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Application of Continuous Group Transforming to Fingero-Imbibition Phenomenon in Porous Media

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ABSTRACT

The non-linear partial differential equation of a parabolic type governing the phenomenon of finger-imbibition in polyphase fluid flow through a homogeneous porous medium with capillary pressure has been discussed using the one parameter continuous group transformation. An analytical solution for average cross sectional area occupied by fingers has been obtained in terms of transcendental functions.

Keywords : - Finger-Imbibition, Transcendental Functions, Homogeneous Porous Medium

I. INTRODUCTION

The present paper discussed the phenomenon of finger-imbibition in polyphase fluid flow through homogeneous porous medium with capillary pressure. This phenomenon arises under certain conditions due to the simultaneous occurrence of two specific phenomenon viz., fingering and imbibition. Here, we have assumed that injection of preferentially wetting, less viscous fluid into a porous medium saturated with resident fluid is initiated under imbibition and in consequence, the resident fluid is pushed by invaling injected fluid which gives arise to fingering; Verma [1, 2] called this conjoint phenomenon as fingero-imbibition.

In this paper the underlying assumptions are that the two fluids are immiscible and the injected fluid is less viscous than native fluid as well as preferentially wetting with respect to porous materials and the effect of capillary pressure is involved. The mathematical formulation of basic equations yields a non-linear partial differential equation governing finger-imbibition in the investigated fluid- fluid displacement problem. An analytical expression for average cross sectional area occupied by fingers has been obtained by similarity method of one parameter continuous group of transformations [3] which is applied to derive the functional relationship between two independent variables which give rise to a similarity transformation. Employing this transformation to the non-linear partial differential equation, it reduce to an ordinary differential equation which is further reduced into the form of Abel's equation of second kind [4] whose solution is obtained in terms of transcendental functions.

II. STATEMENT OF THE PROBLEM

We consider a finite cylindrical piece of homogeneous porous matrix, saturated with native fluid (N) which is completely surrounded by an impermeable surface except for one end, which is termed as the imbibition face (x=0), and this end is exposed to an adjacent formation of injection fluid (e.g. water). It is preferentially wetting and less viscous. This gives rise to a displacement process in which imbibition and the consequent displacement of native liquid (e.g. oil) produces instabilities. This arrangement describes a one-dimensional phenomenon of finger-imbibition.

III. CAPILLARY PRESSURE

The capillary pressure p_0 is defined as the pressure discontinuity between the phases across their common interface and is a function of saturation. For definiteness, here we have assumed P_c to be a continuous linear function of water saturation of the form [5]

$$P_c = \beta\, S_w \qquad (3.1)$$

and

$$p_c = p_o - p_w \qquad (3.2)$$

Where β is a capillary constant. S_w is the water saturation and p_w and p_0 are pressures in water and oil respectively. The case $p_0 = 0$ has been discussed by a number of authors. Here, β is a small positive constant.

IV. RELATION BETWEEN RELATIVE PERMEABILITY AND PHASE SATURATION

An analytical relation between relative permeability and phase saturation, due to Scheidegger and Johnson [6], has been given by,

$$k_w = S_w \qquad (4.1)$$

and

$$k_o = 1 - S_w = S_o \qquad (4.2)$$

Where k_w and k_o are the fictious relative permeability of injected and native fluid respectively. S_w and $S_o$are saturation of the injected and native phase respectively.

V. FUNDAMENTAL EQUATION

We further assume that the flow of two immiscible phases is governed by Darcy's law; we may, therefore, write the seepage velocities injected and native fluids as

$$V_w = -\left(\frac{k_w}{\delta_w}\right) K \frac{\partial p_w}{\partial x} \qquad (5.1)$$

$$V_o = -\left(\frac{o}{\delta_o}\right) K \frac{\partial p_o}{\partial x} \qquad (5.2)$$

Neglecting the variation in phase density the equation of continuity for injected phase is given by

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \qquad (5.3)$$

Where P is the porosity of the medium.

Following Scheidegger [7], the analytical condition governing imbibition is,

$$V_w + V_o = 0 \quad (5.4)$$

VI. FORMULATION OF DIFFERENTIAL EQUATION

Substituting the value of V_w and V_o from (5.1-2) into equation (5.4), we get

$$K\left(\frac{k_0}{\delta_0}\right)\frac{\partial p_o}{\partial x} + K\left(\frac{k_w}{\delta_w}\right)\frac{\partial p_w}{\partial x} = 0 \quad (6.1)$$

Substituting this value of p_o from (3.2) into equation (6.1), we get,

$$\frac{\partial p_w}{\partial x} = -\frac{\left(\frac{k_o}{\delta_o}\right)}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_0}{\delta_0}\right)}\frac{\partial p_o}{\partial x} \quad (6.2)$$

