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Development of Singularities in Radiating Dyon Solution with Cosmological Constant in Higher Dimensional Space-Time

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ABSTRACT

We study here the development of singularities in radiating dyon solution with cosmological constant in higher dimensional space time and cosmic censorship violation in the gravitational collapse of radiating dyon solution in higher dimensional space-time. We study here the final outcome of the collapse does not depends on cosmological constant but it sensitively depends on electric and magnetic charge respectively. We expand the earlier work of gravitational collapse of radiating dyon solution in any arbitrary dimensions. Here is no restriction is adopted on the number of dimensions. This might be important that cosmological constant does not affect on the development of singularities.

Keywords: Gravitational constant, naked singularity, cosmological constant, Dyon.
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I. INTRODUCTION

The final outcome of gravitational collapse is one of the most important issue in the theory and astrophysical applications of advance day black hole physics and cosmic censorship conjecture(CCC)[1]. The CCC postulates states that any physically realistic gravitational collapse of a massive star must end generically into black hole i.e they can not observe by external observer. In recent years, a wide variety of complete gravitational collapse of massive star have been discovered where the dynamical evolution leads to a naked singularity in place of black hole[2]. The physical phenomenon in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final outcome of a massive star, when it collapses under its own gravity at the end of its life cycle, is one of the most important question in astronomy and astrophysics today. A sufficiently massive star of size more than 1.4 times the size of sun would undergo continuous gravitational collapse, on exhausting its nuclear fuel, without achieving an equilibrium state such as a neutron star or white dwarf.

If the occurrence of naked singularities, the final outcome of collapse were non-generic in some appropriately well-defined sense, then it will supported to the CCC. While one can imposes various sets of physically reasonable and regular conditions under which the collapse of massive particle is to be dynamically evolved to examine its final state, again the problem arises here is, the meaning or precise definition of such genericity necessarily not available in gravitation theory. Under such a circumstances, what's actually required is a detailed investigation of the gravitational collapse phenomena within framework of general relativity, from which the only path can provide useful and adequate insight into the final outcome of collapse in terms of black hole or naked singularities.

Over the last forty years, there has been considerable interest in the formation of black hole, naked singularities and cosmic censorship hypothesis(CCH)[2]. Generally naked singularities are singularities that may be observed far away by observers. CCH state that for physically reasonable initial data, the gravitational collapse of space-time can not yield a naked singularity i.e. if singularity is form which is hidden inside the event horizon of the gravity. Now a days this hypothesis has become the most challenging open problem in theory of relativity. A rigorous formation and proof for CCH is not available so far, therefore some examples showing the occurrence of naked singularities remain important for this hypothesis for the formulation. Some important cases of naked singularities studied so far include dust collapse[3-8], radiation collapse[9-16], collapse of perfect fluid[17-20] and strange quark matter[21-22].

In this paper we study the development of naked singularities i.e.cosmic censorship hypothesis violation and effect of cosmological constant on gravitational collapse of radiating dyon solution in higher dimensional space-time. We conclude the paper in section IV by some concluding remarks.

II. GRAVITATIONAL CONSTANT WITH RADIATING DYON SOLUTION IN (N+2)-DIMENSIONAL SPACETIME

The metric in (n+2)-dimensional radiating Dyon space- time can be written as [23]

$$ds^2 = -\left[1 - \frac{2m(u,r)}{(n-1)r^{n-1}}\right] du^2 + 2dudr + r^2 d\omega_n^2 \tag{1}$$

Where u is advanced Eddington time co-ordinate, r is the radial co-ordinate with $0 < r < \infty$ and $m(u,r)$ gives the gravitational mass which is arbitrary inside the sphere of radius r.

$$2m(u,r) = \left[\lambda(n-1)u^{n-1} - \frac{q_e^2(u)+q_m^2(u)}{nr^{n-1}}\right] \tag{2}$$

and $d\omega_n^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2 + \cdots + sin^2\theta_1 sin^2\theta_2 \cdots sin^2\theta_{n-1} d\theta_n^2$ (3) is the line element in a n-sphere in polar co-ordinate and $n = D - 2$, where D denotes the total number of dimensions. Here, q_e^2 and q_m^2 are electric and magnetic charge respectively.

