, it is seen that the effects of non-uniformity caused by solid body rotation of the gas are assentially of third order. Dependence of flow variables on the nonuniformity and the strength of energy iput characterised respectively by the parameter $\mathrm{x}^{2}$ and various evalues has been discussed. It is observed that an increase in the initial angular velocity leads to a decrease in shock velocity.


## INTRODUCTION

Earlier investigations on the propagation of shockwave through a rotating gas include the work of Frenkal who has confined his studies to lower energy input (i.e. sound pulses). Chaturani has discussed the propagation of shock waves through a rotating gas with an instantaneous release of energy. However, the assumption of an instantaneous release of energy, is unrealistic. Chaturani and Kumar have been able to investigate this problem with a time varying energy input of the form $\mathrm{E}_{\alpha}=\mathrm{E}_{\mathrm{c}} \mathrm{T}^{\beta}$.

In this paper, we have to study the propagation of a cylindrical blast wave through a rotating gas with the time varying energy input of the form.

$$
\begin{equation*}
E_{\alpha}-E_{0} t^{\beta}\left(1+e_{1} w+e_{2} w^{2}+\cdots\right) \tag{1}
\end{equation*}
$$

It must be mentioned that the analysis being presented holds only for certain restricted values of $\beta$.,

$$
\begin{equation*}
\beta=\{2 \mathrm{I}(\alpha+1)-(\alpha+3)\} / 2 \mathrm{I} \tag{2}
\end{equation*}
$$

where $I$ is positive integer and $\alpha=0,1$ and 2 rspectively for plane, cylindrical and spherical symmetry of the blast. Using the first permissible value of $\beta=0$.

## Basic Equations and Boundary Conditions :

Let the cylindrical co-ordinate system be denoted by r, $\theta$ and z . Under the assumptions that the gas is inviscid and non-heat conducting, the basic equations describing the cylindrically symmetric flow behind a variable energy blast wave propagating through a gas having solid-body rotation are

$$
\begin{align*}
& \frac{\partial \mathrm{u}}{\partial \mathrm{t}}+\mathrm{u} \frac{\partial \mathrm{u}}{\partial \mathrm{r}}+\frac{1}{\rho} \frac{\partial \rho}{\partial \mathrm{r}}-\frac{\mathrm{v}^{2}}{\mathrm{r}}=0  \tag{3}\\
& \left(\frac{\partial}{\partial \mathrm{t}}+\mathrm{u} \frac{\partial}{\partial \mathrm{r}}\right)(\mathrm{vr})=0  \tag{4}\\
& \left(\frac{\partial}{\partial \mathrm{t}}+\mathrm{u} \frac{\partial}{\partial \mathrm{t}}\right) \rho+\rho\left(\frac{\partial \mathrm{u}}{\partial \mathrm{r}}+\frac{\mathrm{u}}{\mathrm{r}}\right)=0  \tag{5}\\
& \left(\frac{\partial}{\partial \mathrm{t}}+\mathrm{u} \frac{\partial}{\partial \mathrm{r}}\right) \rho+\gamma \rho\left(\frac{\partial \mathrm{u}}{\partial \mathrm{r}}+\frac{\mathrm{u}}{\mathrm{r}}\right)=0 \tag{6}
\end{align*}
$$

where $u$ and $v$ stand for radial and azimuthal components of gas valocity and $p, \rho$ and $\gamma$ are respectively pressure, density and adiabatic index of the gas.

The unperturbed state of gas is indicated by superscript das (') and is specified by $\frac{\partial}{\partial t}=0, u^{\prime}=$ $0, v^{\prime}=\Omega_{0} R, T_{0}$ const. where $\Omega_{0}$ is the constant angular velocity at time $\mathrm{t}=0, \mathrm{~T}_{0}=$ const. where $\Omega_{0}$ is the constant angular velocity at time $t=0, T_{0}$ is temperature and $R$ is the radial distance from the axis of symmetry ( z -axis).

The momentum equation in unperturbed state is $\frac{1}{\rho^{\prime}} \frac{d \rho^{\prime}}{d R}=\Omega_{0}^{2}-R=0$. At the shock front ( $\mathrm{r}=\mathrm{R}$ ), we have the following Rankine-Hugonlot conditions which serve as the boundary conditions to the system of equations