Using (6.2) in equation (5.1) we get,

$$V_w = -K\left(\frac{k_w}{\delta_w}\right)\left[\frac{\frac{k_0}{\delta_0}}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_o}{\delta_o}\right)}\frac{\partial p_o}{\partial x}\right] \quad (6.3)$$

Substituting this value of V_w into equation (5.3), we obtain,

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\frac{k_0}{\delta_0}}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_o}{\delta_o}\right)}\frac{\partial p_o}{\partial x}\right] = 0 \quad (6.4)$$

Finally, using (3.1) in equation (6.4), we get,

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\frac{k_0}{\delta_0}}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_o}{\delta_o}\right)}\frac{\partial p_o}{\partial S_w}\frac{\partial S_w}{\partial x}\right] = 0 \quad (6.5)$$

Again, we use an approximation due to Scheidegger [7],

$$\frac{\frac{k_0}{\delta_0}}{\left(\frac{k_w}{\delta_w}\right)+\left(\frac{k_o}{\delta_o}\right)} \approx \frac{k_o}{\delta_0} \quad (6.6)$$

In view of (3.1), (4.2) and (6.6) equation (6.5) becomes,

$$P\frac{\partial S_w}{\partial t} + \frac{K\beta}{\delta_o}\frac{\partial}{\partial x}\left[(1 - S_w)\frac{\partial S_w}{\partial x}\right] \quad (6.7)$$

Setting $\frac{X}{L} = X, T = \theta t,$ and $1 - S_w = S,$

where $\theta = \left(\frac{K\beta}{P\delta_0 L^2}\right)$, in equation (6.7), we get

$$\frac{\partial S}{\partial T} + S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2 = 0 \quad (6.8)$$

Equation (6.8) is non-linear partial differential equation describing the phenomenon of finger-imbibition in the flow of two immiscible phases through homogeneous porous media.

VII. SIMILARITY ANALYSIS

In this section we analyze the problem (6.8) by studying its invariance under a global transformation. In particular, a family of one parameter continuous group of transformations, has been used to derive the similarity transformation which transformation (6.8) into an ordinary differential equation.

$$\text{Let } S = \theta\,(X, T) \qquad (7.1)$$

by the solution of the problem (6.8). consider the general stretching transformation of (S, X, T) – space,

$$S^* = \delta S, \qquad X^* = \alpha X, \qquad T^* = \beta T \qquad (7.2)$$

with parameter (δ, α, β). If $\delta(\beta)$ and $\beta(\alpha)$ are somehow determined, then (7.2) will be a family of one parameter continuous groups of transformations.

$$\text{Let } S^* = \theta^*(X^*, T^*) \qquad (7.3)$$

be the new solution surface corresponding to (7.2). hence, original solution surface transforms,

$$\left.\begin{aligned} \frac{\partial S}{\partial T} &= \frac{\beta}{\delta}\frac{\partial S^*}{\partial T^*} \\ \frac{\partial S}{\partial X} &= \frac{\alpha}{\delta}\frac{\partial S^*}{\partial X^*} \\ \frac{\partial^2 S}{\partial X^2} &= \frac{\alpha^2}{\delta}\frac{\partial^2 S^*}{\partial X^{*2}} \end{aligned}\right] \qquad (7.4)$$

Since, $\theta(X, T)$ is defined by (6.8), we have,

$$\frac{\beta}{\delta}\frac{\partial S^*}{\partial T^*} + \frac{S^*\alpha^2}{\delta^2}\frac{\partial^2 S^*}{\partial X^{*2}} + \frac{\alpha^2}{\delta^2}\left(\frac{\partial S^*}{\partial X^*}\right)^2 = 0 \qquad (7.5)$$

For invariance, it is necessary that both the operators on the left and right hand sides of (7.5) agree with those in (6.8) multiplied a common factor. Thus, for invariance

$$\delta = 1; \quad \alpha^2 = \beta \Rightarrow \alpha = \sqrt{\beta}.$$

Hence, the one parameter continuous group of transformations, which leaves the problem (6.8) invariant is,

$$S^* = S, X^* = \sqrt{\beta}X, \;\; T^* = \beta T \qquad (7.6)$$

Thus, for S^*, we have

$$\frac{\partial S^*}{\partial T^*} + S^*\frac{\partial^2 S^*}{\partial X^{*2}} + \left(\frac{\partial S^*}{\partial X^*}\right)^2 = 0 \qquad (7.7)$$

