

Solving Transportation Problems Solve Using Various New Approach Method

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ABSTRACT

In this paper, we study the optimization processes in Mathematics, Computer Science and Economics are solving effectively by choosing the best element from set of available element. Finding an initial basic feasible solution to obtain an optimal solution for the Transportation Problems. The most importance and successful application in the optimization refers to Transportation Problem. The main objective of Transportation Problem solution method is to minimize the cost or the time of transportation. Most of the currently use methods for solving is to transportation problem are trying to reach the optimal solution, the method is also illustrated with numerical examples.

Keywords: - Transportation Problem, Transportation Cost, Optima Solution, Solving Optimization, Initial Basic Feasible Solution and Objective Function.

I. INTRODUCTION

Transportation problem was first studied by F.L. Hitchcock[1]. In transportation problem, different sources supply to different destinations of demand in such a way that the transportation cost should be minimized. Transportation problem is famous in operation research for its wide application in real life. This is a special kind of the network optimization problems in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized. The basic transportation problem was originally developed by Hitchcock in 1941. Efficient methods for finding solution were developed, primarily by Dantzig in 1951 [2] and then by Charnes, Cooper and Henderson in 1953 [3]. Basically, the solution procedure for the transportation problem

❖ consists of the following phases:

Phase 1: Mathematical formulation of the transportation problem.

Phase 2: Finding an initial basic feasible solution.

Phase 3: Optimize the initial basic feasible solution which is obtained in Phase 2.

1. The level of supply at each source and the amount of demand at each destination.
2. The unit transportation cost of the commodity from each source to each destination.

Since there is only one commodity, a destination can receive its demand from more than one source. The objective is to determine how much should be shipped from each source to each destination so as to minimize the total transportation cost.

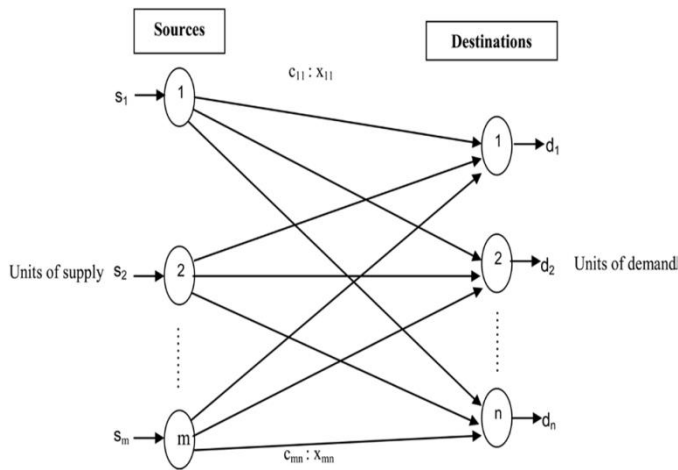


Fig. Network representation of transportation problem

This figure represents a transportation model with m sources and n destinations. Each source or destination is represented by a node. The route between a source and destination is represented by an arc joining the two nodes. The amount of supply available at source i is a_i , and the demand required at destination j is b_j . The cost of transporting one unit between source i and destination j is c_{ij} .

As shown by Figure 1, the problem is to determine an optimal transportation scheme that is to minimize the total of the shipments cost between the nodes in the network model, subject to supply and demand constraints. As well as, this structure arises in many applications such as; the sources represent warehouses and the sinks represent retail outlets.

II. METHODS AND MATERIAL

Mathematical Formulation

Distribution Assumptions

i is the source index for all $i = 1, \dots, m$

j is the destination index for all $j = 1, \dots, n$

Let x_{ij} denote the quantity transported from source i to destination j . The cost associated with this movement is cost \times quantity = $c_{ij}x_{ij}$. The cost of

transporting the commodity from source i to all destinations is given by

$$\sum_{j=1}^n c_{ij}x_{ij} = c_{i1}x_{i1} + c_{i2}x_{i2} + \dots + c_{in}x_{in}$$

Thus, the total cost of transporting the commodity from all the sources to all the destinations is

$$\begin{aligned} \text{Total Cost} &= \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij} \\ &= c_{11}x_{11} + c_{12}x_{12} \dots + c_{1n}x_{1n} + c_{21}x_{21} + \\ & c_{22}x_{22} \dots + c_{2n}x_{2n} + \dots + c_{m1}x_{m1} + \\ & c_{m2}x_{m2} \dots + c_{mn}x_{mn} \end{aligned}$$

In order to minimize the transportation costs, the following problem must be solved:

Minimise

$$z = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij}$$

subject to $\sum_{j=1}^n x_{ij} \leq a_i$ for $i = 1, \dots, m$ and $\sum_{i=1}^m x_{ij} \geq b_j$ for $j = 1, \dots, n$

where $x_{ij} \geq 0$ for all i and j .

