National Conference on Applied Sciences Synergising the Engineering and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
© 2021 IJSRST | Volume 8 | Issue 2

Application of Variational Homotopy Perturbation Method For Schrodinger Equation

Doreswamy H. S[1], Gayathri V[2]

[1]Professor, Department of Mathematics, East Point College of Engineering and Technology, Bengaluru, Karnataka, India

[2]Assitant Professor, Department of Mathematics, East Point College of Engineering and Technology, Bengaluru, Karnataka, India

ABSTRACT

In the present work, we apply the Variational Homotopy Perturbation Method to obtain the solution of linear and nonlinear Schrodinger equation. The Variational Homotopy Perturbation Method (VHPM) deforms a difficult problem into a simple problem which is easy to get the result. The method produces a solution in the form of a convergent series under conditions that are easy to calculate. Some examples are given to show that this method is easy to apply and the results are obtaining very fast.
Keywords : Linear, Nonlinear, Boundary Conditions, VHPM, Schrodinger Equations

I. INTRODUCTION

The Schrodinger equation has been widely used in various areas of physics including optics, mechanics, plasma physics and quantum mechanics. The Homotopy Perturbation Method (HPM) and the Variational iteration method (VIM) was introduced by Chinese mathematician J. Huan He [1-5]. In recent years, various methods have been proposed such as Adomian decomposition method [ADM], Laplace decomposition method [LDM], Finite difference technique [FDT], Variational method [VIM], but all these methods have some limitations.

The VHPM is a general analytic approach to get series solutions of various types of equations. The VHPM provides us a simple way to ensure the convergence of series solution by introducing a small parameter to ideal results. In this case, the parameter p takes the values from 0 to 1. Suppose, at $p = 0$, the problem usually reduces to simplified form. As parameter p slowly increases to 1, the problem goes through a sequence of deformation. Suppose, at $p = 1$, the problem takes the original form and the final stage of deformation gives the exact solution.

In the recent decade, several scholars in the fields of partial differential equations have paid attention in showing the existence and the solutions of linear and nonlinear Schrodinger equations were developed [6-10].

The main output of the present paper is details of VHPM in solving the linear and nonlinear Schrodinger equations to show the simplicity and straight forwardness are explained.

II. BASIC IDEA OF VARIATIONAL HOMOTOPY PERTURBATION METHOD

Consider a nonlinear Partial Differential Equations (PDE) with the initial conditions

$$L[u(x,t)]+N[u(x,t)]=g(t) \qquad (1)$$

where $L =$ linear differential operator

N =general nonlinear differential operator

$g(t) =$ source term

According to Variational Iteration Method (VIM), we write a correctional functional as

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda(x,t)\left[Lu_n(x,t)+N\tilde{u}_n(x,t)-g(t)\right]dt \qquad (2)$$

where λ represents the general Lagrangian multiplier which can be identified optimally,

$u_n =$ n[th] approximate solution,

$\tilde{u}_n =$ restricted variation, i.e., $\delta\tilde{u}_n = 0$.

By Homotopy perturbation method (HPM), we can construct an equation is as follows

$$\sum_{i=0}^{\infty} p^i u_i = u_0(x)+p\int_0^t \lambda(x,t)\left[\sum_{i=o}^{\infty} L\left(p^i u_i(x,t)\right)+N\left(p^i\tilde{u}_i(x,t)\right)\right]dt-\int_0^t \lambda(x,t)g(t)\,dt \qquad (3)$$

an approximation to the solution usually will be obtained by identical powers of p and taking the limit as $p \to 1$, we get

$$u(x,t)=\lim_{p\to 1}\sum_{i=0}^{\infty} p^i u_i(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+.......... \qquad (4)$$

III. APPLICATIONS

3.1 The linear Schrodinger equation

Example 1: Consider the linear Schrodinger equation

$$u_t + iu_{xx} = 0 \qquad (5)$$

Initial condition is given by

$$u(x,0)=1+\cosh 2x$$

This is solved by VHPM.