Now, due to uniqueness, θ must be same function of (X^*, T^*) as θ^* is of (X, T), i.e.,

$$\theta^*(X, T) = \theta(X^*, T^*) \qquad (7.8)$$

As a consequence of transformation (7.6) and the invariance condition (7.8), we obtain a functional equation which must be satisfied by the solution; therefore, (7.8) implies,

$$\theta(\sqrt{\beta}X, \beta T) = \theta(X, T) \tag{7.9}$$

This functional relation (7.9) holds true for all values of β. To obtain a functional form that the solution $\theta(X, T)$ must have consider $\partial/\partial\beta$ of (7.9) near the identity $\beta = 1$; we then have

$$\frac{X}{2\sqrt{\beta}}\frac{\partial}{\partial X}\left[\theta(\sqrt{\beta}\,X, \beta T)\right] + T\frac{\partial}{\partial T}\left[\theta(\sqrt{\beta}X, \beta T)\right] = 0 \tag{7.10}$$

as $\beta \rightarrow 1, \theta(X, T)$ satisfies a first order p.d.e.

$$\frac{X}{2}\frac{\partial\theta}{\partial X} + T\frac{\partial\theta}{\partial T} = 0 \tag{7.11}$$

The general solution of (7.11) involves an arbitrary function. The characteristic equation associated with (7.11) are

$$\frac{dX}{\left(\frac{X}{2}\right)} = \frac{dT}{T} = \frac{dS}{O} \tag{7.12}$$

The integral of the first two in (7.12) implies $\eta = \frac{X}{\sqrt{T}}$ (7.13)

and that of for the last two in (7.12) implies

$$S = constant = F(\eta) \tag{7.14}$$

Thus, the similarity transformation has the form

$$S = F(\eta); \quad \eta = \frac{X}{\sqrt{T}} \tag{7.15}$$

Applying this similarity transformation to equation (6.8) it reduces to,

$$FF'' + (F')^2 - \frac{1}{2}\,\eta\, F' = 0 \tag{7.16}$$

Where $F = F(\eta)$ and a prime denotes differentiation with respect to η

The non-linear ordinary differential equation (7.10) is obtained by a similarity transformation of the problem (6.8).

VIII. ANALYTIC SOLUTION

To solve the equation (7.16), we get

$$F(\eta) = \eta^2\, u(z); z = \log\eta\,, u'(z) = p \tag{8.1}$$

into equation (7.16); it reduced to

$$upp'(u) + p^2 + \left(7u - \frac{1}{2}\right)p + u(6u - 1) = 0 \tag{8.2}$$

Equation (8.2) is Abel's equation of second kind [4]; to solve it let $up = \log v(z) = M(z)$. then (8.2) becomes

$$\frac{dM}{dz} + \left(7u - \frac{1}{2}\right)\frac{M}{u} + (6u-1)u = 0 \qquad (8.3)$$

Equation (8.3) is a linear o.d.e. of first order. The integrating factor of (8.3) is

$$exp.\left|\int \left(7 - \frac{1}{2u}\right) dz\right| \qquad (8.4)$$

Therefore the solution of the problem (8.3) is,

$$M\, exp.\left[\int \left(7 - \frac{1}{2u}\right) dz\right]$$

$$= \int \left[u(6u-1)exp.[\int \left(7 - \frac{1}{2u}\right) dz]\right] dz \ + c$$

(8.5)

Where C is constant of integration. Hence, in terms of η and $F(\eta)$ equation (8.5) can be written as

$$M\, exp.\left(\int \left(\frac{7}{\eta} - \left(\frac{\eta}{2F}\right)\right) d\eta \right)$$

$$= \int \left[\left(\frac{F}{\eta^3}\right)\left[1 - \left(\frac{6F}{\eta^2}\right)\right] \times exp.\left(\int \left|\left(\frac{7}{\eta}\right) - \left(\frac{\eta}{2F}\right)\right| d\eta\right)\right] d\eta + c$$

(8.6)

The constant C of integration can be determined by considering the boundary condition at $t = 0$. Since at t=0, $\eta = \infty$ and $F(\eta) = \infty$ the equation (8.6) gives C=0 and (8.6) reduced to

$$M\, exp.\left(\int \left(\frac{7}{\eta} - \left(\frac{\eta}{2F}\right)\right) d\eta \right)$$

$$= \int \left[\left(\frac{F}{\eta^3}\right)\left[1 - \left(\frac{6F}{\eta^2}\right)\right] \times exp.\left(\int \left|\left(\frac{7}{\eta}\right) - \left(\frac{\eta}{2F}\right)\right| d\eta\right)\right] d\eta$$

(8.7)

The equation (8.7) gives the solution of the problem in terms of transcendental function which is an approximate solution of the problem (6.8) which governs the phenomenon.

IX. CONCLUSION

In this section, we conclude the results of the previous sections. We have obtained an analytical solution of the finger-imbibition phenomenon of two immiscible phases in homogeneous porous media. The similarity transformation has been derived by using a family of one parameter continuous group of transformation. The similarity transformation thus obtained has been used to reduce the original partial differential equation to an ordinally differential equation known as Abel's equation of second kind. The solution of this o.d.e. has been obtained in terms of transcendental function which represent the distribution of injected fluid in porous which was saturated by native fluid.

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