# Bianchi Type- $\mathrm{VI}_{\mathrm{h}}$ Cosmological Model with Quadratic EOS in $\mathrm{f}(\mathrm{R}, \mathrm{T})$ Theory 

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#### Abstract

In this paper, we study the spatially homogenous and anisotropic Bianchi type-VIh cosmological model in $f(R, T)$ modified theory with variable cosmological term $\Lambda$. Solutions are found by assuming forms of the function $f(R, T)$ as $f(R, T)=f_{1}(R)+f_{2}(T)$ and with a quadratic equation of state. It is found that some of which are decelerating and others accelerating. Here, the cosmological parameter is not constant, but it is taken as variable, which can solve the cosmological constant problem.


Keywords: Cosmological Model, Equation of State, $f(\mathrm{R}, \mathrm{T})$ Theory, Bianchi Type VIh.

## I. INTRODUCTION

The most successful theory in application to cosmology is the theory of General relativity (GR). Until recently, our mental picture of the universe was based more on our philosophical prejudices (or religious beliefs) than on observational data [1]. Cosmology refers to study of the origin of the universe, it's structure evolution, and future of the universe as a whole based upon the interpretations of astronomical observations at different wave-lengths through laws of physics. Relativistic cosmological models are described as the exact solutions of the EFEs that help in understanding the important features of our universe. Many generalizations of EFEs have been proposed in last few decades. Einstein's general theory of relativity (GR) is considered as one of the most beautiful structures of theoretical physics. Among several theories of gravitation, GR has been designated as the most
successful one. In fact, GR is regarded as a geometric theory of gravitation. Mathematical elegance and outstanding formal beauty using tools of Riemannian geometry are the characteristics of Einstein's theory of gravitation. It leads to gravitational action. In 1917, Einstein introduced the cosmological constant L as the universal repulsion to make the universe static in accordance with a generally accepted picture of that time.
For better understanding, researchers proposed various generalizations of general theory relativity, viz., bimetric theory, scalar tensor theory, $f(G), f(R, G), f(R), f(T)$ and $f(R, T)$ gravitational theories. From these, some theories have been studied to learn the behavior of dark energy and its aspects related to the cosmological constant problem. One of the most popular alternatives to Einstein's theory of gravitation is the $f(R, T)$ theory and was proposed by

Harko et al. (2011) [2]. He introduced an arbitrary function of the Ricci scalar $R$ and the trace $T$ of the energy-momentum tensor. The field equations are obtained from the Hilbert-Einstein type variational principle [3, 4]. In $f(R, T)$ theory it is assumed that the gravitational part of the action still depends on a generic function of the Ricci scalar R, but also presents a generic dependence on $T$ [5]. This kind of dependence on T may come because of the consideration of quantum effects[6]. [7, 8] gave modifications in $f(R, T)$ theory.The $f(R, T)$ theory provides An alternative way is given in $f(R, T)$ theory, which explains the cosmic acceleration as presented by current observations. In this theory there remains no need to introduce either the existence of extra spatial dimension or a component like dark energy [9, 10]. Application of this theory will help in exploring various issues which are of current interests and may give rise to some good inferences [11].

Generally many researchers use a linear equation of state (EOS). which give a linear relation between pressure and energy density. But, recent studies shows that there is a accleratd expansion of the universe. To explain this, one can use a more general EOS's. The generalization to the linear EOS is the quadratic EOS:

$$
\begin{equation*}
p=p_{0}+\beta \rho+\alpha \rho^{2} \tag{1.1}
\end{equation*}
$$

where $p_{0}, \alpha$ and $\beta$ are constants. Here, if we express pressure as a function of density $p=p(\rho)$ in a Taylor's series about $\rho=0$, this gives equation (1.1) to second order.

In the present study, an attempt has been made to solve Binchi Type $\mathrm{VI}_{\mathrm{h}}$ cosmological model by considering the following quadratic EOS of the type
$p=\alpha \rho^{2}-\rho$
where $\alpha \neq 0$. Here we consider variable cosmological parameter. In this paper, Section 2, briefly explains the $f(R, T)$ theory, Section 3, gives the field equations and Section 4, discusses the solutions. Conclusion is presented in Section 5.