Now, by exerting the VHPM, it is possible to obtain the equation as follows,

$$u_0 + pu_1 + p^2u_2 + p^3u_3 + ... = (1+\cosh 2x) - ip\int_0^t \left(u_0 + pu_1 + p^2u_2 + p^3u_3 + ...\right)_{xx} dt \qquad (6)$$

Comparing the powers of p from both sides, we have

$$p^0 : u_0(x,t) = 1+\cosh 2x$$

$$p^1 : u_1(x,t) = -4it\cosh 2x$$

$$p^2 : u_2(x,t) = \frac{(-4it)^2}{2!}\cosh 2x$$

$$p^3 : u_3(x,t) = \frac{(-4it)^3}{3!}\cosh 2x$$

$$p^4 : u_4(x,t) = \frac{(-4it)^4}{4!}\cosh 2x$$

....

$$u(x,t) = 1 + \cosh 2x + (-4it)\cosh 2x + \frac{(-4it)^2}{2!}\cosh 2x + \frac{(-4it)^3}{3!}\cosh 2x + \frac{(-4it)^4}{4!}\cosh 2x + \ldots\ldots\ldots\ldots$$

$$= 1 + \cosh h2x\left[1 + (-4it) + \frac{(-4it)^2}{2!} + \frac{(-4it)^3}{3!} + \frac{(-4it)^4}{4!} + \ldots\ldots\ldots\right]$$

$$= 1 + \cosh 2xe^{-4it} \quad (7)$$

This is the exact solution which is readily obtained upon using the Taylor's series expansion of e^{-4it}.

Example 2: Consider the linear Schrodinger equation

$$u_t + iu_{xx} = 0 \quad (8)$$

Initial condition is given by

$$u(x,0) = e^{3ix}$$

This is solved by VHPM.

Now, by exerting the VHPM, it is possible to obtain the equation as follows,

$$u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots\ldots\ldots = e^{3ix} - ip\int_0^t \left(u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots\ldots\ldots\right)_{xx} dt \quad (9)$$

Comparing the powers of p from both sides, we have

$$p^0 : u_0(x,t) = e^{3ix}$$

$$p^1 : u_1(x,t) = 9ite^{3ix}$$

$$p^2 : u_2(x,t) = \frac{(9it)^2}{2!}e^{3ix}$$

$$p^3 : u_3(x,t) = \frac{(9it)^3}{3!}e^{3ix}$$

....

Substituting this value in equation (4), we get

$$u(x,t) = e^{3ix} + 9ite^{3ix} + \frac{(9it)^2}{2!}e^{3ix} + \frac{(9it)^3}{3!}e^{3ix} + \ldots\ldots\ldots$$

$$= e^{3ix}\left[1 + 9it + \frac{(9it)^2}{2!} + \frac{(9it)^3}{3!}\ldots\ldots\ldots\ldots\ldots\right]$$

$$= e^{3ix}.e^{9it}$$

$$= e^{3i(x+3t)} \quad (10)$$

This is the exact solution.

3.2 The Nonlinear Schrodinger equation

Example 3: Consider the nonlinear Schrodinger equation

$$iu_t + u_{xx} + m|u|^2 u = 0 \quad \text{or} \quad (11)$$

$$u_t = iu_{xx} + im|u|^2 u$$

Initial condition is given by

$$u(x,0) = e^{nix}$$

This is solved by VHPM.

Now, by exerting the VHPM, it is possible to obtain the equation as follows

$$u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots\ldots.. = e^{nix} + ip\int_0^t \left(u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots.\right)_{xx} dt$$
$$+imp\int_0^t \left(u_0 + pu_1 + p^2u_2 + \ldots.\right) dt \quad (12)$$

Comparing the powers of p from both sides, we have

$$p^0 : u_0(x,t) = e^{nix}$$

$$p^1 : u_1(x,t) = i(m-n^2)te^{nix}$$

$$p^2 : u_2(x,t) = i^2(m-n^2)^2 \frac{t^2}{2!} e^{nix}$$

$$p^3 : u_3(x,t) = i^3(m-n^2)^3 \frac{t^3}{3!} e^{nix}$$

....

Substituting this value in equation (4), we get

$$u(x,t) = e^{nix} + i(m-n^2)te^{nix} + i^2(m-n^2)^2 \frac{t^2}{2!} e^{nix} + i^3(m-n^2)^3 \frac{t^3}{3!} e^{nix} + \ldots\ldots..$$

$$= e^{nix}\left[1 + i(m-n^2)t + i^2(m-n^2)^2 \frac{t^2}{2!} + i^3(m-n^2)^3 \frac{t^3}{3!} + \ldots\ldots..\right]$$

$$= e^{nix}.e^{i(m-n^2)t}$$

$$= e^{i\left[nx + \left(m-n^2\right)t\right]} \quad (13)$$

This is the exact solution.