The non-vanishing components of the Einstein tensor are given by

$$G_{00} = \frac{n\dot{m}}{(n-1)r^n} - \frac{nm'}{(n-1)r^n}\left[1 - \frac{2m}{(n-1)r^{n-1}}\right], \quad G_{01} = -\frac{nm'}{(n-1)r^n}, \quad G_{22} = \frac{m"}{(n-1)r^{n-3}},$$
$$G_2^2 = G_3^3 = \cdots\cdots = G_{n+1}^{n+1} \tag{4}$$

Here dash and dot denote derivative with respect to r and u respectively.

The energy momentum tensor for type II fluid is given by [13-15].

$$T_{ik} = \mu l_i l_k + (\rho + p)(l_i \eta_k + l_k \eta_i) - p g_{ik} \tag{5}$$

Where,

$$l_i l^i = \eta_i \eta^i = 0, \quad l_i \eta^i = 1 \tag{7}$$

The null vector l_i is twice null eigen vector of T_{ik}. Physically occurring distribution is null radiation flowing in the radial direction corresponding to $\rho = p = 0$, the Vaidya space-time of radiating star. When $\mu = 0$, T_{ik} reduces to degenerate type I fluid and further it represents string dust for $\mu = 0 = p$. The energy condition for such a distribution are as follows [16,17].

(a) Weak and strong energy condition

$$\mu > 0, \quad \rho > 0, \quad p \geq 0 \tag{8}$$

(b) Dominant energy condition

$$\mu > 0, \quad \rho \geq 0, \quad p \geq 0 \tag{9}$$

In the case of $\mu = 0$, the energy conditions would be,

(c) Weak condition

$$\rho + p \geq 0, \quad \rho \geq 0 \tag{10}$$

(d) Strong condition

$$\rho + p \geq 0, \quad p \geq 0 \tag{11}$$

(e) Dominant condition

$$\rho \geq 0, \quad -\rho \leq p \leq \rho \tag{11}$$

The energy momentum tensor (5) has support along both the two future pointing null vectors l_i and η_i , and it is exactly, as we shall show later, in the form to give Bonnor-Vaidya metric in higher dimensions [18-22]. We also note that $T_{ik}l^il^k = 0$ and $T_{ik}\eta^i\eta^k = \mu$.

For the metric (1) can be written as,

$$l_i = g_i^0, \quad \eta_i = g_i^1 + \frac{1}{2}\left(1 - \frac{2m}{(n-1)r^{n-1}}\right) g_i^0 , \tag{12}$$

Now the Einstein field equations

$$G_{ik} = -T_{ik} \tag{13}$$

Equation (5) satisfies the condition (11)

Substituting (5) in (13) we obtain,

$$\sigma = \mu = -\frac{n\dot{m}}{(n-1)r^n}, \quad \rho = \frac{nm'}{(n-1)r^n}, \quad p = -\frac{m''}{(n-1)r^{n-1}} \tag{14}$$

Let us introduce the term with cosmological constant in the above mass function. Hence the mass function $m(u,r)$ can be written as

$$2m(u,r) = \left[\frac{\Lambda r^{n-1}}{3} + \lambda(n-1)u^{n-1} - \frac{\left(q_e^2(u)+q_m^2(u)\right)}{nr^{n-1}}\right] \tag{15}$$

Let us consider the electric charge parameter $q_e^2(u) = \alpha^2 u^{2n-2}$

and magnetic charge parameter $q_m^2(u) = \beta^2 u^{2n-2}$ (16)

Using the equation (15) and (16) equation (1) becomes,

$$ds^2 = -\left[1 - \frac{\Lambda r^{n-1}}{3(n-1)r^{n-1}} - \frac{\lambda u^{n-1}}{r^{n-1}} + \frac{\alpha^2 u^{2n-2}}{n(n-1)r^{2n-2}} + \frac{\beta^2 u^{2n-2}}{n(n-1)r^{2n-2}}\right] du^2 + 2dudr + r^2 d\omega_n^2$$

i.e $$ds^2 = -\left[1 - \frac{\Lambda}{3(n-1)} - \frac{\lambda u^{n-1}}{r^{n-1}} + \frac{\alpha^2 u^{2n-2}}{n(n-1)r^{2n-2}} + \frac{\beta^2 u^{2n-2}}{n(n-1)r^{2n-2}}\right] du^2 + 2dudr + r^2 d\omega_n^2 \tag{17}$$

III. NATURE OF THE SINGULARITY

To identify the structure of the singularity either it may black hole or naked, we follow the method given in reference [23] i.e. $ds^2 = 0$. The singularity is said to be naked, then the radial null geodesic equation admits at least one real and positive root.

