International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
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Using Some Transform Techniques to Find Thermal Stresses and Temperature of An Annular Disc.

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ABSTRACT

In this paper, an attempt has been made to solve problems of thermoelasticity and determine the unknown temperature, displacement and stress components. In this problem the zero temprerature is maintained on the curved surface and third kind boundary condition is maintained on lower and upper surface. The governing heat conduction has been solved by using finite Hankel transform technique. The results are obtained in series form in terms of Bessel's functions and have been computed numerically and illustrated graphically.

Keywords : Hankel transform, Thermoelastic problem, Annular Disc, Third kind boundary value Problem, Steady- state.

I. INTRODUCTION

During the second half of the twentieth century, non –isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their many applications in diverse fields. First, the high velocity of modern aircrafts give rise to an aerodynamic heating, which produce intense thermal stresses reducing the strength of aircrafts structure.

Three-Dimensional Thermoelastic Problem Under Two-Temperature Theory studied by (Abhik et al., 2016). A Study on the Generalized Thermoelastic Problem for an Anisotropic Medium studied by (Ghosh et al. 2018)Two dimensional transient problems for a thick annular disc in thermoelasticity studied by (Dange et al., 2009). An inverse temperature field of theory of thermal stresses investigated by (Grysa et al; 1981) while A note of quasi –static thermal stresses in steady state thick annular disc and an inverse quasti-static thermal stresses in thick annular disc are studied by (Gaikwad et al; 2010).

In this paper, the problem of third kind boundary condition is maintained on lower and upper surface of an annular disc. The governing heat conduction equation has been solved by using Hankel transform technique. The results are obtained in series form in terms of Bessel's functions and illustrated graphically.

This paper contains new and novel contribution of thermal stresses in an annular disc under steady state. The above results were obtained under steady state field. The result presented here are useful in engineering problem particularly in the determination of the state of strain in an annular disc constituting foundations of containers for hard gases or liquids, in the foundations for furnaces etc.

II. STATEMENT OF PROBLEM

Consider an annular disc of thickness 2h occupying the space D: $a \le r \le b$, $-h \le z \le h$. The thermoelastic displacement function as in [(Nowacki; 1962) is governed by Poisson's equation

$$\nabla^2 U = (1+\nu)a_t T \tag{2.1}$$

$$\text{with } U_r = 0 \text{ at } r = a \text{ and } r = b \tag{2.2}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$

ν and a_t are the poisson's ratio and the linear coefficient of thermal expansion of the material of the disc and T is the temperature of the disc satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{2.3}$$

Subject to the boundary conditions

$$T(r,z) = 0 \text{ at } r = a, -h \le z \le h \tag{2.4}$$

$$T(r,z) = 0 \text{ at } r = b, -h \le z \le h \tag{2.5}$$

$$\frac{\partial T}{\partial z} + k_1 T = f(r), \text{at } z = h, a \le r \le b \tag{2.6}$$

$$\frac{\partial T}{\partial z} - k_2 T = g(r), \text{at } z = -h, a \le r \le b \tag{2.7}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by,

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{2.8}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{2.9}$$

Where μ *is* the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz} and $\sigma_{\theta z}$ are zero within the disc in the plane state of stress.

The equations (2.1) to (2.9) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF PROBLEM

On applying the finite Hankel transform defined in (Sneddon;1972) to Eq. (2.3), one obtain

$$\frac{d^2 \bar{T}}{dz^2} - \xi^2 \bar{T} = 0 \tag{3.1}$$

where $\bar{T}$ is the Hankel transform of T.

On applying Eq. (3.1) under the conditions given in Eq.(2.6) and Eq.(2.7), one obtains

$$\bar{T} = \sum_{n=1}^{\infty} \bar{f}(\xi_n) \times \left[\frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right]$$
$$- \sum_{n=1}^{\infty} \bar{g}(\xi_n) \times \left[\frac{\xi_n \cosh[\xi_n(z-h)] - k_1 \sinh[\xi_n(z-h)]}{(\xi_n^2 + k_1 k_2)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \quad (3.2)$$

Applying the inverse Hankel transform to the equation (3.2), one obtain the expression for the temperature as

$$T = \sum_{n=1}^{\infty} \bar{f}(\xi_n) \left[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)\right] \times \left[\frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right]$$
$$- \sum_{n=1}^{\infty} \bar{g}(\xi_n) \left[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)\right] \times \left[\frac{\xi_n \cosh[\xi_n(z-h)] - k_1 \sinh[\xi_n(z-h)]}{(\xi_n^2 + k_1 k_2)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \quad (3.3)$$

where

$$\bar{f}(\xi_n) = \int_a^b f(r) r \left[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)\right] \quad (3.4)$$

$$\bar{g}(\xi_n) = \int_a^b g(r) r \left[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)\right] \quad (3.5)$$

Equation (3.3) is the desired solution of the given problem.