## II. THE BASIC EQUATIONS

The $f(R, T)$ theory is a modified theory of General Relativity. The field equations of $f(R, T)$ gravity are formulated from a Hilbert-Einstein type variational principle as follows
$S=\int\left(\frac{f(R, T)}{16 \pi G}+L_{m}\right) \sqrt{g} d^{4} x$
where $R$ is the Ricci scalar and $T$ is the trace of the energy momentum tensor $T_{a b}$. Here $L_{m}$ is matter Lagrangian density. The energy-momentum tensor is:
$T_{\mu \nu}=\frac{-2}{\sqrt{g}} \frac{\delta\left(\sqrt{g} L_{m}\right)}{\delta g^{\mu \nu}}$
where, $L_{m}$ is the Lagrangian density of the matter and it depends only on $g_{a b}$. Thus
$T_{\mu v}=L_{m} g_{\mu v}-2 \frac{\partial L_{m}}{\partial g^{\mu v}}$
Now, varying (2.1) with respect to $g_{a b}$, the following field equations are obtained
$f_{R}(R, T) R_{\mu v}-\frac{1}{2} f(R, T) g_{\mu \nu}+\left(g_{\mu \nu} \square\right.$
$\left.-\nabla_{\mu} \nabla_{v}\right) f_{R}(R, T)=8 \pi-f_{T}(R, T) T_{\mu v}-f_{T}(R, T) \Theta_{\mu v}$

Where
$\Theta_{\mu \nu}=-2 \mathrm{~T}_{\mu v}+L_{m} \mathrm{~g}_{\mu v}-2 g^{i j} \frac{\partial^{2} L_{m}}{\partial g^{\mu v} \partial g^{i j}}$
Here $\mathrm{T}_{\mu \nu}$ is the energy-momentum tensor associated with the Lagrangian $L_{m}$ and $\square=\nabla^{\mu} \nabla_{v}$ is the d' Alembert's operator, $f_{T}(R, T)=\frac{\partial f(R, T)}{\partial T}, f_{R}(R, T)=$ $\frac{\partial f(R, T)}{\partial R}$. Now, by contracting (2.4) leads to
$f_{R}(R, T) R+3 \square f_{R}(R, T)-2 f(R, T)=8 \pi T-(T+$ $\Theta) f_{T}(R, T)$
where, $\Theta=g^{\mu v} \Theta_{\mu \nu}$. Using equations (2.4) and (2.6), the gravitational equations can be derived as

$$
\begin{array}{r}
f_{R}(R, T)\left(R_{\mu v}-\frac{1}{3} R g_{\mu v}\right)+\frac{1}{6} f(R, T) g_{\mu v} \\
=\left(8 \pi-f_{T}(R, T)\right)\left(T_{\mu v}-\right. \\
\left.\frac{1}{3} g_{\mu v}\right)-f_{T}(R, T)\left(\Theta_{\mu v}-\frac{1}{3} \Theta g_{\mu v}\right)+\nabla_{\mu} \nabla_{v} f_{R}(R, T) \tag{2.7}
\end{array}
$$

For perfect fluid
$\mathrm{T}_{\mu \nu}=(\rho+p) u_{\mu} u_{v}-p g_{\mu v}$
with $u^{\mu}=(1,0,0,0)$ being the four velocity for which
$u^{\mu} u_{\mu}=1, u^{\mu} \nabla_{v} u_{\mu}=0$. From equation (2.5), we get
$\Theta_{\mu v}=-2 \mathrm{~T}_{\mu v}-p \mathrm{~g}_{\mu v}$

There is freedom of choice for $f(R, T)$, following, Harko et al. (2011) [4] to construct different kinds $f(R, T)$ modified cosmological models by specifying following three forms of $f(R, T)$ as,

$$
f(R, T)=\left\{\begin{array}{c}
R+2 f(T)  \tag{2.10}\\
f_{1}(R)+f_{2}(T) \\
f_{1}(R)+f_{2}(R) f_{3}(T)
\end{array}\right.
$$

In this paper, we choose $f(R, T)=f_{1}(R)+f_{2}(T)$. Now, using equations (2.8) and (2.9) equation (2.4) becomes

$$
\begin{array}{r}
f_{1}^{\prime}(R) R_{\mu v}-\frac{1}{2} f_{1}(R) g_{\mu v}+\left(g_{\mu v} \boxtimes-\nabla_{\mu} \nabla_{v}\right) f_{1}^{\prime}(R) \\
=(8 \pi- \\
\left.f_{2}^{\prime}(T)\right) T_{\mu v}+\left(f_{2}^{\prime}(T) p+\frac{1}{2} f_{1}(T)\right) \mathrm{g}_{\mu v} \quad(2.11 \tag{2.11}
\end{array}
$$