The first constraint says that the sum of all shipments from a source cannot exceed the available supply. The second constraint specifies that the sum of all shipments to a destination must be at least as large as the demand. The above implies that the total supply $\sum_{i=1}^m a_i$ is greater than or equal to the total demand $\sum_{j=1}^n b_j$. When the total supply is equal to the total demand then the Transportation model is said to be balanced. In a balanced transportation model, each of the constraints is an equation:

$$\sum_{j=1}^n x_{ij} = a_i \text{ for } i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \text{ for } j = 1, \dots, n$$

A transportation model in which the total supply and total demand are unequal is called Unbalanced. It is always possible to balance an unbalanced transportation problem.

Algorithms for solving: there are several algorithms for solving transportation problems which are based on different of special linear programming methods, among these are:

1. Northwest Corner method

2. Minimum cost method
3. Genetic algorithm
4. Vogel's approximation method
5. Row Minimum Method
6. Column Minimum Method

Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields smaller objective value. In this study, we used the Vogel's approximation method, since it generally produces better starting solutions than other solving methods; as well we have used the BCM solution steps [8].

Vogel's Approximation Method (VAM):

Generally, VAM can be summarized by the following three main steps [9]:

- i. The result of subtracting the smallest unit cost element in the row/column (cell) from the immediate next smallest unit cost element in the same row/column is determining a penalty measure for the target row/column.
- ii. This step includes the following sub-steps:
 - a. Identify the row or the column that includes the largest penalty.
 - b. Break ties arbitrarily.
 - c. As much as possible, the lowest cost row/column (cell) in the row or column should be allocated with the highest difference.
 - d. Adjust the supply and demand, and then cross out the satisfied row or column.
 - e. If a row and column are satisfied simultaneously, then only one of them is crossed out, as well the remains rows or columns are assigned to supply as zero (demand).
- iii. Finally, the result should be computed as follows:
 - a. If a row or a column is assigned as zero supply, or demand remains uncrossed out, then stop the process.
 - b. If one row/column with positive supply (demand) remains uncrossed out, then

determine the basic variables in the row/column by the lowest cost method, and then stop.

- c. If all the uncrossed out rows and columns have (remaining) zero supply and demand then determine the zero basic variables by the lowest cost method and stop.
- d. Otherwise, go to step (i).

The Best Candidates Method (BCM):

BCM process includes three steps, these steps are shown as follows [8]:

- Step1: Prepare the BCM matrix, If the matrix unbalanced, then the matrix will be balanced without using the added row or column candidates in solution procedure.
- Step2: Select the best candidates, that is for minimizing problems to the minimum cost, and Maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as ,the columns must be checked such that if it is not have candidates so that the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.
- Step3: Find the combinations by determining one candidate for each row and column, this should be done by starting from the row that have the least candidates, and then delete that row and column. If there is situation that has no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step 3 by determining the next candidate in the row that started from. Compute and compare the summation of candidates for each combination. This is to determine the best combination that give the optimal solution.

III. RESULTS AND DISCUSSION

3. PROPOSED METHOD

In this study, we proposed a new solving method for transportation problems by using BCM. The proposed method must operate the as following:

- Step1: We must check the matrix balance, If the total supply is equal to the total demand, then the matrix is balanced and also apply Step 2. If the total supply is not equal to the total demand, then we add a dummy row or column as needed to make supply is equal to the demand. So the transportation costs in this row or column will be assigned to zero.
- Step2: Applying BCM to determine the best combination that is to produce the lowest total weight of the costs, where is one candidate for each row and column.
- Step3: Identify the row with the smallest cost candidate from the chosen combination. Then allocate the demand and the supply as much as possible to the variable with the least unit cost in the selected row or column. Also, we should adjust the supply and demand by crossing out the row/column to be then assigned to zero. If the row or column is not assigned to zero, then we check the selected row if it has an element with lowest cost comparing to the determined element in the chosen combination, then we elect it.
- Step4: Elect the next least cost from the chosen combination and repeat Step 3 until all columns and rows is exhausted.

Types of Transportation Problem in Operation Research

The transportation problem is classified into two types. They are balanced transportation problem and unbalanced transportation problem. If the number of sources is equal to number of demands, then it is called balanced transportation problem. If not, it is called unbalanced transportation problem. If the source of item is greater than the demand, then we

should add dummy column to make the problem as balanced one. If the demand is greater than the source, then we should add the dummy row to convert the given unbalanced problem to balanced transportation problem.

