International Journal of Scientific Research in Science and Technology
Print ISSN: 2395-6011 | Online ISSN: 2395-602X (www.ijsrst.com)
doi : https://doi.org/10.32628/IJSRST218477

Analysis of Ideas Changing in the History of Mathematical Analysis

Ming-Xing Hu*, De-Peng Kong
Department of Mathematics, Huaibei Normal University, Huaibei, Anhui, PRC, China

Article Info
Volume 8, Issue 4
Page Number : 505-510

Publication Issue
July-August-2021

Article History
Accepted : 06 Aug 2021
Published : 12 Aug 2021

ABSTRACT

Analysis is a branch of mathematics that deals with continuous change and with certain general types of processes that have emerged from the study of continuous change, such as limits, differentiation, and integration. In the history of mathematics, analysis is the first subject became epidemic, the development of analysis originated from the British mathematician and physicist, the Sir Isaac Newton, and the German mathematician, Gottfried Wilhelm Leibniz, who developed the theory of Calculus, with hundred-years developing, the modern analysis is now very ample and has widely applications, it has grown into an enormous and central field of mathematical research, with applications throughout the sciences and in areas such as finance, economics, and sociology. In this paper, we investigated in some detail with the changing of the ideas in mathematical analysis. By numerating historical facts and the mathematical ideas, we concluded the result that the ideas changing is because of the changing of the studying objects, the conclusion are studied detailly in the paper.

Keywords : Analysis, Mathematical History, Asymptotic Analysis, Representation

I. INTRODUCTION

CALCULUS & INFINITESIMAL ANALYSIS

In the middle-late periods of 17th century, due to the urgency of physics, it forced Newton to find a completely new method to solve the physics problems, for example, how to find the velocity $v(t_0)$ at the time t_0 of a motion $x = x(t): \mathbb{R} \longrightarrow \mathbb{R}^n$, and how to find the total displacement during the time $[t_0, t_1]$ via the velocity function $v(t)$, Newton then established the method of calculus [1], meanwhile, due to the study of some geometric problem, such as how to find the area covered by a curve, the German mathematician Leibniz establish the same method at the same time [2].

The main idea of the calculus is to use the "straight objects" to approximated the "curly objects" in an infinitesimal area, for example, to find the velocity $v(t_0)$ at the time $t = t_0$, it entails to consider in an infinitesimal neighbourhood of t_0, namely,

$[t_0,\ t_0+\Delta t]$, then we can consider the motion as a uniform motion, then by taking the limits

$$v(t_0)=\lim_{\Delta t\to 0}\frac{\|x(t_0+\Delta t)-x(t_0)\|}{\Delta t}$$

that is the velocity, which is also called the differentiation, and to compute the total displacement, it entails to cut the interval into infinitesimal pieces, then by regarding the motion as the uniform motion on each piece, then adding them together, that is desired result, which is also called the integration.
But there came out some natural questions, what is the "limit"? And does the result independent to the way of cutting?

The answer to these questions was not answered until 1821, French mathematician August Louis Cauchy, who introduced the $\epsilon-\delta$ language to study the analysis rigorously [2].
However, after establishing the calculus theory, it developed rapidly, especially during the 17th-18th century, due to the urgency in the study of physics, engineering, military, geometry, probability and statistics, calculus and their applications became an epidemic subject in scientific research, as for its application to the mathematics itself, it yields a method so called the "hard analysis".

In this period, people mainly matured in technical [3-4], they developed some technologies incisively and vividly, some of the most representative mathematicians such as J. Fourier [5], L. Euler, S. Poisson, K.F. Gauss, and also G.H. Hardy, J.E. Littlewood who appeared in the late era [6-7]. Some of the most outstanding works [7], such as the Poisson's kernel formula

$$\sum_{n=-\infty}^{\infty} r^{|n|}e^{inx}=\frac{1-r^2}{1-2r\cos x+r^2}$$

The Gauss' elliptic integration formula

$$\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}}=\frac{\pi}{AG(a,b)}$$

Since there are no rigorously definition of the limit, most of those works are working for computation [4].

