

# Implementation of Mathematical Models of Buck Converter using MatLab/Simulink

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## ABSTRACT

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The study of the dynamic behavior, transient response and the characteristics of the converters require deep knowledge in mathematical models. Mathematical models have been adequately used in the design and control of DC-DC converters and also the mathematical models are more suitable than the physical models. This paper describes the four modeling techniques of a buck converter, circuit model, mathematical model, state space model and transfer function model, and their implementation in Simulink environment.

**Keywords :** Buck converter, State space, Transfer function, mathematical model

## I. INTRODUCTION

Mathematical equations which have been used to describe the behaviour and performance of any particular converters are the foundation of models. The power electronic converters are developed either as a system model or a circuit model. The switching function concept can be introduced along with the system model to reach out the expected outputs [1-3]. Hence the mathematical models are used to predict the performance and these predictions might help the design Engineers to think various control strategies [4].

Mathematical modelling of a circuit has complete control over the simulation process which takes a smaller execution time but circuit model consumes less initial setup time. The transfer function model is a

simple one, and also applicable to both time domain and frequency domain analysis. But, it is only suitable for single input and single output model. But, the state space model overcomes this difficulty [5-7].

The simple buck converter is considered in this paper to detail about the mathematical models. Buck converter is a second order converter which consists of a diode, one inductor, a semiconductor switch, one capacitor, one load resistor and a DC input voltage source. It converts high voltage into low voltage and finds applications that require fast line and load transient responses as in voltage regulators [5].

## II. METHODS AND MATERIAL

The buck converter topology is shown in Fig. 1 where the switch can be any one of MOSFET, BJT, IGBT or

GTO. The switch is triggered by an external pulse width modulation scheme whose pulse period is T and duty cycle is D.

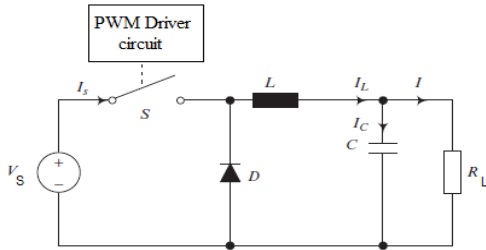


Figure 1: Buck converter

The equivalent circuits when the switch is ON from 0 to DT and OFF from DT to T are shown in Fig. 2.

Based on the assumption of continuous conduction mode (CCM) of operation, the average inductor current over one complete cycle of period T is zero and the equations can be derived.

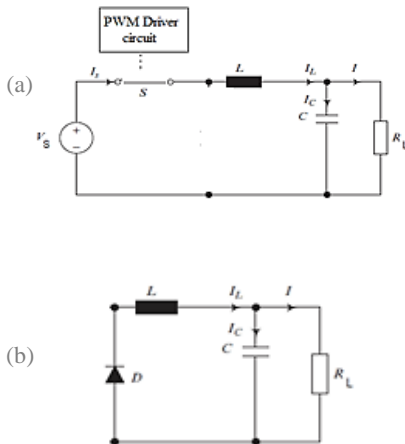


Figure 2: When the switch is (a) ON (b) OFF

$$\frac{(V_s - V_o)DT}{L} + \frac{(-V_o)(1-D)T}{L} = 0 \quad (1)$$

$$\frac{V_o}{V_s} = D \quad (2)$$

The buck converter waveform in CCM is shown in Fig. 3. During the switch ON period, the inductor current increases to  $I_{Lmax}$  and drops to  $I_{Lmin}$  linearly during switch OFF period. From Fig 3.a, the average inductor current  $I_{Lavg}$  is derived as in equation (3)-(4).