1. Balanced Transportation Problem
2. Unbalanced Transportation Problem

Algorithm:

- **Step 1:** Check whether the given transportation problem is balanced or not. If not, balance or by adding dummy row or column. Then go to the next step.
- **Step 2:** Find the harmonic mean for each row and each column. Then find the maximum value among that.
- **Step 3:** Allocate the minimum supply or demand at the place of minimum value of the related row or column.
- **Step 4:** Repeat the step 2 and 3 until all the demands are satisfied and all the supplies are exhausted.
- **Step 5:** Total minimum cost = sum of the product of the cost and its corresponding allocated values of supply or demand.

TRANSPORTATION PROBLEM AND ITS APPLICATIONS IN OPERATION RESEARCH

Transportation problem arises in various applications of Sample Surveys and Operation Research. For details see [10]. The cost matrices associated with these transportation problems are of special structure. Now, we raise the following question. What is the structure of the cost matrix for which North West corner solution produces an optimal solution? We consider some of the structures of the cost matrix which arise in some of the applications in the literature. Ho_man [11] studied transportation problem in the context of North West Corner Rule. Burkard et al. [12] mentioned Monge properties in connection with the transportation problem. Szwarc [13] developed direct methods for solving transportation problems with cost coefficient of the

form $c_{ij} = x_i + x_j$ having applications in shop loading and aggregate scheduling.

Example 1. Balanced Transportation Problem

Consider the following problem with 4 Company and 6 products:

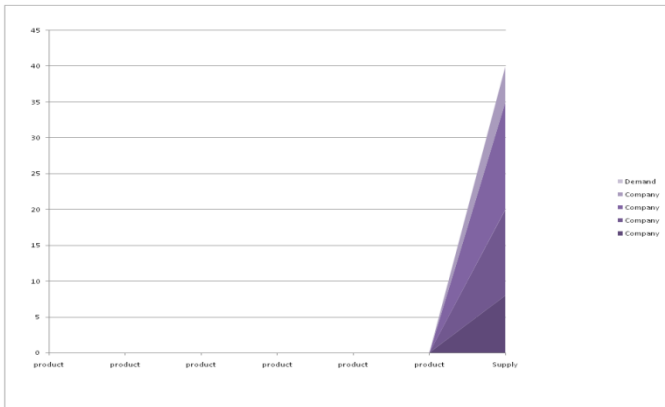
	product	product	product	product	product	product	Supply
Company	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	8
Company	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	12
Company	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	15
Company	c_{41}	c_{42}	c_{43}	c_{44}	c_{45}	c_{46}	5
Demand	4	12	9	8	7	10	

Total supply = $08+12+15+05 = 40$

Total demand = $04+12+09+08+07+10 = 40$

= Total supply

Since Total supply = Total demand, the problem is balanced.



Unbalanced Transportation Problem

Total quantity available \neq total quantity required

i.e., Total supply \neq Total demand

The total quantity available at all the sources is equal to the total quantity required the destinations. If they do not match each other, dummy sources or dummy destination are added to make it a standard transportation problem.

There are 2 situations leading to this unbalanced condition

(i). Total Supply > Total Demand

(ii). Total supply < Total demand

(I). Total Supply > Total Demand

I.e., the total quantity available > total quantity required

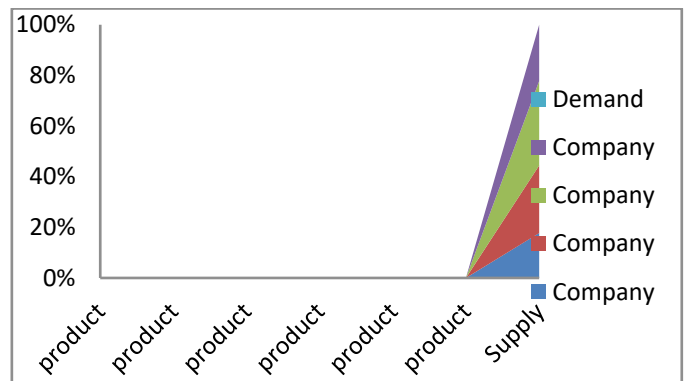
Example 1. Unbalanced Transportation Problem

	product	product	product	product	product	product	Supply
Company	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	8
Company	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	12
Company	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	15
Company	c_{41}	c_{42}	c_{43}	c_{44}	c_{45}	c_{46}	10
Demand	4	12	9	8	7	10	

Total supply = $08+12+15+10 = 45$

Total demand = $04+12+09+08+07+10 = 40$

Since Total supply \neq Total demand, the problem is Unbalanced.



IV.CONCLUSION

At the outset this paper considers some structured transportation problems which arise in sample surveys and other areas of Operation Research. Explain the various methods in transportation problem. In today's highly competitive market, various organizations want to deliver products to the customers in a cost effective way, so that the market becomes competitive. To meet this challenge, transportation model provides a powerful framework to determine the best ways to deliver goods to the customer and also Most of the currently use methods for solving is to transportation problem are trying to reach the optimal solution, the method is also illustrated with numerical examples.

V. REFERENCES

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