The outgoing radial null geodesic equation for metric (17) is given by

$$\frac{dr}{du} = \frac{1}{2}\left[1 - \frac{\Lambda}{3(n-1)} - \frac{\lambda u^{n-1}}{r^{n-1}} + \frac{\alpha^2 u^{2n-2}}{n(n-1)r^{2n-2}} + \frac{\beta^2 u^{2n-2}}{n(n-1)r^{2n-2}}\right] \tag{18}$$

It is note that the equation (18) has a singularity at $r \to 0, u \to 0$ In order to differentiate the radial and non-radial outgoing non-space like geodesics ending at the singularity in the past, we consider limiting value of $X = \frac{u}{r}$ along a singular geodesic at the singularity is approached [24-25].

Let us consider $X_0 = \lim_{\substack{u\to 0\\ r\to 0}} X = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r} = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr}$

Therefore $X_0 = \frac{2}{\left[1 - \frac{a}{(n-1)} - \frac{\lambda u^{n-1}}{r^{n-1}} + \frac{\delta u^{2n-2}}{n(n-1)r^{2n-2}} + \frac{\Upsilon u^{2n-2}}{n(n-1)r^{2n-2}}\right]}$ (19)

i.e. $\frac{(\alpha^2+\beta^2)}{n(n-1)} X_0^{2n-1} - \lambda X_0^n + (1 - \frac{\Lambda}{3(n-1)}) X_0 - 2 = 0$ (20)

The above equation gives the nature of the singularity. If this equation has at least one real and positive root, then the final outcome of the collapse will be naked. If this will not happen then the collapse ends into black hole i.e observer can see the singularity.

For this we study the equation (20) , now considering some different values of $n, \alpha^2, \beta^2, \lambda$ & Λ.

Case I: Consider $n = 3$, then the equation (20) reduces to five dimensional space-time. In particular we select $n = 3, \alpha^2 = 0.05, \beta^2 = 0.01, \lambda = 0.01$ *and* $\Lambda = 0.5$ then equation (20) transform to

$$\frac{(0.05+0.01)}{6} X_0^5 - (0.01)X_0^3 + \left(1 - \frac{0.1}{6}\right) X_0 - 2 = 0 \quad (21)$$

then one of the real and positive root of equation (21) is $X_0 = 1.9531$, which shows that naked singularity is obtained.

If we varies the value of β^2, λ and fixed $\alpha^2 = 0.003, \Lambda = 0.7$ to get different roots in tabular form as

Table 1 Values of X_0 for different values of λ *and* β^2

λ	X_0		
	$\beta^2 = 0.01$	$\beta^2 = 0.05$	$\beta^2 = 0.09$
0.1	3.3427	2.5315	2.2491
0.3	6.5561	4.5717	3.7080
0.5	8.5835	6.0301	4.9000
0.7	10.1985	7.1848	5.8488
0.9	11.5847	8.1721	6.6590

