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Connectedness in Bitopological Spaces

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In the present section we discuss connectedness in bitopological spaces. The notion of connectedness in bitopological spaces was introduced by William J. Pervin in 1967 (P. 2).

Definition : (1.1) : A bitopological space (X, T_1, T_2) is said to be connected if X cannot be expressed as the union of two non-empty disjoint sets A and B such that

$[A \cap (T_1 - \text{cl } B)] \cup [T_2 - \text{cl } A) \cap B] = \phi$,....... (1.1)

If there exist subsets A and B of X with $X = A \cup B$ where A and B satisfy condition (1.1) then X is said to be disconnected. In this case we write X = A|B and cell it a separation of X.

NOTE (1.1) : It is easy to see that every com-elementary bitopological space is disconnected.

Now we mention characteristic property of connectedness for a bitopological space as given in [Pervin, 1967]

THEOREM (1.1) : Let (X, T_1, T_2) be a bitopological space, then the following conditions are equivalent :
(i) (X_0, T_1, T_2) is connected,
(ii) X cannot be expressed as the union of two non empty disjoint sets A and B such that A is T_1 = open and B is T_2 – open.
(iii). X contains no non empty proper subset of X which is both T_1 = open and T_2 = closed.

NOTE (1.2) : Condition (iii) given in the above theorem is very useful in the sense that it helps in determining connectedness of a bitopological spaces.

To discuss connectedness in bitopological space we need some examples with we mention below :

Example : (1.1), Let D = (0, 1), $P_1 = \{\phi, \{0\}, D\}$ and $P_2 = \{\phi, \{1\}, D\}$. Then (D, P_1) and (D, P_2) are connected topological spaces but the bitopological space (D, P_1, P_2) is not connected because {0} is P_1 – open and P_2 – closed.

In the above example 0 and 1 are symbols which can be replaced by any two objects.

Example (1.2) : Let X = {a, b, c}, $T_1 = \{\phi, \{a\}, \{b, c\}, X\}$ and $T_2 = \{\phi, \{b\}, \{a, c\}, X\}$.

Then both (X, T_1) and (X, T_2) are disconnected but the bitopological space (X, T_1, T_2) is connected.

Example (1.3) : Let Y = {a, b, c}, $P_1 = \{\phi, \{a\}, Y\}$ and $P_2 = \{\phi, \{b\}, Y\}$. Then (Y, P_1), (Y, P_2) as well as (Y, P_1, P_2) are connected spaces.

Example (1.4) : We recall example (4.4) of chapter I where X = {a, b, c, d},

T_1 = { ϕ , {a}, {a, b}, {c, d}, {a, c, d}, X}

T_2 = { ϕ , {b}, {c}, {b, c}, X}.

Then (X, T_1) is disconnected, (X, T_2) is connected and (X, T_1, T_2) is not connected.

REMARK (1.1) : From the above example we see that the connectedness of a bitopological space (X, T_1, T_2) is independent from the connectedness of topological spaces (X, T_1) and (X, T_2).

We recall the definition of a continuous map from δ 1 of chapter III (Def. (1.1).

Pervin [1967] characterises connectedness in terms of continues mappings. Results are as follows :

THEOREM (1.2) : A bitopological space (X, T_1, T_2) is connected if every continuous mapping of (X, T_1, T_2) into (D, P_1,P_2) [Example (1.1)]is constant.

TEHOREM (1.3) : A bitopological space (X, T_1, T_2) is connected if every continues mapping of (X, T_1, T_2) into (R, P_1, P_2) has the Darbour property (i.e., its range is an interval) where P_1 and P_2 are left hand and right hand topologies on R [Example (1.4), chapter-II].

The basic relationship between continuity and connectedness remains the same as given in the following result [P_2, 1967].

THEOREM (1.4) : The connectedness of a bitopological space remains invariant under a continues mapping.

NOTE (1.3) : It is important to note that the image of a disconnected bitopological space may be connected under continues mapping.

For this we have the following example :

Example : (1.5) : Let F : (X, T_1, T_2) → (Y, P_1, P_2) be defined as

f (a) = a, f (b) = b, f (c) = f (d) = c.

where (X, T_1, T_2) and (Y, P_1, P_2) are bitopological spaces of examples (1.4) and (1.3) respectively. Here f is continuous, (X, T_1, T_2) is disconnected and (Y, P_1, P_2) is connected.

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