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Rund Like Recurrent Connection

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ABSTRACT

In this paper we will find the relation for curvature tensor of Rund Like Recurrent Connection.

Keywords: Rund like Recurrent Connection, Curvature tensor.

1. INTRODUCTION

Let M^n be an n-dimensional differentiable Manifold. (M^n, a_{ij} (x)) be Riemannian Space equipped with fundamentals metric tensor. From a Physical view poin S. Ikeda. Introduced line element space of M^n with a metric tensor $g_{ij}(x,y) = e^{2\rho}\, a_{ij}$ (x), ($\rho = \rho$ (y), $y^i = dx^i$) It is however easily that this form of $g_{ij}(x,y)$ depends on a Particular choice of Co-ordinate (x^i). The theories of connection on Finsler Space have been studied by many author from their own stand point, A well known connection are the Berwald Connection the Cartan Connections, Rund Connection, in all these connections the defluxion tensor and torsion tensor vanish. Prasad etal [1], [2], [3] have introduced a Finsler Connection with respect to which metric tensor is h-recurrent or v-recurrent.

(1.1) $\quad g_{ij|k} = a_k g_{ij}$ (h-recurrent)

(1.2) $\quad g_{ij\,k} = b_k g_{ij}$ (v -recurrent)

Several curvature tensor have been defined and studied in Finsler Space with help of different Finsler connections. The Rund's Curvature tensor is K^i_{jhk} defined.

(1.3) $\quad K^i_{jhk} = \theta\,(h,k)\,(\frac{\delta\, \Gamma^{*i}_{jh}}{\delta x^k} + \Gamma^{*m}_{jh}\quad \Gamma^{*i}_{mk})$

Where $\theta\,(h,k)$ throughout denote the interchange of indices h, k and subtraction. In this paper we will study Rund like Recurrent Finsler connection and their curvature tensor.

2. RUND LIKE RECURRENT CONNECTION :-

From the Cartan like recurrent Connection $RR\Gamma = (F^i_{jk}, N^i_k, C^i_{jk})$ we drive a Rund like recurrent Connection.

$RR\Gamma = (\dot{F}^i_{jk} = F^i_{jk,}\, \dot{N}^i_{k}, = N^i_k\; \dot{C}^i_{jk} = 0)$ by c-Process.

The hand v covariant derivatives with respect to $RR\Gamma$ will be denoted by (I) and (II) respectively. Ricci identities with respect to $RR\Gamma$ written as -

(2.1) (a) $x^h_{|j||k} - x^i_{|k|j} = x^r\, \dot{R}^h_{rjk} - x^h_{||r}\, \dot{R}^r_{jk}$

(b) $x^h_{||j||k} - x^h_{||k||j} = x^r\, \dot{P}^h_{rjk} - x^h_{||r}\, \dot{P}^r_{jk}$

(c) $x^h_{||j||k} - x^h_{||k||j} = 0$

Similar to case of $RC\Gamma$ Ricci identities with respect to $RR\Gamma$ given by –

PROPOSITION 2 :-

(2.2) (a) $\dot{R}^i_{ojk} = \left(1 - \frac{1}{2}\ bo\right) \dot{R}^i_{jk}$

(b) $\dot{P}^i_{jk} = \left(1 - \frac{1}{2}\ bo\right) \dot{P}^i_{jk}$

(c) $\dot{R}_{ijhk} + \dot{R}_{jihk} = g_{ij}[\ (a_{k|h} - a_{h|k}) - 2\rho_{|r}\ \dot{P}^r_{hk}]$

(d) $\dot{P}_{ijhk} + \dot{P}_{jihk} = g_{ij}\ \left[(-a_{h||k} - b_{k|h}\right) - 2\rho_{||r}\ P^r_{hk}]$

3. CURVATURE TENSORS :-

We continue to consider generalized metric space $\overline{M^n}$ with a conformably Riemannian metric. In this section we will find the expression of curvature tensors $\dot{P}^h_{ijhk}, \dot{R}^h_{ijk}$ of $RR\Gamma$ under assumption n ≥3

We deal with the Rund like recurrent connection

$RR\Gamma(\dot{P}^i_{jk} = P^i_{jk}, \dot{N}^i_k = N^I_K, \dot{c}^i_{jk} = 0)$ Differentiating

(3.1) (a) $\Gamma^h_{ik} + \delta^h_i\ \rho_{ik} + \delta^h_k\ \rho_{|l} - g_{ik}\ \rho^h_{|} - F^h_{ik} = \frac{1}{2}(a_k\ \delta^h_i + a_i\ \delta^h_k - a^h\ g_{ki})$

by y^l and paying attention to

$(g_{ik}\ g^{hr})_{||l} = (a_{ik}\ a^{hr})_{||l} = 0$ we get

(3.2) $\dot{P}^h_{ikl} = (\dot{\partial}_l\ F^h_{ik}) = (\rho_{|k||l} - \frac{1}{2} a_{k||l})\ \delta^h_i$