II. REPRESENTATION & APPROXIMATION

The idea of representation is to use the objects which are familiar to us to represents the objects which are unfamiliar to us, as the objects people studied became more and more strange, people begun to find some connections between those strange stuffs and the well-known stuffs, the first outstanding discover should belong to the British mathematician Brook Taylor, who used the polynomial function to represent a smooth function

$$f(x)=\sum_{k=n}^{n}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k+o((x-x_0)^n)$$

That help us to reduce many other problems to the problems of polynomials.
Other applications can be found in the theory of complex analysis in one variable, it is a quiet useful tool to investigate the holomorphic functions, for example, the Cauchy's integral formula [8]

$$u(z_0)=\frac{1}{2\pi\sqrt{-1}}\left(\int_{\partial\Omega}\frac{u(z)dz}{z-z_0}+\int_{\Omega}\frac{\partial u/\partial\bar{z}}{z-z_0}dz\wedge d\bar{z}\right)$$

Where $z_0\in\Omega\subset\mathbb{C}$ is a domain, whose boundary $\partial\Omega$ consists of finitely many C^1 Jordan curves, and $u\in C^1(\Omega;\mathbb{C})$.
Another outstanding contributor to this idea is the French mathematician J. Fourier [9], he found the trigonometric representation of a function when he studied the heat conduction, the expansion is

$$f(x)\sim\sum_{n=-\infty}^{\infty}\hat{f}(n)e^{inx}$$

Where the coefficients are

$$\hat{f}(n)=\frac{1}{2\pi i}\int_{-\pi}^{\pi}f(x)e^{-inx}dx$$

Which are called the Fourier coefficients. This is an era-crossing discovery, because in nowadays viewpoint, the sequence $e_n = \{e^{inx}\}_{n=-\infty}^{\infty}$ forms a basis of the function space $L^2([-\pi,\pi])$ [10] [12], hence the properties of functions in that space can be reduced to the properties of base functions, for example, Weyl used this idea proved the Weyl's equi-distribution theorem.

Fourier then used this tool solved a lot of partial differential equations, such as the heat equation [9]

$$\frac{\sigma}{\kappa}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

In the late of 19th century, a branch of analysis which is so called the Harmonic Analysis born [10], the theory is based on the Fourier's discovery to study how to construct functions via the harmonic vibrations, with the developing of algebra, people found it attaches tightly with the representation theory (which is a branch of algebra) [13], for example, the famous Gel' fend representation theorem [12] is just an operator algebraic version of the Fourier's theory, in modern analysis there is a subject named "Abstract Harmonic Analysis" which studies the harmonic analysis on some topological groups [14], which offers great helps in number theory.

The idea of approximation is to use a family of objects with good properties to approximate an object with bad non-good properties, for example, the well-known Weierstrass approximation theorem states that any continuous function can be uniformly approximated by a polynomial or trigonometric polynomial. This idea will allow us to transfer the problems of a function into the problems of its approximation function, a canonical application is in the proof of the Fubini's theorem.

This idea, in the topological viewpoints, is called the "density" [11-12]. With the development of functional analysis, people learn to make all the objects a collection, and usually this collection is a vector space (of infinite dimension), then by adding some inner-product or norm structures, it becomes a metric space, if some family with good properties, that is a subset of that metric space, can approximate any member in that metric space in sense of the norms, this family, in the topological way, is dense in that space, for example [11], if f is a bounded function, then f can be approximated by a family of continuous functions $\{f_n\}$ in sense of L^1 norm

$$\int_E |f_n - f| \longrightarrow 0$$

III. ASYMPTOTIC

In the late 19th century, people begun to aware that one just need to compute in a limit process instead of the whole one, meanwhile, some objects were becoming incomputable, so people wanted to find some computable object as an asymptotic of an object in a limit process, this idea due to two German mathematician, Paul Gustav Bachmann and Edmund Landau, who introduced a so called $O-$symbol in 1894 to do the asymptotic analysis [15], this method was first used to study the analytic number theory, because a lot of functions in number theory are incomputable and the definitions are usually tautological, for example, the function $\tau(s) = \Sigma_p 1/p^s$, where p takes through all the primes, we don't know what it really looks like, but we can know what does it look like at infinity

$$\sum_p \frac{1}{p^s} = \log\zeta(s) + O(1)$$

Which implies $\tau(s)$ grows as $\log\zeta(s)$ at infinity differs by a bounded function.

It can also be used to study differential equations and iteration sequences, for instance, $\sin x_n = x_{n+1}$, we know from the fixed-point theorem the sequence $\{x_n\}$ tends to 0, but how it tends? By the asymptotic analysis we know that

$$x_n = \sqrt{\frac{3}{n}} - C\frac{\log n}{n\sqrt{n}} + O\left(\frac{1}{n^2}\right)$$

In all, it is a quite useful idea in analysis.