## III. METRIC AND FIELD EQUATIONS

We consider generalized space metric which will cover spatially homogeneous Bianchi type $\mathrm{VI}_{\mathrm{h}}$ represented by $d s^{2}=d t^{2}-A^{2}(t) d x^{2}-B^{2}(t) e^{2 x} d y^{2}-$ $C^{2}(t) e^{2 h x} d z^{2}$
Where $h$ arbitrary constants and $A, B, C$ are metric functions of cosmic time.

In equation (2.11) we take $f_{1}(R)=\lambda R$ and $f_{2}(T)=\lambda T$ then the Einstein field equations take the form
$G_{i j}=\left(\frac{8 \pi+\lambda}{\lambda}\right) T_{i j}+\left(\frac{\rho-p+2 \Lambda}{2}\right) g_{i j}$
(3.2)
where the symbols have their usual meaning.
Einstein field equations (3.2) for the metric (3.1) can be written as
$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B} \dot{C}}{B C}-\frac{h}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda$
(3.3)
$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A} \dot{C}}{A C}-\frac{h^{2}}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda$
(3.4)
$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A} \dot{B}}{A B}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda$
(3.5)
$\frac{\dot{A} \dot{B}}{A B}+\frac{\dot{B} \dot{C}}{B C}+\frac{\dot{A} \dot{C}}{A C}-\frac{1+h+h^{2}}{A^{2}}=-\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) \rho+\frac{p}{2}-\Lambda$
(3.6)
$(1+h) \frac{\dot{A}}{A}-\frac{\dot{B}}{B}-h \frac{\dot{C}}{C}=0$
(3.7)

The energy conservation equation is given by $T_{i j} ; j=$ 0 gives
$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0$

Where overhead dots denote differentiation with respect to time $t$.

Now, the field equations (3.3) - (3.7) form a system of four independent equations with six unknown parameters , $B, C, p, \rho, \Lambda$. Hence in order to find explicit solution of the system, two additional conditions are to be considered.
So following (Yadav and Sharma 2013;Yadav 2012;Pradhan et al.2013d )first we consider the average scale factor is an increasing function of time as given below
$a=\left(t^{n} e^{t}\right)^{\frac{1}{k}}$

Where $k>0$ and $\mathrm{n} \geq 0$ are constants.
Secondly we consider quadratic barotropic equation of state (EOS) relating pressure and density given as follows.
$p=\alpha \rho^{2}-\rho$
(3.10)

Where $\alpha \neq 0$
Spatial volume for the model (3.1) is written as
$V=A B C$

We define average scale factor $a$ of anisotropic model as
$a=V^{\frac{1}{3}}=(A B C)^{\frac{1}{3}}$
(3.12)

Generalized mean Hubble parameter $H$ is given by
$H=\frac{1}{3}\left(H_{1}+H_{2}+H_{3}\right)$

Where $H_{1}=\frac{\dot{A}}{A}, H_{2}=\frac{\dot{B}}{B}, H_{3}=\frac{\dot{C}}{C}$ are the directional Hubble parameters in the direction of $x, y, z$ resp.

Expansion scalar $(\theta)$, anisotropy parameter $\left(A_{m}\right)$ and From (3.23)-(3.25) and (3.11) we obtain shear scalar $(\sigma)$ are defined as usual as follows

$$
\theta=3 H
$$

$$
A_{m}=\frac{1}{3} \sum_{i=1}^{3}\left(\frac{H_{i}-H}{H}\right)^{2}
$$

$$
\begin{equation*}
\sigma^{2}=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^{2}+\left(\frac{\dot{B}}{B}\right)^{2}+\left(\frac{\dot{C}}{C}\right)^{2}\right]-\frac{\theta^{2}}{6} \tag{3.29}
\end{equation*}
$$