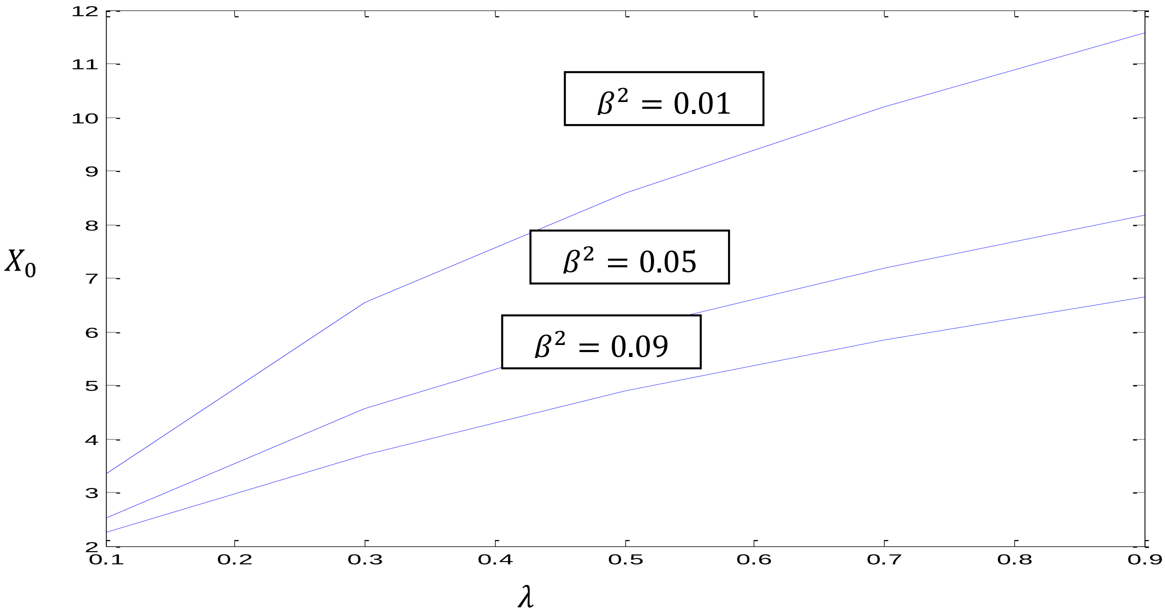

Figure 1: Graph of the values of X_0 against the value of λ.

From the above graph we may observe that the values of X_0 have positive real roots. It is also observe that if we increase the value of λ then X_0 also increases for particular charge simultaneously when we increase β^2 then X_0 decreases with positive roots, if for large values of β^2 then X_0 may be negative.

For $n = 4$ (i.e. for 6D) and fixed $\alpha^2 = 0.001, \lambda = 0.2$ then equation (20) can be written as

$$\frac{(0.001+\beta^2)}{12}X_0^7 - 0.2X_0^4 + \left(1 - \frac{\Lambda}{9}\right)X_0 - 2 = 0 \quad (22)$$

from this we will get real and positive roots for different values of β^2 and Λ i.e. to get naked singularities in six dimensional radiating dyon solution with cosmological constant are shown in following table.

Table 2 Values of X_0 for different values of Λ *and* β^2

Λ	X_0		
	$\beta^2 = 0.03$	$\beta^2 = 0.05$	$\beta^2 = 0.07$
0.1	4.2126	3.5522	3.1716
0.4	4.2159	3.5570	3.1776
0.6	4.2181	3.5601	3.1815
0.8	4.2203	3.5631	3.1854
1.2	4.2246	3.5693	3.1931

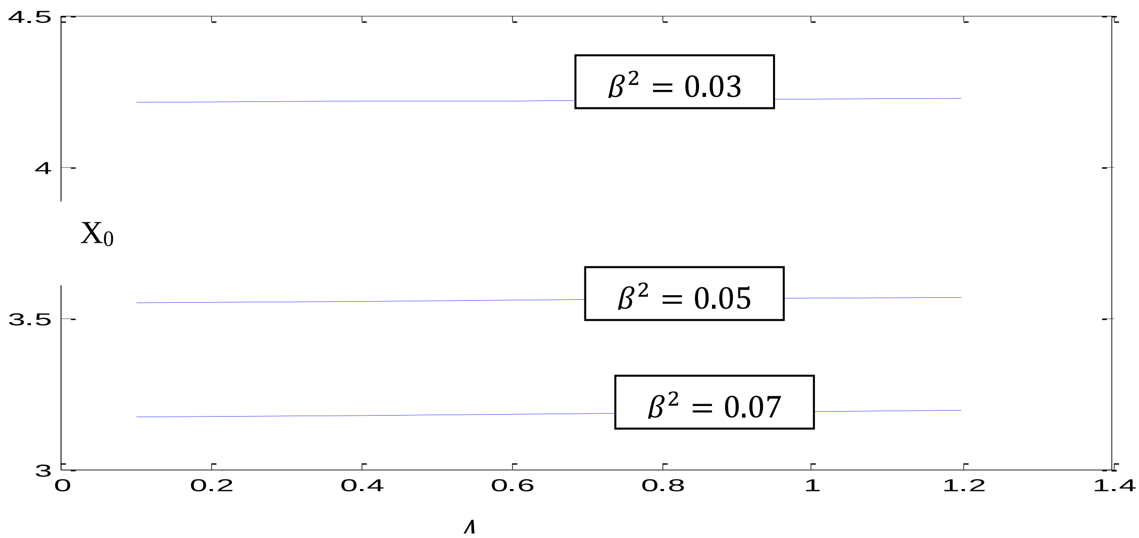

Figure 2: Graph of the values of X_0 against the value of Λ.