$$I_{Lavg} = \frac{\text{Area 1} + \text{Area 2} + \text{Area 3}}{T} \quad (3)$$

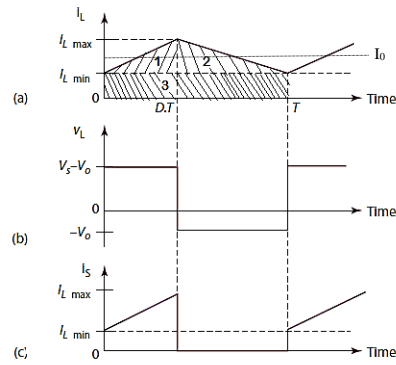


Figure 3: Buck converter waveforms

$$= \frac{(\frac{1}{2} \times DT \times (I_{Lmax} - I_{Lmin})) + \frac{1}{2} \times (1-D)T \times (I_{Lmax} - I_{Lmin}) + I_{Lmin} \times T}{T}$$

$$I_{Lavg} = \frac{1}{2} \times (I_{Lmax} + I_{Lmin}) \quad (4)$$

The average load current is given by Equation (5).

$$I_o = \frac{V_o}{R_L} \quad (5)$$

From Fig. 3a,

$$I_{Lmax} = I_o + \frac{\Delta i_L}{2} \text{ and } I_{Lmin} = I_o - \frac{\Delta i_L}{2} \quad (6)$$

$$\text{where } \Delta i_L = \frac{(V_s - V_o) \times DT}{L} \quad (7)$$

Using equation (2),

$$I_{Lmax} = DV_s \times \left[ \frac{1}{R_L} + \frac{(1-D)T}{L} \right] \quad (8)$$

$$I_{Lmin} = DV_s \times \left[ \frac{1}{R_L} - \frac{(1-D)T}{L} \right] \quad (9)$$

The critical inductance to retain CCM is obtained by equating  $I_{Lmin}$  to zero.

$$L_{crit} = \frac{(1-D)T \times R_L}{L} \quad (10)$$

### III. MATHEMATICAL MODELS AND SIMULATION

The mathematical models are carried out for the following design parameters. The input voltage,  $V_s = 48V$ , the output voltage,  $V_o = 12V$ , the nominal output power,  $P_o = 200W$ , the switching frequency,  $f_s = 50 \text{ kHz}$ , the maximum current ripple in the output inductor  $\Delta I_{Lo} =$

$10\% I_o$ , and the maximum output voltage ripple  $\Delta V_o = 0.5\% V_o$ .

For an output voltage of 48V, the corresponding duty cycle  $D = V_o/V_s = 12/48 = 0.25$  and the output current  $I_o = 200/12 = 16.6$  A. Based on the specifications, the required output inductance  $L$  and output capacitance  $C$  can be obtained as follows.

$$L \geq \frac{(1-D)T \times R_L}{L} = 109 \mu H$$

$$C \geq \frac{\Delta I_L \times T}{8 \times \Delta V_0} = 69 \mu F$$

**A. Circuit Model**

It is a graphical method of representing the circuit by using the physical components available in the Simulink library. It is a simple and a straight forward method. Fig. 4 shows the circuit model of Buck converter for the chosen design parameters and the output voltage is shown in Fig. 5.

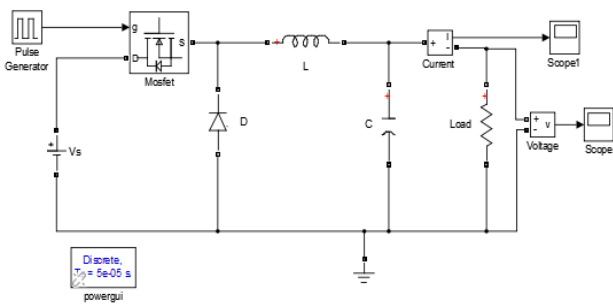


Figure 4: Circuit model of Buck converter

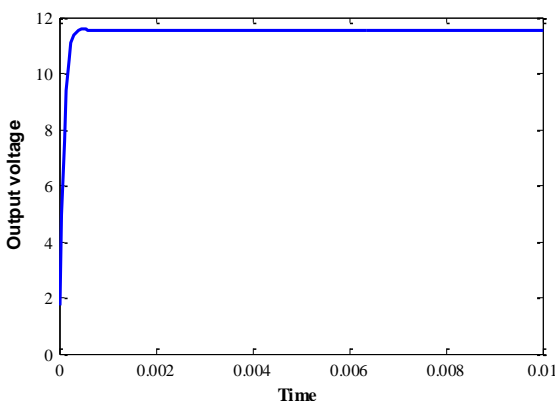


Figure 5: Output voltage of circuit model

**B. Mathematical Model**

According to the Kirchhoff's current and Kirchhoff's voltage laws, the basic differential equations can be

derived by referring the circuit shown in Fig. 2 for the ON and OFF states.