Deceleration parameter $q$ is defined as

$$
q=-\frac{a \ddot{a}}{\dot{a}^{2}}=-\left(\frac{\dot{H}+H^{2}}{H^{2}}\right)
$$

$$
\text { Case I) } h=\mathbf{1}
$$

We consider $h=1$ then the field eqns (3.3) - (3.7) take the form

$$
\begin{equation*}
\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B} \dot{C}}{B C}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda \tag{3.31}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A} \dot{C}}{A C}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda \tag{3.32}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A} \dot{B}}{A B}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda \tag{3.19}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\dot{A} \dot{B}}{A B}+\frac{\dot{B} \dot{C}}{B C}+\frac{\dot{A} \dot{C}}{A C}-\frac{3}{A^{2}}=-\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) \rho+\frac{p}{2}-\Lambda \tag{3.21}
\end{equation*}
$$

$$
\begin{equation*}
2 \frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{3.22}
\end{equation*}
$$

From (3.22) we obtain

$$
\begin{equation*}
A^{2}=B C \tag{3.23}
\end{equation*}
$$

Solving (3.18) - (3.21) by usual technique we obtain

$$
\frac{A}{B}=d_{1} \exp \left(q_{1} \int \frac{d t}{V}\right)
$$

$$
\begin{equation*}
\frac{B}{C}=d_{2} \exp \left(q_{2} \int \frac{d t}{V}\right) \tag{3.46}
\end{equation*}
$$

$$
\frac{A}{C}=d_{3} \exp \left(q_{3} \int \frac{d t}{V}\right)
$$

Where $d_{1}, d_{2}, d_{3}$ and $q_{1}, q_{2}, q_{3}$ are constants of integration.

$$
\begin{align*}
& A(t)=L_{1} a \exp \left(Q_{1} \int a^{-3} d t\right) \\
& B(t)=L_{2} a \exp \left(Q_{2} \int a^{-3} d t\right)  \tag{3.14}\\
& C(t)=L_{3} a \exp \left(Q_{3} \int a^{-3} d t\right) \tag{3.15}
\end{align*}
$$

Where $L_{1} L_{2} L_{3}=1$ and $Q_{1}+Q_{2}+Q_{3}=0$
In particular

$$
\begin{equation*}
L_{1}=\left(\frac{l_{3}{ }^{2}}{l_{2}}\right)^{\frac{1}{3}}, L_{2}=\left(\frac{l_{2}}{l_{1}}\right)^{\frac{1}{3}}, L_{3}=\left(\frac{l_{1}}{l_{3}{ }^{2}}\right)^{\frac{1}{3}} \tag{3.16}
\end{equation*}
$$

And

$$
\begin{equation*}
Q_{1}=\frac{2 q_{3}-q_{2}}{3}, Q_{2}=\frac{q_{2}-q_{1}}{3}, Q_{3}=\frac{q_{1-}-2 q_{3}}{3} \tag{3.17}
\end{equation*}
$$

From eqn (3.26) - (3.28) and (3.8) we obtain

$$
\begin{equation*}
A(t)=L_{1}\left(t^{n} e^{t}\right)^{\frac{1}{k}} \exp \left(Q_{1} F(t)\right) \tag{3.30}
\end{equation*}
$$

$$
B(t)=L_{2}\left(t^{n} e^{t}\right)^{\frac{1}{k}} \exp \left(Q_{2} F(t)\right)
$$

$$
\begin{equation*}
C(t)=L_{3}\left(t^{n} e^{t}\right)^{\frac{1}{k}} \exp \left(Q_{3} F(t)\right) \tag{3.18}
\end{equation*}
$$

Where

$$
\begin{equation*}
F(t)=\int\left(t^{n} e^{t}\right)^{-\frac{3}{k}} d t=\sum_{i=0}^{\infty} \frac{(-3)^{i-1} t^{i-\frac{3 n}{k}}}{n^{i-2}(n i-3 k)(i-1)!} \tag{3.20}
\end{equation*}
$$

Hence from (3.29)-(3.31) the metric takes the form
$d s^{2}=d t^{2}-L_{1}{ }^{2}\left(t^{n} e^{t}\right)^{\frac{2}{k}} \exp \left(2 Q_{1} F(t)\right) d x^{2}-$
$e^{2 x}\left[L_{2}{ }^{2}\left(t^{n} e^{t}\right)^{\frac{2}{k}} \exp \left(2 Q_{2} F(t)\right) d y^{2}+\right.$
$\left.L_{3}{ }^{2}\left(t^{n} e^{t}\right)^{\frac{2}{k}} \exp \left(2 Q_{3} F(t)\right) d z^{2}\right]$