DETERMINATION OF THERMELASTIC DISPLACEMENT

Substituting the value T(r, z) from Eq. (3.3) in Eq. (2.1) one obtains the thermoelastic displacement function U(r, z) as,

$$U(r,z) = -(1+\nu)a_t \sum_{n=1}^{\infty} \left(\frac{\bar{f}(\xi_n)}{\xi_n^2}\right) [J_0(r\xi_n) G_0(b\xi_n) - J_0(b\xi_n) G_0(r\xi_n)]$$

$$\times \left[\frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1k_2)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)}\right]$$

$$+(1+\nu)a_t \sum_{n=1}^{\infty} \frac{\bar{g}(\xi_n)}{(\xi_n^2)} [J_0(r\xi_n) G_0(b\xi_n) - J_0(b\xi_n) G_0(r\xi_n)]$$

DETERMINATION OF STRESSES

Using Eq. (3.6) in Eq. (2.8) and Eq. (2.9), one obtains the stress function σ_{rr} and $\sigma_{\theta\theta}$ as,

$$\sigma_{rr} = -\frac{2\mu}{r}(1+\nu)a_t \sum_{n=1}^{\infty} \left(\frac{\bar{f}(\xi_n)}{\xi_n}\right) [J_1(r\xi_n) G_0(b\xi_n) - J_0(b\xi_n) G_1(r\xi_n)]$$

$$\times \left[\frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1k_2)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)}\right] \quad (3.7)$$

$$\sigma_{\theta\theta} = -2\mu(1+\nu)a_t \sum_{n=1}^{\infty} \bar{f}(\xi_n) [J_1'(r\xi_n) G_0(b\xi_n) - J_0(b\xi_n) G_1'(r\xi_n)]$$

$$\times \left[\frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1k_2)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)}\right] \quad (3.8)$$

SPECIAL CASE AND NUMERICAL RESULTS

Set

$f(r) = (r-a)(r-b)e^{h}$, $g(r) = (r-a)(r-b)e^{-h}$, $\alpha = (1-a-b)$ in (3.3) one obtains

$$\frac{T(r,z)}{\alpha} = \sum_{n=1}^{\infty} e^{h}(G_0(b\xi_n) - J_0(b\xi_n))\left\{\frac{b^2}{\xi_n^2}[2J_0(b\xi_n) + (b\xi_n - \frac{4}{b\xi_n})J_1(b\xi_n)]\right.$$

$$-\frac{a^2}{\xi_n^2}\left[2J_0(a\xi_n)+\left(a\xi_n-\frac{4}{a\xi_n}\right)J_1(a\xi_n)\right]$$

$$+\frac{ab}{(1-a-b)}[bJ_1(b\xi_n)-aJ_1(a\xi_n)]\}[J_0(r\xi_n)G_0(b\xi_n)-J_0(b\xi_n)G_0(r\xi_n)]$$

$$\times\left[\frac{\xi_n\cosh[\xi_n(z+h)]+k_2\sinh[\xi_n(z+h)]}{(\xi_n^2+k_1k_2)\sinh(2\xi_n h)+\xi_n(k_1+k_2)\cosh(2\xi_n h)}\right]$$

$$-\sum_{n=1}^{\infty}e^h(G_0(b\xi_n)-J_0(b\xi_n))\{\frac{b^2}{\xi_n^2}\left[2J_0(b\xi_n)+\left(b\xi_n-\frac{4}{b\xi_n}\right)J_1(b\xi_n)\right]$$

$$-\frac{a^2}{\xi_n^2}\left[2J_0(a\xi_n)+\left(a\xi_n-\frac{4}{a\xi_n}\right)J_1(a\xi_n)\right]$$

$$+\frac{ab}{(1-a-b)}[bJ_1(b\xi_n)-aJ_1(a\xi_n)]\}[J_0(r\xi_n)G_0(b\xi_n)-J_0(b\xi_n)G_0(r\xi_n)]$$

$$\times\left[\frac{\xi_n\cosh[\xi_n(z-h)]-k_1\sinh[\xi_n(z-h)]}{(\xi_n^2+k_1k_2)\sinh(2\xi_n h)+\xi_n(k_1+k_2)\cosh(2\xi_n h)}\right] \quad (3.9)$$

