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Generalization of Mittag-Leffler Function to Represent the Series $(1+x)^{-1}$ and Paper $(1-x)^{-1}$ this Series Converges for$|x|<1$

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ABSTRACT

In this paper we use the mittag leffler function of real variable x which is special case of mittag leffler function of complex variable zAnd we can add some factor to mittag leffler function of real variable And represents the two series in $(1+x)^{-1}$ and paper $(1-x)^{-1}$ this series converges for $|x|<1$,in this mittag leffler function which is generalization of mittag leffler function.i.e mittage leffler function is special case of this generalized function.

Keywords: Mittag-leffler function ,generalized mittag leffler function,mittage leffler function for real variables,

I. INTRODUCTION

Fractional derivative is as old as calculus. L'Hospital in 1695 asked what does it meanif $\frac{d^n f}{dx^n}$.if n=$\frac{1}{2}$ Since then, manyresearchers tried to put a definition of a fractional derivative. Most of them used an *integral form* for the fractional derivative.

Two of which are the most popular ones.

1) Riemann liouville definition. For $\alpha \in [n-1, n)$ the α derivative of f is

$$D_a{}^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}} dx$$

2) Coputo definition. For $\alpha \in [n-1, n)$ the α derivative of f is

$$D_a{}^{\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(x)}{(t-x)^{\alpha-n+1}} dx$$

II. METHODS AND MATERIAL

One parameter mittage leffler function

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)} \alpha > 0$$

Two parameter mittage leffler function

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \alpha > 0, \beta > 0$$

It follows from definition

$$E_{1,1}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k + 1)} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z$$

$$E_{1,2}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k + 1)} = \frac{e^z - 1}{z}$$

$$E_{1,3}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k + 1)} = \frac{e^z - 1 - z}{z^2}$$

In general

$$E_{1,m}(z) = \frac{1}{z^{m-1}} \left\{ e^z - \sum_{k=0}^{m-2} \frac{z^k}{k!} \right\}$$

$$E_{2,1}(z^2) = \sum_{k=0}^{\infty} \frac{z^{2k}}{\Gamma(2k + 1)} = Cosh(z)$$

$$E_{2,2}(z^2) = \sum_{k=0}^{\infty} \frac{z^{2k+1}}{\Gamma(2k + 2)} = \frac{\sinh(z)}{z}$$

The generalized

$$E_{(\alpha_1 \alpha_2 \ldots \alpha_m),\beta}(z_1 z_2 \ldots\ldots z_m) = \sum_{k=0}^{\infty} \sum_{l_1 + l_2 + \cdots \ldots l_m = k, l_1 > 0, l_2 > 0 \ldots\ldots l_m > 0} \frac{(k; l_1, l_2 \ldots\ldots l_m) \prod_{i=1}^{m} {z_i}^{l_i}}{\Gamma(\beta + \sum_{i=1}^{m} \alpha_i l_i)}$$

Where $(k; l_1, l_2 \ldots\ldots l_m)$ are multinomial coefficient.

III. RESULTS AND DISCUSSION

In this paper generalization of mittage leffler function is derive and acommoded two imp series in this paper $(1 + x)^{-1}$ and paper $(1 - x)^{-1}$ this series converges for $|x| < 1$,

So main aim is to accomoded this two series in the generalied mittage leffler function

The two series is given by

$$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \cdots \ldots\ldots\ldots (-1)^n x^n \ldots\ldots\ldots converges\ for\ \ldots |x| < 1 \ldots.$$

$$(1 - x)^{-1} = 1 + x + x^2 + x^3 + \cdots \ldots\ldots + x^n \ldots\ldots\ldots\ldots converges\ for |x| < 1$$

The mittage leffler function for real variable x is

$$E_{\alpha}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + 1)}, \alpha > 0$$

The generalized mittag leffler function.

$$\delta E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+1)} . (\Gamma(\alpha k+1))^\delta$$

Where $\delta \in N_1 = \{0,1\}$

Case I.when $\delta = 0$ then original mittage leffler function of one parameter of real variable x is obtained i.e

$$0E_\alpha(z) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+1)} . (\Gamma(\alpha k+1))^0$$

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k+1)}$$

CaseII:- When $\delta = 1$ then mittage leffler function of parameter family reduces to

$$1E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+1)} . (\Gamma(\alpha k+1))^1$$

$$1E_\alpha(z) = \sum_{k=0}^{\infty} z^k$$

Therefore

$$1E_\alpha(z) = \sum_{k=0}^{\infty} z^k = 1 + x + x^2 + x^3 + \cdots \dots \dots + x^n + \cdots \dots . = (1-x)^{-1}$$

In a similar manner,

$$1E_\alpha(-z) = \sum_{k=0}^{\infty} (-z)^k = 1 - x + x^2 - x^3 + \cdots \dots \dots + (-1)^n x^n + \cdots \dots . = (1-x)^{-1}$$

IV. CONCLUSION

One can also generalize mittage leffler function.

V. REFERENCES

[1]. K.S. Miller, An Introduction to Fractional Calculus and Fractional Differential Equations, J. Wiley and Sons, New York, 1993.

[2]. K. Oldham, J. Spanier, The Fractional Calculus, Theory and Applications of Differentiation and Integration of Arbitrary Order, Academic Press, USA, 1974.

[3]. A. Kilbas, H. Srivastava, J. Trujillo, Theory and Applications of Fractional Differential Equations, in: Math. Studies., North-Holland, New York, 2006.

[4]. I. Podlubny, Fractional Differential Equations, Academic Press, USA, 1999.