$$
\begin{aligned}
& \frac{\mathrm{U}_{1}}{\mathrm{U}}=\frac{2}{\gamma+1}\left\{1-(\mathrm{C} / \mathrm{U})^{2}\right\} \\
& \frac{\mathrm{p}_{1}}{\mathrm{p}_{0}}=\frac{2 \gamma}{\gamma+1}\left(\frac{\mathrm{U}}{\mathrm{C}}\right)^{2}-\frac{\gamma-1}{\gamma+1} \\
& \frac{\rho_{1}}{\rho_{0}}=\frac{\gamma+1}{\gamma-1}\left\{1+\frac{2}{\gamma-1}(\mathrm{C} / \mathrm{U})^{2}\right\}^{-1}, v_{0}=v_{1} .
\end{aligned}
$$

where the subscript 0 and 1 respectively donote the state immediately ahead and behind the shock front, U is shock velocity defined by $U=\frac{d R}{d t} \mathrm{C}^{2}=\gamma \mathrm{p}^{\prime} / \rho^{\prime}$. here R is shock radius at time $\mathrm{t}, \mathrm{C}$ is sound velocity in unperturbed state, given by $C^{2}=\gamma P^{\prime} / \rho^{\prime}$.

Since there are four boundary conditions and five unknown variables
( $u_{1}, p_{1}, p_{1} v_{1}$ and $U$ ), we must have one more equation. The required equation is obtained from the energy balance. This equation is
$\mathrm{E}_{\alpha}=\int_{\mathrm{R}_{\mathrm{p}}}^{\mathrm{R}}\left\{\frac{1}{2}\left(\mathrm{u}^{2}+v^{2}\right)+\frac{\mathrm{p}}{(\gamma-1) \rho}\right\} 2 \pi \mathrm{r} \rho \mathrm{dr}-\int_{0}^{\mathrm{R}}\left\{\frac{1}{2} \mathrm{v}^{\prime 2}+\frac{\mathrm{p}^{\prime}}{(\gamma-1) \rho^{\prime}}\right\} 2 \pi \rho^{\prime} \mathrm{R} d \mathrm{~d}$,
where $\mathrm{R}_{\mathrm{p}}$ is the piston position. At inner expanding surface the following kinematic condition is satisfied;
$\left(\frac{\mathrm{dR} \rho}{\mathrm{dt}}=\mathrm{u}\right)_{\mathrm{r}}=\mathrm{R}_{\mathrm{p}}$.
The energy input $\mathrm{E}_{\alpha}$ is assumed to vary with time t in a manner given by equation (2).

## Energy Integral Coefficients:

Needless to reproduce the non-dimensionalization and similarity transformation of the basic equations and boundary conditions (Chaturani \& $\mathrm{Kumar}^{5}$ ), we proceed directly for derivation of energy integral coefficients. The non-dimensional from of energy balance equation can be written as :
$w^{\beta}\left(1+e_{1} w+\ldots\right)=z^{2} D\left(\frac{U_{0}}{C}\right)^{2} \gamma h f^{2}$
$\int_{X_{p}}^{1}\left(\frac{1}{2}+\frac{g}{\gamma-1}+\frac{1}{2}\right)\left(\frac{h v_{0}^{2}}{U^{2}} w^{2}\right) x d x-\int_{0}^{z}\left(\gamma \frac{D v_{0}^{2}}{C_{0}^{2}}+\frac{\rho}{\gamma-1}\right) z d z$.
It is assumed that the functions $f, g$, $h$ and $w$ can be expressed in power series of $y$ as : $\mathrm{f}(\mathrm{x}, \mathrm{y})=\mathrm{f}_{0}(\mathrm{x})+\mathrm{f}_{1}(\mathrm{x}) \mathrm{y}+\ldots \ldots . ; \mathrm{g}(\mathrm{x}, \mathrm{y})=\mathrm{g}_{0}(\mathrm{x})+\mathrm{g}_{1}(\mathrm{x}) \mathrm{y}+\ldots \ldots . ; \mathrm{h}(\mathrm{x}, \mathrm{y})=\mathrm{h}_{0}(\mathrm{x})+\mathrm{h}_{1}(\mathrm{x}) \mathrm{y}+\ldots . . ; \mathrm{w}(\mathrm{x}, \mathrm{y})=\mathrm{w}_{0}(\mathrm{x})+\mathrm{w}_{1}(\mathrm{x}) \mathrm{y}+\ldots \ldots$
(10) and following Freeman let us write ;
$\mathrm{z}=\mathrm{z}_{0} \mathrm{y}^{1 / \lambda_{0}}\left(1+\mathrm{z}_{1} \mathrm{y}+\mathrm{z}_{2} \mathrm{y}^{2}+\ldots ..\right) ;$
$\mathrm{w}=\mathrm{z}_{0} \mathrm{Y} \frac{\lambda_{0}+2}{2 \lambda_{0}}\left(\frac{2}{\lambda_{0}+2}\right)\left(1+\mathrm{w}_{1} \mathrm{z}_{1} \mathrm{y}+\mathrm{w}_{2} \mathrm{z}_{2} \mathrm{y}^{2}+\ldots \ldots ..\right) ;$
$\mathrm{J}=\mathrm{J}_{0}\left(1+\sigma_{1} \mathrm{y}+\sigma_{2} \mathrm{y}^{2}+\ldots \ldots ..\right)$,
Where
$\lambda_{0}=2(\alpha+1-\beta) /(2+\beta) ; \mathrm{w}_{1}=\left(\lambda_{0}+1\right)\left(2+\lambda_{0}\right) /\left(3 \lambda_{0}+2\right), \mathrm{W}_{2}=\frac{\left(\lambda_{0}+2\right)\left(2 \lambda_{0}+2\right)}{\left(5 \lambda_{0}+2\right)}$
$\mathrm{w}_{3}=\left(\lambda_{0}+2\right)\left(3 \lambda_{0}+1\right) /\left(7 \lambda_{0}+2\right)$
its power series from provided $\left(\lambda_{0}+2\right) / 2 \lambda_{0}$ is positive integer, i.e. $1 / 2+1 / \lambda_{0}=I$ (where I is positive integer) or this is equivalent to $\beta=[2 I(\alpha+1)(\alpha+3)] / 2 I$. And as the non-dimensional radius z contains the term and it appears in power series expressions for $\mathrm{p}(\mathrm{z}), \mathrm{D}(\mathrm{z})$ etc., (see eq $\mathrm{n}_{\mathrm{s}} .26$ and 27) the requirement that 1 / $2+1 \backslash \lambda_{0}$ should be an integer restricts the use of the present approach to those non-uniformities where the series representing the density variation contains only even powers of z . Besides this, the approach is workable only for those values of $\beta$ which are given by (2).