## Case II) $h=-1$

In this section we have considered $h=-1$, then the set of field equations (3.3) - (3.7) reduce to

$$
\begin{align*}
& \frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B} \dot{C}}{B C}+\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda  \tag{3.24}\\
& \frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A} \dot{C}}{A C}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda  \tag{3.25}\\
& (3.47) \\
& \frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A} \dot{B}}{A B}-\frac{1}{A^{2}}=\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) p-\frac{\rho}{2}-\Lambda  \tag{3.26}\\
& \text { (3.48) }
\end{align*}
$$

$\frac{\dot{A} \dot{B}}{A B}+\frac{\dot{B} \dot{C}}{B C}+\frac{\dot{A} \dot{C}}{A C}-\frac{1}{A^{2}}=-\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right) \rho+\frac{p}{2}-\Lambda$
$\frac{\dot{C}}{C}-\frac{\dot{B}}{B}=0$
(3.50)

From eqn (3.50) we obtain

$$
B=C
$$

(3.51)

Eqn (3.46) - (3.49) with the use of eqn (3.50) form the system of 3 independent equations with five unknowns $A B, p, \rho, \Lambda$. So to find explicit solutions for the system we consider two conditions given by eqn (3.9) and (3.10) .

Solving eqn. (3.46)-(3.47) in usual way we obtain

$$
\begin{aligned}
& \frac{A}{B}=c_{2} \exp \left[c_{1} F(t)\right] \\
& (3.52) \\
& \frac{A}{C}=c_{4} \exp \left[c_{3} F(t)\right] \\
& (3.53)
\end{aligned}
$$

Where

$$
\begin{aligned}
& F(t)= \int \frac{e^{t}}{a^{3}} d t=\int t^{-\frac{3 n}{k}} e^{t\left(1-\frac{3}{k}\right)} d t \\
&=\sum_{i=0}^{n} \frac{(k-3)^{i-1} t^{i-\frac{3 n}{k}}}{k^{i-2}(k i-3 n)(i-1)!}
\end{aligned}
$$

And $c_{1}, c_{2}, c_{3}, c_{4}$ are constant of integration.
From (3.11), (3.52) and (3.53) we obtain

$$
\begin{equation*}
A=c_{2}{ }^{\frac{2}{3}}\left(t^{n} e^{t}\right)^{\frac{1}{k}} \exp \left[\frac{2 c_{1}}{3} F(t)\right] \tag{3.54}
\end{equation*}
$$

$$
\begin{equation*}
B=C=c_{2}-\frac{1}{3}\left(t^{n} e^{t}\right)^{\frac{1}{k}} \exp \left[-\frac{c_{1}}{3} F(t)\right] \tag{3.55}
\end{equation*}
$$

Now the metric (3.1) can be rewritten as
$d s^{2}=d t^{2}-c_{2}{ }^{\frac{4}{3}}\left(t^{n} e^{t}\right)^{\frac{2}{k}} \exp \left[\frac{4 c_{1}}{3} F(t)\right] d x^{2}-$
$c_{2}{ }^{-\frac{2}{3}}\left(t^{n} e^{t}\right)^{\frac{2}{k}} \exp \left[-\frac{2 c_{1}}{3} F(t)\right]\left(e^{2 x} d y^{2}+e^{-2 x} d z^{2}\right)$

## IV. RESULTS AND DISCUSSION

For both the cases expression for energy density and pressure, Hubble parameter $(H)$, expansion scalar $(\theta)$ and deceleration parameter $(q)$ are obtained as

$$
\begin{equation*}
\rho=\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1} \tag{4.1}
\end{equation*}
$$

$$
\begin{align*}
& p=\alpha\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-2}-\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1} \\
& H=\frac{1}{k}\left(\frac{n}{t}+1\right) \tag{4.3}
\end{align*}
$$

$$
\theta=\frac{3}{k}\left(\frac{n}{t}+1\right)
$$

$$
\begin{equation*}
q=-1+\frac{k n}{(n+t)^{2}} \tag{4.4}
\end{equation*}
$$

(4.5)

Other physical parameters such as cosmological parameter $(\Lambda)$, mean shear scalar $(\sigma)$ and an anisotropy parameter $\left(A_{m}\right)$ are obtained as