$$I_c(t) = I_L(t) - I_0(t)$$

$$V_c(t) = \frac{1}{C} \int I_c(t) dt = \frac{1}{C} \int (I_L(t) - I_0(t)) dt$$

$$I_0(t) = \frac{V_0(t)}{R_L}$$

$$V_L(t) = V_s(t) - V_0(t) \times DT - (V_0(t)) \times (1 - D)T \times \text{sign}(I_L)$$

The inductor current may take either positive or negative value based on the circuit elements. The equations (11) – (13) are used to implement the mathematical model in Simulink and as shown in Fig. 6 and the output voltage is shown in Fig.7.

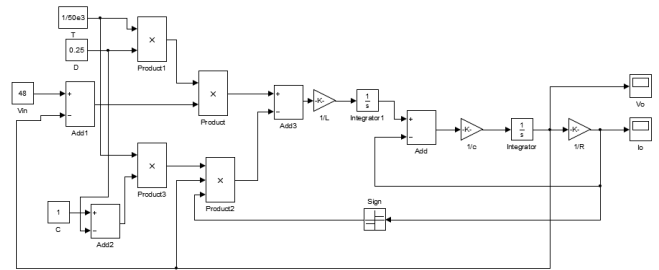


Figure 6: Mathematical model of buck converter

When the switch is ON for the period of  $DT$ ,

$$\frac{dI_L(t)}{dt} = \frac{V_s - V_0}{L}$$

$$\frac{dV_C(t)}{dt} = \frac{I_L - I_0}{C}$$

$$V_0(t) = V_c(t)$$

When the switch is OFF for the period of  $(1-D)T$ ,

$$\frac{dI_L(t)}{dt} = \frac{-V_0}{L}$$

$$\frac{dV_C(t)}{dt} = \frac{I_L - I_0}{C}$$

$$V_0(t) = V_c(t)$$

The output parameters over one complete period  $T$  can be calculated by

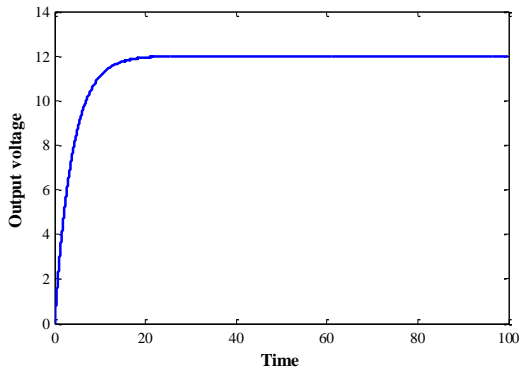


Figure 7: Output voltage

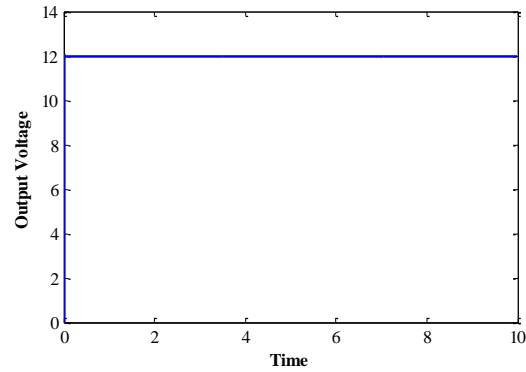


Figure 9: Output voltage

**A. Transfer Function Model**

From the Fig 1, the differential equations can be written for the inner mesh and outer mesh as follows.

$$\begin{aligned}
 V_s(t) &= L \frac{dI_L(t)}{dt} + \frac{1}{C} \int I_C(t) dt \\
 V_s(t) &= L \frac{dI_L(t)}{dt} + I_0(t)R \\
 V_s(t) &= L \frac{dI_L(t)}{dt} + (I_L(t) - I_C(t))R
 \end{aligned} \tag{14}$$