IV. MEASURES

In 1902, Dr. Lebesgue passed his doctorate dissertation in Nancy University, France, his dissertation *Integral, Length and Area* led the analysis into a new age, the measure theory [16].

It came out when people wanted to study the concept "almost", that entails one to think how to define "almost" rigorously, an intuitively way is to take the length or area, for example, when we are saying $f:\mathbf{R}\longrightarrow\mathbf{R}$ is almost 0, which is to say the length of set $\{x\in\mathbf{R}:f(x)\neq 0\}$ is 0, and that "length" is what we called the measure.

With the method of measures, it allows one to discuss analysis more detailly, for example [11], if we have

$$\int_E f = 0$$

Then one cannot deduce that $f=0$, because one can find a counterexample

$$f(x)=\begin{cases} 1 & x=0 \\ 0 & x\neq 0 \end{cases}$$

but one can deduce that $f=0$ almost everywhere. Another example is that we know Fourier series may not converge to the function itself, but it was in 1990, Carleson proved that it almost converges to function itself.

V. ABSTRACTION

With the changing of studying objects, mathematicians then wanted to study on some arbitrary domains beyond the classical domain $\mathbf{R}^n$, such as analysis on some manifolds, analysis on some topological groups, this entails mathematician to use idea of abstraction, to abstract the essential of the analysis from the $\mathbf{R}^n$ case, then generalize them to the usual case.

In the early of 20th century, two great mathematicians established the notion of abstract topological space, the one is the Polish mathematician Felix Hausdorff, the other one is the French mathematician Henri Poincaré, they found all things we need to study the limits is the definition of open sets [17], the open sets on $\mathbf{R}^n$ has some particular properties, that is the finite intersection and countably union of open sets are still open sets, moreover, $\varnothing$ and $\mathbf{R}^n$ are both open, generalize this notion on an arbitrary set X, if one can define a family of subsets $\mathcal{T}$ of X, then this family is called a topology of X, the space $(X,\mathcal{T})$ is called a topological space, it allows us to discuss limits on an arbitrary set with a topology.

Then people define the abstract measure which allows us to discuss integration on an abstract space, with the various space chosen, the corresponding analysis will applicant variously.

VI. IDEAS FROM ALGEBRA

The algebraic idea was first appeared in analysis in 1834, the distinguished French mathematician Joseph Liouville who first introduce the so called differential Galois' theory to prove some functions such as $1/\log x$, $\sin x/x$ has no elementary primitive functions [18], and he gave the classification of all elementary functions, motivated by Liouville, some mathematicians, for example Picard, begun to this method to investigate the differential equations, and they constructed a very useful branch which crosses analysis and algebra, namely Differential Algebra [19].

In the late 20th century till now, two Japanese mathematician Kashiwara and Sato, who developed a new branch which crosses analysis and algebra, that is the Microlocal Analysis [20], which is also called by their constructor, the algebraic analysis, it studies the differential equations algebraically.

VII. IDEAS FROM TOPOLOGY

In 1925, American mathematician Harold Morse first introduced the topological method in analysis, especially nonlinear analysis, which was the beginning of the global analysis [21].

The main aim of nonlinear analysis is to find the solution of nonlinear equations, the variation method is to transfer the solution into the critical points of functionals, then lifting its regularity, but this method is not detailly, they can only prove the existence of the solution but cannot distinguish the multiplicity, then Morse theory came to help, specifically, we observe some relative homology groups near a critical point, Morse theory tells us if relative homology group is nontrivial, then this critical point will change the topological structures of the horizontal set, and the dimension of the relative homology group relates to the Morse index of this critical points.

VIII. CONCLUSIONS

By numerating historical examples and mathematical ideas so far, we can observe that the ideas changing is the consequence of the changing of the studying objects, if one wants to investigate detailly, locally, such as the structures of some particular functions, equations, we need the hard analysis, that involves inequalities, asymptotic methods, representation methods, approximation methods, some formulae and some other specific technical, if one wants to investigate some abstract properties, such as to justify whether the solution of a differential equation elementary, that will entail some algebraically methods, if one wants to applicant the analysis on some other territory, one will need the abstraction methods, to generalize the analysis notions on the desired scope, if one concerns more about the global aspects of the analysis, one will need the topological idea to think globally.

Various branches of mathematics are developed rapidly nowadays, different ideas methods from other branches are coming to the study of the analysis, such as combinations, probability dynamic systems, and analysis, as an idea itself, is applicated in various territory, the future will as beautiful as the history, but what different is, it will be more excited.

