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Temperature and Thermal Stress Problem of Hollow Cylinder on a Certain Steady-State

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ABSTRACT

In this paper, an attempt has been made to solve the problem of thermo elasticity and determine the unknown temperature, displacement and stress components. In this problem the zero temperature is maintained on the lower surface and third kind boundary condition is maintained on lower and upper surface. The governing heat conduction has been solved by using finite Hankel transform technique unknown temperature found on the lower surface of the hollow cylinder. The results are obtained in series form in terms of Bessel's functions and have been computed numerically and illustrated graphically.

Key words: Steady- state, Thermoelastic problem, Hankel transform, Hollow cylinder.

I. INTRODUCTION

In 1967, Lord and Shulman [7] introduced the theory of generalized thermoelasticity with one relaxation time for an isotropic body. This theory corrects the unrealistic conclusions of the older theories (the uncoupled and the coupled theories of thermoelasticity) that heat waves travel with infinite speeds. In 1972 Green and Lindsay [1] developed the theory of generalized thermoelasticity with two relaxation times, based on a generalized inequality of thermodynamics. In this theory both the equations of motion and of heat conduction are hyperbolic. The heat conduction law is the same as Fourier's law when the system has a centre of symmetry. Among the contributions to this theory are the works in [8, 9].

In view of some experimental evidence available in favour of finiteness of heat propagation speed, generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes at short intervals, like those occurring in laser units and energy channels [2].

Two dimensional transient problems for a thick annular disc in thermoelasticity studied by (Dange et al., 2009). An inverse temperature field of theory of thermal stresses investigated by (Grysaet al; 1981) while A note of quasi –static thermal stresses in steady state thick annular disc and an inverse quasti-static thermal stresses in thick annular disc are studied by (Gaikwad et al; 201s0).

In this paper, in the first problem, an attempt is made to determine the unknown temperature, displacement and stress functions on curved surfaces, where an arbitrary heat is applied on the lower surface (z = -h) and

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maintained zero on upper surface (z = h) . The governing heat conduction equation has been solved by using Hankel transform technique. The results are obtained in series form in terms of Bessel's functions and illustrated graphically.

This paper contains a new and novel contribution of thermal stresses in an annular disc under steady state. The above results were obtained under a steady state field. The results presented here are useful in engineering problems particularly in the determination of the state of strain in an annular disc constituting foundations of containers for hard gases or liquids, in the foundations for furnaces etc.

II. STATEMENT OF PROBLEM

Consider an annular disc of thickness 2h occupying the space$D: a \leq r \leq b, -h \leq z \leq h.$

The thermoelastic displacement function as in (Nowacki; 1962) is governed by poison's equation

$\nabla^2 U = (1+\nu)a_t T$ (2.1)

with$U_r = 0$ at $r = a$ and$r = b$ (2.2)

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$$

νanda_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the disc and T is the temperature of the disc satisfying the differential equation

$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0$ (2.3)

subject to the boundary conditions

$T(r,z) = 0$ at$r = a, -h \leq z \leq h$ (2.4)

$T(r,z) = 0$ at$r = b, -h \leq z \leq h$ (2.5)

$\frac{\partial T}{\partial z} - k_1 T = g(r),$ at$z = -h, a \leq r \leq b$ (2.6)

$\frac{\partial T}{\partial z} + k_2 T = 0,$ at$z = h, a \leq r \leq b$ (2.7)

where k_1 and k_2 are the radiation constants on the two plane surfaces.

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by,

$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r}$ (2.8)

$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2}$ (2.9)

whereμisthe Lame's constant, while each of the stress functions σ_{rz} ,σ_{zz} and $\sigma_{\theta z}$ are zero within the disc in the plane state of stress.

The equations (2.1) to (2.9) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF PROBLEM

On applying the finite Hankel transform defined in (Sneddon; 1972) to Eq. (2.3), one obtain

$$\frac{d^2 \overline{T}}{dz^2} - \xi_n^2 \overline{T} = 0 \quad (3.1)$$

where $\overline{T}$ is the Hankel transform of T.

On applying Eq. (3.1) under the conditions given in Eq. (2.6) and Eq. (2.7), one obtains

$$\overline{T} = \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left[\frac{\xi_n \cosh\left[\xi_n(z+h)\right] + k_2 \sinh\left[\xi_n(z+h)\right]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)} \right] \tag{3.2}$$

On applying the inverse Hankel transform to Eq. (3.2), one obtain the expression for the temperature as

$$T(r,z) = \sum_{n=1}^{\infty} \underline{f}(\xi_n)[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)] \times \left[\frac{\xi_n cosh[\xi_n(z+h)] - k_2 sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2) sinh(2\xi_n h) + \xi_n(k_1+k_2) cosh(2\xi_n h)} \right] \tag{3.3}$$

where $\underline{f}(\xi_n) = \int_a^b f(r) r [J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)]$ (3.4)

Equation (3.3) is the desired solution of the given problem.

DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value T(r, z) from Eq. (3.3) in Eq. (2.1) one obtains the thermoelastic displacement function U(r, z) as,

$$U(r,z) = -(1+\nu)a_t \sum_{n=1}^{\infty} \left(\frac{\underline{f}(\xi_n)}{\xi_n^2}\right) [J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)] \times \left[\frac{\xi_n cosh[\xi_n(z+h)] - k_2 sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2) sinh(2\xi_n h) + \xi_n(k_1+k_2) cosh(2\xi_n h)} \right] \tag{3.5}$$

DETERMINATION OF STRESSES

Using Eq. (3.5) in Eq. (2.8) and Eq. (2.9), one obtains the stress functions σ_{rr} and $\sigma_{\theta\theta}$ as,

$$\sigma_{rr} = -\frac{2\mu}{r}(1+\nu)a_t \sum_{n=1}^{\infty} \left(\frac{\underline{f}(\xi_n)}{\xi_n}\right) [J_1(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_1(r\xi_n)] \times \left[\frac{\xi_n cosh[\xi_n(z+h)] - k_2 sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2) sinh(2\xi_n h) + \xi_n(k_1+k_2) cosh(2\xi_n h)} \right] \tag{3.6}$$

$$\sigma_{\theta\theta} = -2\mu(1+\nu)a_t \sum_{n=1}^{\infty} \underline{f}(\xi_n)[J_1'(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_1'(r\xi_n)] \times \left[\frac{\xi_n cosh[\xi_n(z+h)] - sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2) sinh(2\xi_n h) + \xi_n(k_1+k_2) cosh(2\xi_n h)} \right] \tag{3.7}$$

SPECIAL CASE AND NUMERICAL RESULTS

Set

$f(r) = (r+a)(r+b)e^h$, $\alpha = (1-a-b)$ in (3.3) one obtains

$$\begin{aligned} \frac{T(r,z)}{\alpha} = \sum_{n=1}^{\infty} \; & e^h\big(G_0(b\xi_n) - J_0(b\xi_n)\big)\{\frac{b^2}{\xi_n^2}\left[2J_0(b\xi_n) + \left(b\xi_n - \frac{4}{b\xi_n}\right)J_1(b\xi_n)\right] \\ & - \frac{a^2}{\xi_n^2}\left[2J_0(a\xi_n) - \left(a\xi_n - \frac{4}{b\xi_n}\right)J_1(a\xi_n)\right] + \frac{ab}{(1-a-b)}[bJ_1(b\xi_n) \\ & - aJ_1(a\xi_n)]\}[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)] \\ & \times \left[\frac{\xi_n cosh[\xi_n(z+h)] - k_2 sinh[\xi_n(z+h)]}{(\xi_n^2 + k_1 k_2) sinh(2\xi_n h) + \xi_n(k_1+k_2) cosh(2\xi_n h)} \right] \end{aligned} \tag{3.8}$$

The numerical calculation have been carried out for steel (SN 50 C) plate with parameters$a = 1m$,$b = 2m$,$h = 0.5m$.thermal diffusivity$k = 15.9 \times 10^{-6}(m^2 s^{-1})$ and poisons ratio $\nu = 0.281$, while $\xi_1 = 5.95$,$\xi_2 = 7.23$,$\xi_3 = 9.45$, $\xi_4 = 11.52$, $\xi_5 = 13.79$ being the positive roots of transcendental equation $[J_0(r\xi_n)G_0(b\xi_n) - J_0(b\xi_n)G_0(r\xi_n)] = 0$ as in (Ozisik;1968).

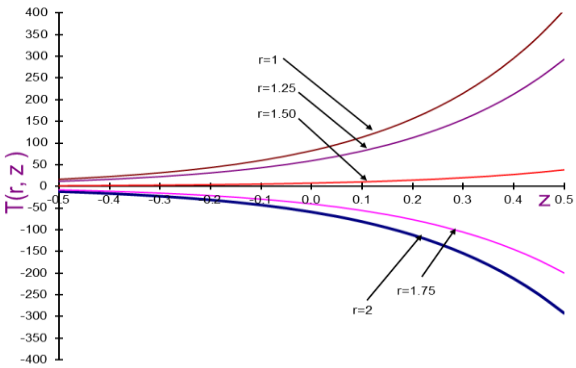
IV. DISCUSSION

In this paper, equation (3.8) have been calculated and shown graphically by using MATLAB, and the conclusion is as under:

Initially the temperature of the Hollow cylinder has been determined by using the conditions given in both the problems and applying finite Hankel transform technique and its inverse.

Thus the value of stress function of the material of annular disc is found using temperature T, linear coefficients of the thermal expansiona_t, and Poisson's ratio of the material. Finally, the displacement component has been arrived at using the stress function; and lastly, the stress component in terms of U has been found. Now, thermal diffusivity and thermal conductivity are two important thermal properties that enter the differential equation of heat conduction. Therefore accuracy of the value chosen for these properties affects the accuracy of the results in heat conduction problems.

In first problem of a certain steady-state problems of temperature and thermal stresses of an hollow cylinder , the condition that has been given kept at zero on the curved surfaces and third kind boundary condition is maintained on the lower surfaces and while temperature of the cylinder has been kept at zero on the upper boundary surface.



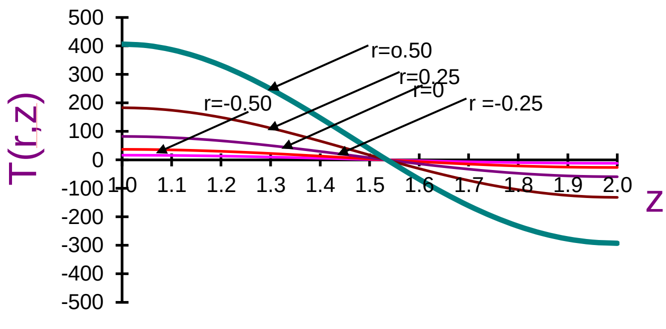


Fig1. The temperature distribution T(r, z)in axial directionFig2. The temperature distribution T(r, z)in axial direction

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