Now using first permissible value of $\beta=0$ and comparing the coefficients of zeroth, first and second powers of $y$ and both sides of equation (9), we get :
For the zeroth power of $y$
$Z_{0}=\mathrm{J}_{0}^{-\frac{1}{2}}$
For the first power of $y$
$\sigma_{1}=\frac{\mathrm{e}_{1} \mathrm{z}_{0}}{\mathrm{z}}-2 \mathrm{z}_{1}-\frac{\chi^{2} \mathrm{z}_{0}^{2}}{2}+\frac{\mathrm{z}_{0}^{2}}{2(\gamma-1)}$
For the second power of $y$
$\sigma_{1}=\frac{e_{2} z_{0}^{2}}{4}--\frac{e_{1} z_{0} z_{1}}{4}+3 z_{1}^{2}-2 z_{2}-\frac{1}{4} \chi^{2} e_{1} z_{0}^{3}+\frac{y}{8} z_{0}^{4}-M_{0} z_{0}^{4}+\frac{\chi^{2} z_{0}^{2}}{8} \frac{\left(y^{2}-2+y\right)}{\gamma+1}$
where $M_{0}=\int_{x p}^{1} h_{0} w_{0}^{2} x d x$. Similarly values for $z_{0} \sigma_{1} \sigma_{2}$ etc., for other permissible value of $\beta$ can be calculated.
Thus the effects of solid body rotation of the gas $\left(x^{2}\right)$ are of second and fourth order respectively, For the first and second permissible value of $\beta$. In general for then ${ }^{\text {th }}$ permissible value of $\beta$, the effects appear first in (20)th order term. Since the successive permissible value of $\beta$ are obtained on equating $1 / \lambda_{0}+1 /$ $2=\mathrm{I}=1,2,3$ etc. and the dominent term involving the non-uniformity of the medium is $\mathrm{z}^{2}$ term, it would, appear in terms of $y$ as if the effect of the non-uniformity were arising in second, fourth, sixth order terms respectively, Such a description is essentially a result of the nature of inter-relationship between $z$ and $y$, though actually the term representing the non-uniformity has always remained the $z^{2}$ term i.e., the third order term.
The numerical integration is carried by runge-kutta method, we get
$\mathrm{Z}_{1}=\frac{\mathrm{J}_{0} \sigma_{\pi}-\int_{\mathrm{x}_{\mathrm{p}}}^{1} \frac{\gamma}{2}\left\{2 \mathrm{f}_{0} \mathrm{~h}_{0} \mathrm{f}_{11}+\mathrm{f}_{0}^{2} \mathrm{~h}_{11}+\frac{\mathrm{g}_{11}}{\gamma-1}\right\} x d x}{\int_{\mathrm{x}_{\mathrm{p}}}^{1}\left\{\frac{\gamma}{2}\left(2 \mathrm{f}_{0} \mathrm{~h}_{0} \mathrm{f}_{12}+\mathrm{f}_{0}^{2} \mathrm{~h}_{12}\right)+\frac{\mathrm{h}_{12}}{\gamma-1}\right\} x d x-\mathrm{J}_{0} \sigma_{12}}$
where $\sigma_{11}=\frac{\mathrm{e}_{1} \mathrm{z}_{0}}{2}-\frac{\chi^{2} \mathrm{z}_{0}^{2}}{2}+\frac{\mathrm{z}_{0}^{2}}{2(\gamma-1)}$. and $\sigma_{12}=-2.0$
$\mathrm{z}_{2} \frac{\mathrm{~J}_{0} \sigma_{21}-\int_{\mathrm{x}_{\mathrm{p}}}^{1}\left\{\frac{\gamma}{2}\left(2 \mathrm{f}_{0} \mathrm{~h}_{0} \mathrm{f}_{21}+\mathrm{f}_{0}^{2} \mathrm{~h}_{21}+\mathrm{h}_{0} \mathrm{f}_{1}^{2}+2 \mathrm{f}_{0} \mathrm{f}_{1} \mathrm{~h}_{1}\right)+\frac{\mathrm{g}_{21}}{\gamma-1}\right\} \mathrm{x} d \mathrm{dx}}{\int_{\mathrm{x}_{\mathrm{p}}}^{1} \frac{\gamma}{2}\left\{\left(2 \mathrm{f}_{0} \mathrm{~h}_{0} \mathrm{f}_{21}+\mathrm{f}_{0}^{2} \mathrm{~h}_{22}\right) \frac{\mathrm{g}_{22}}{\gamma-1}\right\} \mathrm{xdx}-\mathrm{J}_{0} \sigma_{22}}$
where $\sigma_{21}=\frac{e_{2} z_{0}^{2}}{4}-\frac{e_{1} z_{0} z_{1}}{4}+3 z_{1}^{2}-\frac{\chi^{2}}{4} e_{1} z_{0}^{3}+\frac{\gamma}{8} \chi^{4} z_{0}^{4}-M_{0} Z_{0}^{4}+\left(\gamma^{2}-2+\gamma\right) \frac{\pi^{2}}{8} z_{0}^{4}$, $\sigma_{22}=-2.0$ and $M_{0}=\frac{\gamma}{2} \chi^{2} \int_{\mathrm{x}_{\mathrm{p}}}^{1} \mathrm{~h}_{0} \mathrm{w}_{0}^{2} \mathrm{x} d \mathrm{~d}$.