IX. ACKNOWLEDGEMENTS

The authors would like to thank to Professor Cooper of New York University for supervising us in writing, and Dr. Kurosov of HSE for offering the books and papers we needed, and also thank to Mr. Lau, our friend who gave the necessary help, you will become a great mathematician someday.

X. REFERENCES

[1]. I. Newton, "Philosophiae Naturalis Principia Mathematica," Cambridge University Press, 1687, 1st ed. Cambridge, U.K

[2]. E. I. Gordon, A. G. Kusraev, Kutateladze S.S. (2002) Excursus into the History of Calculus. In: Infini-tesimal Analysis. Mathematics and Its Applications, vol 544. Springer, Dordrecht. https://doi.org/10.1007/978-94-017-0063-4_1

[3]. A. Stubhaug. (2000) Into Mathematical History. In: Niels Henrik Abel and his Times. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-662-04076-8_22

[4]. M. Otte. (2007) Mathematical history, philosophy and education. Educ Stud Math 66, 243–255. https://doi.org/10.1007/s10649-007-9079-z

[5]. J. B. J. Fourier. (1822) Théorie Analytique de la Chaleur, Didot, Paris, 499-508.

[6]. G. H. Hardy. (1908) Mendelian Proportions in a Mixed Population. Science, 28(706), 49-50.

[7]. G. H. Hardy, A Course of Pure Mathematics, Cambridge University Press, 2010, 10th ed. Cambridge, U.K

[8]. D. Huybrechts, Complex Geometry, Springer-Verlag, 2005, 1st ed.

[9]. J. B. J. Fourier. (1808) Mémoire sur la propagation de la Chaleur dans les corps solides, Nouveau Bulle-tin des Sciences par la Société Philomathique, tome 1, no. 6, pp. 112-116

[10] . E.M. Stein, R. Shakarich, Fourier Analysis: An Introduction, Princeton University Press, 2014, 2nd ed. New Jersy, U.S.A

[11] . E. M. Stein, R. Shakarich, Real Analysis, Princeton University Press, 2014, 2nd ed. New Jersy, U.S.A

[12] . E. M. Stein, R. Shakarich, Functional Analysis, Princeton University Press, 2014, 2nd ed. New Jersy, U.S.A

[13] . J. J Kulikowski, P. O. Bishop. (1981) Fourier analysis and spatial representation in the visual cortex. Experientia 37, 160–163. https://doi.org/10.1007/BF01963207

[14] . E. Hewitt, K. A. Ross, Abstract Harmonic Analysis, Springer-Verlag, 1979, 1st ed.

[15] . E. Landau. (1926) Die Bedeutungslosigkeit der Pfeiffer'schen Methode für die analytische Zahlentheo-rie. Monatsh. f. Mathematik und Physik 34, 1–36. https://doi.org/10.1007/BF01694887

[16] . H. Lebesgue. (1902) Intégrale, Longueur, Aire. Annali di Matematica, Serie III 7, 231–359. https://doi.org/10.1007/BF02420592

[17] . I. M. James, History of Topology, North-Holland, 1999, 1st ed.

[18] . J. Peiffer, (1983) Joseph Liouville (1809-1882): ses contributions à la théorie des fonctions d'une variable complexe, Rev. Hist. Sci. 36, 209-248.

[19] . J. F. Pommaret. (1991) Intrinsic differential algebra. In: Jacob G., Lamnabhi-Lagarrigue F. (eds) Alge-braic Computing in Control. Lecture Notes in Control and Information Sciences, vol 165. Springer, Berlin, Heidelberg. https://doi.org/10.1007/BFb0006943

[20] . Homan, Andrew J, "Applications of microlocal analysis to some hyperbolic inverse problems" (2015). Open Access Dissertations. 473. https://docs.lib.purdue.edu/open_access_dissertations/47

[21] . Floer, A. (1988). Morse theory for Lagrangian intersections. Journal of Differential Geometry, 28(3), 513-547, DOI: 10.4310/jdg/1214442477

Cite this article as :

Ming-Xing Hu, De-Peng Kong, "Analysis of Ideas Changing in the History of Mathematical Analysis ", International Journal of Scientific Research in Science and Technology (IJSRST), Online ISSN : 2395-602X, Print ISSN : 2395-6011, Volume 8 Issue 4, pp. 505-510, July-August 2021. Available at doi : https://doi.org/10.32628/IJSRST218477

Journal URL : https://ijsrst.com/IJSRST218477