Laplace transform is applied to equation (14) to derive the transfer functions of  $I_L(s)$  and  $V_0(s)$ .

$$\begin{aligned}
 V_s(s) &= LSI_L(s) + (I_L(s) - I_C(s))R \\
 I_C(s) &= \frac{(LS + R)I_L(s) - V_s(s)}{R} \\
 V_s(s) &= LSI_L(s) + \frac{1}{CS} \left[ \frac{(LS + R)I_L(s) - V_s(s)}{R} \right] \\
 \frac{V_s(s)}{I_L(s)} &= \frac{RLCS^2 + LS + R}{1 + RCS} \\
 \frac{V_0(s)}{V_s(s)} &= \frac{R}{RLCS^2 + LS + R}
 \end{aligned} \tag{15}$$

Equation (15) is implemented in Simulink and shown in Fig. 8 and its output voltage is shown in Fig. 9.

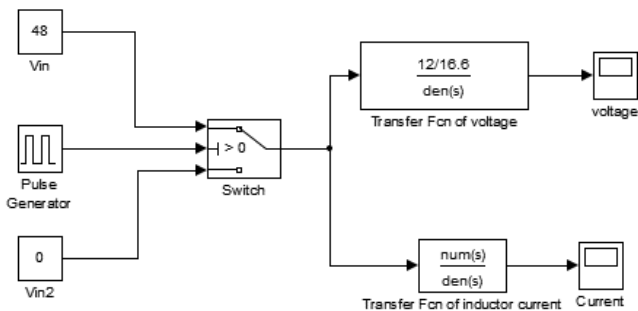


Figure 8: Transfer function model of buck converter

**B. State space average Model**

The state equations are written for the buck converter as follows.

$$\begin{aligned}
 \dot{X} &= AX + BU \\
 Y &= CX + DU
 \end{aligned} \tag{16}$$

Here, X is the state variable and Y is the output. A, B, C and D are state Matrices. From equation (11) and (12),

$$\begin{aligned}
 \dot{X} &= \begin{bmatrix} \frac{dI_L(t)}{dt} \\ \frac{dV_C(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_s \\
 Y &= [0 \quad 1] \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + 0
 \end{aligned} \tag{17}$$

Equation (17) is implemented in Simulink for both the switch ON and OFF conditions and shown in Fig.10. The output voltage is shown in Fig. 11.

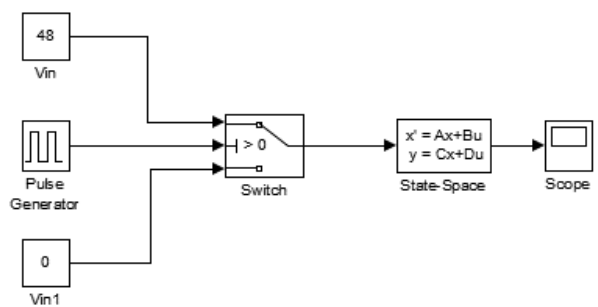


Figure 10: State space model of buck converter

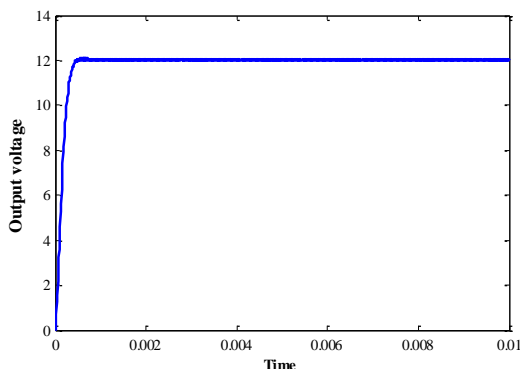


Figure 11: Output voltage

#### IV. CONCLUSION

The mathematical equations are derived and modified based on the models selected. The simulation is implemented in Simulink and the output voltages for each model are presented in this paper. The physical model looks very simple to construct but the result is not matched with the designed value but the other three models produce the designed output voltage. The transfer function model produces the output voltage with zero rise time than three models. The rise time to reach the steady state value is larger in case of mathematical model and this may be due to the delays created by each block used in the circuit. Hence, the mathematical models are more suitable in the design of controller.

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