For $h=1$
$\Lambda=-\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right)\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1}+$ $\frac{1}{2}\left\{\alpha\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-2}-\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1}\right\}-\frac{3}{k^{2}}\left(\frac{n}{t}+\right.$

1) $)^{2}-$

$$
Q_{4} t^{-\frac{6 n}{k}} e^{\left(-\frac{6 t}{k}\right)}+
$$

$\frac{3}{l_{1}{ }^{2}}\left(t^{n} e^{t}\right)^{-\frac{2}{k}} \exp \left[-Q_{1} F(t)\right]$

$$
\begin{equation*}
A_{m}=k^{2} Q\left(t^{n} e^{t}\right)^{-\frac{6}{k}}\left(\frac{n}{t}+1\right)^{-2} \tag{4.6}
\end{equation*}
$$

where $Q=Q_{1}{ }^{2}+Q_{2}{ }^{2}+Q_{3}{ }^{2}$

$$
\begin{equation*}
\sigma^{2}=\frac{Q}{2}\left(t^{n} e^{t}\right)^{-\frac{6}{k}} \tag{4.8}
\end{equation*}
$$

$$
\begin{gathered}
\text { For } \boldsymbol{h}=-\mathbf{1} \\
\Lambda=-\left(\frac{16 \pi+3 \lambda}{2 \lambda}\right)\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1}+ \\
\frac{1}{2}\left\{\alpha\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-2}-\left[\frac{3 \alpha}{k} \log \left(t^{n} e^{t}\right)\right]^{-1}\right\}-\frac{3}{k^{2}}\left(\frac{n}{t}+\right. \\
1)^{2}- \\
Q_{4} t^{-\frac{6 n}{k}} e^{\left(-\frac{6 t}{k}\right)}+
\end{gathered}
$$

$\frac{3}{l_{1}{ }^{2}}\left(t^{n} e^{t}\right)^{-\frac{2}{k}} \exp \left[-Q_{1} F(t)\right]$
$\sigma^{2}=\frac{c_{1}{ }^{2}}{3} t^{-\frac{6 n}{k}} e^{2 t\left(1-\frac{3}{k}\right)}$
(4.10)

$$
\begin{aligned}
& A_{m}=\frac{2 c_{1}{ }^{2}}{9\left(\frac{1}{\bar{k}}\left(\frac{n}{t}+1\right)\right)^{2}} t^{-\frac{6 n}{k}} e^{2 t\left(1-\frac{3}{k}\right)} \\
& (4.11)
\end{aligned}
$$

From the above result, we can note that the volume of the universe increases as there is an increase in cosmic time t . We observe that energy density ( $\rho$ ) and barotropic pressure ( $p$ ) of the fluid is a decreasing function of time.From above eqns we can observe that at $\mathrm{t}=0$, the spatial volume scale factor $(\mathrm{V})$ vanishes. While the parameters such as scalar of expansion ( $\theta$ ), mean Hubble parameter ( H ) and shear scalar ( $\sigma$ ) are infinite, this is big bang scenario. As $t \rightarrow \infty, V$ diverges to $\infty$ whereas $\theta, \mathrm{H}$ and $\sigma$ approach to zero whenever $k>0$ and $n \in \mathbb{R}$
Anisotropy parameter $A_{m}$ is a decreasing function of time t and tends to 0 for large values of time t in both the cases whenever $k>0$ and $Q, n \in \mathbb{R}$. Hence, at late times, the Bianchi type $V I_{h}$ model is isotropic for positive values of k .
The deceleration parameter for this model is positive if $\mathrm{t}<\sqrt{n k}-n$ which shows that the universe is in a decelerating phase and, and negative for $\mathrm{t}>\sqrt{n k}-n$ which indicates that the universe is accelerating.

## V. CONCLUSION

In this paper, we have presented new exact solutions of the Bianchi $\mathrm{VI}_{\mathrm{h}}$ cosmological model. The model a spatially homogenous and anisotropic in $f(R, T)$ gravitation theory along with variable cosmological parameter $\Lambda$. The solutions of the field equations in $f(R, T)$ are obtained by considering that $f(R, T)=$ $f_{1}(R)+f_{2}(T)$, the quadratic equation of state. We have considered two variations of $h, h=1$ and $h=-1$. For both the values we have obtained exact solutions of the field equations almost identical observations.

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