From the above graph when we change the values of β^2 and Λ that there is slight change in the values of X_0, but the roots are positive approximately same real values. It insures that cosmological constant in higher dimension doesn't affects on the structure of singularities.

IV. CONCLUDING REMARKS

Cosmic censorship conjecture has become most challenging and a significant open problem in general relativity. In the present work we have analyzed the development of singularities in radiating dyon solution with cosmological constant in higher dimensional space-time. We used here electric and magnetic charge parameter in mass function, It has been clear that the structure of the singularities arise in this space-time are not hidden inside the event horizon. It means that cosmological constant does not affects the development of singularities in higher dimensional space time.

We may also argue that the dimensions of the space-time with cosmological constant does not play a fundamental role in the formation of naked singularities. Occurrence of naked singularities in higher dimensional in the presence of cosmological constant and charge parameter the cosmic censorship conjecture would be violated so that singularity can be observed by an external observer. Thus we may suggest that the introduction of cosmological constant to gravity can't prevent to vanishing the apparent horizon i.e. naked singularity for forming completely. It reflects that cosmic censorship conjecture is actually violets.

V. REFERENCES

[1] . J.R Oppenheimer and H.Snyder, Phys. Rev. 56, 455 (1939); S. Datt, Zs. f. Phys. 108, 314 (1938).
[2] . R. Penrose, Riv, Nuovo. Cimento. 1, 252 (1969)
[3] . D. M. Eardley and L. Smar, Phys. Rev. D 19, 2239 (1979).
[4] . D. Christodoulou, Commun. Math. Phys., 93, 171 (1984).
[5] . R. P. A. C. Newman, Class. Quantum. Grav., 3, 527 (1986).
[6] . P. S. Joshi and I. H. Dwivedi Phys. Rev. D 47, 5537 (1993).
[7] . K. D. Patil, Phys. Rev. D 67, 024017 (2003).
[8] . K. D. Patil, Int. J. Mod. Phys. D 15(2), 251 (2006).
[9] . W. A. Hiscok, L. G. Williams and D. M. Eardley, Phys. Rev.D 26, 751 (1982).
[10] . K. Rajagopal and K. Lake Phys. Rev.D 35, 1531 (1987).
[11] . I. H. Dwivedi and P. S. Joshi, Class. Quantum. Grav. 6, 1599 (1989).
[12] . K. Lake and T. Zannias, Phys. Rev. D 43, 1798 (1991).
[13] . J. P. S. Lemos, Phys. Rev. D 59, 044020 (1999).
[14] . A. Papapetrou, in A Random Walk in Relativity and Gravitation, eds. N. Dad-hich, J. K. Rao, J. V. Narlikar and C. V. Vishveshwara (Wiley, New York, 1985) pp 184-189.
[15] . K. D. Patil and S. S. Zade, Int. J. Mod. Phys. D 15, 1359(2006).
[16] . K. D. Patil and U. S. Thool, Int. J. Mod. Phys. D 15(11), 1977 (2006).
[17] . A. Ori and T. Piran, Phys. Rev. Lett. 59, 2137 (1987).
[18] . A. Ori and T. Piran, Phys. Rev. D 42, 1068 (1990).
[19] . T. Harada and H. Maeda, Phys. Rev. D 63, 0804022 (2001).
[20] . H. Iguchi, T. Harada and Filipe C. Mena, Class. Quantum Grav. 22, 841-854 (2005).
[21] . T. Harko and K. S. Cheng, Phys. Lett. A 266, 249 (2000).
[22] . S. G. Ghosh and N. Dadhich, Gen. Relativ. Gravit. 35, 359 (2003).
[23] . L.K.Patel and Dadhich Naresh, arXiv:grqc/9909068v1, Sept 21 (1999).
[24] . P. S. Joshi , Clarendon, Oxford,1993.
[25] . S. G. Ghosh and N. Dadhich, arXiv: gr-qc/0211019 v 1, Nov 6 (2002).