Example 4: Consider the nonlinear Schrodinger equation

$$iu_t + u_{xx} + 2|u|^2 = 0 \quad (14)$$

or

$$u_t = iu_{xx} + 2i|u|^2$$

Initial condition is given by

$$u(x,0) = 2\operatorname{sech}(2x)$$

This is solved by VHPM.

Now, by exerting the VHPM, it is possible to obtain the equation as follows,

$$u_0 + pu_1 + p^2u_2 + p^3u_3 + ... = 2\operatorname{sech}(2x) + ip\int_0^t \left(u_0 + pu_1 + p^2u_2 + p^3u_3 + ...\right)_{xx} dt \quad (15)$$

Comparing the powers of p from both sides, we have

$$p^0 : u_0\left(x,t\right) = 2\operatorname{sech} 2x$$

$$p^1 : u_1\left(x,t\right) = 8it\operatorname{sech} 2x$$

$$p^2 : u_2\left(x,t\right) = 16i^2t^2\operatorname{sech} 2x$$

$$p^3 : u_3\left(x,t\right) = \frac{64}{3}i^3t^3\operatorname{sech} 2x$$

....

Substituting this value in equation (4), we get

$$u\left(x,t\right) = 2\operatorname{sech} 2x + 8it\operatorname{sech} 2x + 16i^2t^2\operatorname{sech} 2x + \frac{64}{3}i^3t^3\operatorname{sech} 2x + ...$$

$$u\left(x,t\right) = 2\operatorname{sech} 2x\left[1 + 4it + \frac{(4it)^2}{2!} + \frac{(4it)^3}{3!} + ...\right]$$

$$= 2\operatorname{sech} 2x.e^{4it} \quad (16)$$

This is the exact solution.

IV. CONCLUSION

In this paper, the Variational Homotopy Perturbation Method has been successfully applied to find the solution of the linear and nonlinear Schrodinger equations. The present method is easy and reliable to use. The outcomes show that VHPM is very powerful and efficient technique in finding analytical solutions for wider problems. The convergence accuracy of this method was examined in several examples. This method is capable of reducing the computational work.

Acknowledgements:

The authors would like to thank the reviewers for their valuable comments and suggestions to improve this paper. The authors would like to express their gratitude to chairman, principal and staff members of East Point College of Engineering and Technology, Bengaluru.

V. REFERENCES

[1]. J.H. He, Homotopy perturbation technique, computer methods in applied mechanics and engineering, 178, 257-262, 1999.

[2]. J.H. He, A review on some new recently developed nonlinear analytical techniques, International journal of nonlinear sciences and numerical simulation,1,51-70, 2000.

[3]. J. H. He, Homotopy perturbation method: a new nonlinear analytical technique, Applied Mathematics and Computation, 135, 73-79, 2003.

[4]. J.H. He, Homotopy perturbation method for solving boundary value problems, physics letters A 350:87-88, 2006.
[5]. J.H. He, Recent development of the homotopy perturbation method, Topological Methods in Nonlinear Analysis, vol.31,2,205-209,2008.
[6]. Sh. Sadigh Behzadi, Solving Schrodinger equation by using modified variational iteration and homotopy analysis methods, J. Appl. Anal. Compute, 1(4), 427-437,2011.
[7]. R Hao, Lu Li, Zhonghao Li, W Xue, G Zhou, A new approach to exact soliton solutions and soliton interaction for nonlinear Schrodinger equation for nonlinear Schrodinger equation with variable coefficients, Opt. Commun, 236 (1-3), 79-86, 2004.
[8]. Biao Li, Yong Chen, On exact solutions of the nonlinear Schrodinger equations in optical fiber, Chaos Solitons Fractals, 21(1), 241-247,2004.
[9]. S.Z. Rida, H.M. El-Sherbiny, A.A.M. Arafa, on the solution of the fractional nonlinear Schrodinger equation, Physical Letters, A, 372 (5),553-558, 2008
[10]. A M Wazwaz, Study on linear and nonlinear Schrodinger equations by the variational iteration method, Chaos Solitons Fractals, 37 (4), 1136-1142,2006.