The numerical calculation have been carried out for steel (SN 50 C) plate with parameters $a = 1m$, $b = 2m$, $h = 0.5m$.thermal diffusivity $k = 15.9 \times 10{-}6(m2s{-}1)$ and poisons ratio $\nu = 0.281$, while $\xi 1 = 3.1965$, $\xi 2 = 6.3123$, $\xi 3 = 9.4445$, $\xi 4 = 12.5812$, $\xi 5 = 15.7199$ being the positive roots of transcendental equation $[J0(r\xi n)G0(b\xi n) - J0(b\xi n)G0(r\xi n)] = 0$ as in (Ozisik;1968).

IV. DISCUSSION

In this paper, equations (3.8) and (3.9) have been calculated and shown graphically by using Matlab, and the conclusion is as under:

Initially the temperature of the annular disc has been determined by using the conditions given in the problem and applying finite Hankel transform technique and its inverse. In this problem the temperature of the disc has been kept at zero on the curved surfaces and third kind boundary condition has been kept on lower and upper surface of the disc.Here, we have considered steel plate (SN 50C) as the metal and hence the graph shows a particular pattern resembling the properties of steel plate.

In an annular disc the change of temperature does not lead to any change in shear angles, except the stress and strain of the disc. These properties are clearly reflected in the plotted graph given below. The following conclusion can be drawn:

From fig.1 we see that temperature decreases from lower surface to outer curved surface in axial direction. Form fig.2 we observe that temperature decreases from upper surfaces to lower surface in axial direction.

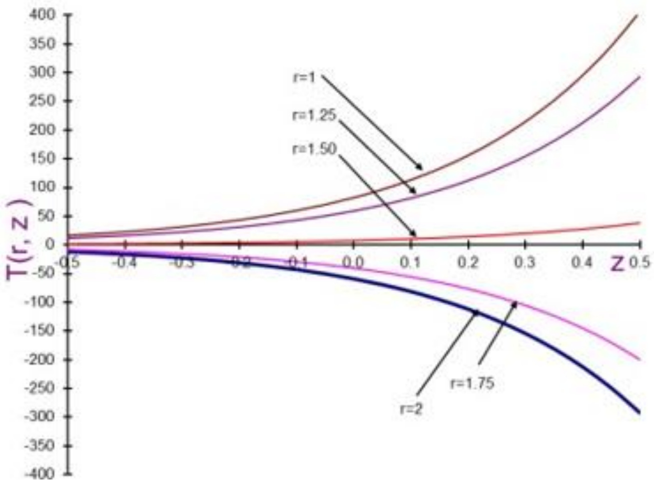

Fig1. The temperature distribution T(r, z) in axial direction

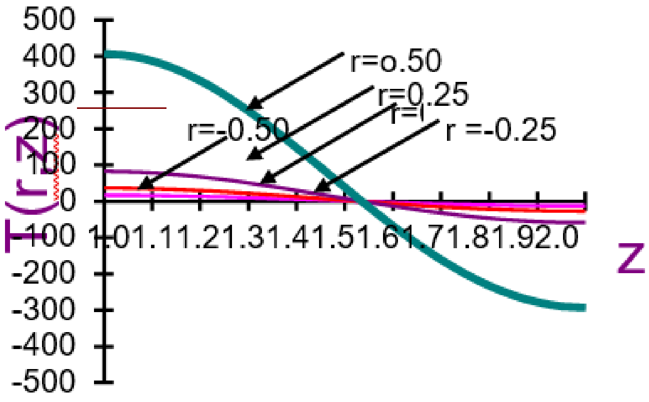

Fig2. The temperature distribution T(r, z)in axial direction

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