## Results and Discussion

Since the first approximation solutions do not include the effects of ambient gas pressure and solid body rotation of the gas, these may be regarded as solutions of the problem concerning the propagation of a very strong cylindrical blast wave through a uniform medium. Further, these solutions being for $\beta=0$ (i.e.,
the first permissible value), this corresponds to the case of propagation of cylindrical blast wave through a uniform medium with an instantaneous release of energy.

The second approximation solution of the present analysis is affected by (i) the parameter $\mathrm{X}^{2} \mid$ and (ii) constant $e_{1}$. It is evident that an increase in the value of $c_{1}$ results in an increase in velocity and pressure. However, the change in density with $e_{1}$ is inappreciable and the trend is reversed as the position is approached. We find that velocity decreaes and pressure and density increase with an increase of shock velocity by decrease with the increase of $\mathrm{e}_{1}$.

The solutions upto second approximations of the present analysis essentially correspond to Chaturani's thrid approximation solutions. As expected, for $\mathrm{e}_{1}=0$ our second approximation results reduce to those obtained by Chaturani ${ }^{4}$ for the case of instantaneous release of energy.

The third approximation solutions are affected by (1) the parameter $\mathrm{x}^{2}$, (ii) constant $\mathrm{e}_{1}$ and (iii) constant $e_{2}$. It may be mentioned here that the constant $e_{2}$ has litle influence on the flow variables, which is very much expected (16). We must, therefore, confine our discussion to the dependence of flow variables upon $x^{2}$ and $e_{1}$ only. It is observed that an increase of $e_{1}$ causes a decrease of velocity, pressure and density ; the variation in velocity and density with $\mathrm{e}_{1}$ is relatively very small. The reversal of $\mathrm{e}_{1}$, dependence of velocity and pressure in the third approximation an compared to second approximation must be particularly noted.

Here, it is worth while to indicate, that our third approximation results actually correspond to the fifth approximation solutions of Chaturani or Freeman. It is also essential to mention that prior to applying this analysis to variable energy blast weve propagation through a non-uniform medium one should make sure that nergy input form and the non-uniformity of pre-explosion medium are compatible.

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