$+ (\rho_{|i||l} - \frac{1}{2} a_{i||l})\ \delta^h_k - g_{ik}\ g^{hr}\ (\rho_{|r||l} - \frac{1}{2} a_{r||l})$

Which gives after contraction with hand K

(3.3) $\dot{P}_{il} = \dot{P}^h_{ihl} = (\rho_{|r||l} - \frac{1}{2} a_{il})\ n$

So equation (3.2) becomes –

(3.4) $\dot{P}^h_{ikl}$ - $(\delta^h_i \dot{P}_{kl} + \delta^h_k\ \dot{P}_{il} - g_{ik}\ g^{hr}\ \dot{P}_{rl})$/n = 0

In order to consider h-curvature tensor $\dot{R}^h_{ikl}$ of $RR\Gamma$ we shall find relation between tensor $\overbrace{R^h_{ikl}}^{(r)}$ and $\dot{R}^h_{ikl}$. Now differentiating (3.1) with respect to x^l we get

(3.5) $\partial_l\ \overbrace{\Gamma^h_{ik}}^{(r)} = \delta_l\ F^h_{ik} - \delta^h_i\ (A_{k|l} + A_r\ F^r_{kl}) - \delta^h_k\ (A_{i|l} + A_r\ F^r_{il})$

$+ g_{ik}(A^h_{|l} - A^r\ F^h_{rl}) + A^h(a_l\ g_{ik} + F_{ikl} + F_{ilk}) = 0$

Where $A_k = \frac{1}{2} a_k - \rho_{|k}$

(3.6) $\partial_l\ \overbrace{\Gamma^h_{ik}}^{(r)} - \delta_k\ \overbrace{\Gamma^h_{il}}^{(r)} = (\delta_l\ F^h_{ik} - \delta_k\ F^h_{il}) + \delta^h_i\ (A_{|l} - A_{l|k})$

$+ \delta^h_k\ A_{|l} - \delta^h_e\ (A_{i|k} - A_r\ F^r_{ik}) - g_{ik}\ (A^h_{|l} - A_r\ F^h_{rl})$

$+ g_{il}\ (A^h_{i|k} - A^r\ F^h_{rk}) - A^h\ (a_l\ g_{ik} - a_k\ g_{il})$

on other hand (3.1) ...

(3.7) $\theta_{(kl)} = \overbrace{\Gamma^s_{ik}}^{(r)}\ \overbrace{\Gamma^h_{al}}^{(r)} = \theta_{(kl)}\ [F^S_{ik}\ F^h_{al} - \delta^h_k\ (A_i\ \ A_l + F^S_{il}\ A_S)$

$- g_{ik}\ A^a\ (A_S\ \delta^h_l + F^h_{Sl})$

$+ A^h\ (\ A_l\ g_{ik} + F_{ikl})\ ]$

Where $\theta_{(kl)}$ denotes Subtraction after interchanging the indices

$$(3.8)\ \overset{(r)}{\tilde{\bar{R}}}{}^{h}_{ik} = \dot{R}^{h}_{ikl} + \delta^{h}_{i}(A_{k}|_{l} - A_{l}|_{k}) + \delta^{h}_{k}\,(A_{i}|_{l} - A_{i}\,A_{l} - A_{s}\,F^{S}_{il})$$
$$- \delta^{h}_{i}\ (A_{i}|_{k} + A_{r}\,F^{r}_{ik} - A_{i}\,A_{k} - A_{S}\,F^{S}_{ik}) - g_{ik}\,(A^{h}|_{l} - A^{r}\,F^{h}_{rl} + A^{S}\,A_{S}\,\delta^{h}_{l}$$
$$+ F^{h}_{Sl}\ A^{S} + (a_{l} - A_{l})\,A^{h})$$

Now contracting (3.8) by δ^{l}_{h} we get

$$(3.9)\ (a)\ \overset{(r)}{\tilde{\bar{R}}}{}_{ik} = \dot{R}_{ik} + A_{k}|_{i} - A_{i}\,A_{k} - A_{S}\,F^{S}_{ik} - \mathrm{n}\,(A|_{k} + A_{r}\ F^{r}_{ik} - A_{i}\,A_{k} - A_{S}\,F^{S}_{ik})$$
$$- g_{ik}\,(A^{h}|_{h} - A^{r}\,F^{h}_{rh} + nA^{S}\,A_{S} + F^{h}_{sh}\,A^{S} + (a_{l} - A_{l})\,A^{l})$$
$$+ g_{ih}\,(A^{h}|_{k} - A^{r}\,F^{h}_{rk} + A^{S}\,A_{S} + \delta^{h}_{k} + F^{h}_{Sk}\,A^{S} + (a_{k} - A_{k})\,A^{h}).$$

Theorem 1 :

Let $\bar{M}^{n} = (M^{n}, g_{ij} = e^{2\rho(x,y)}\,a_{ij(x)})\ n \geq 3$ be generalized metric space with conformally Riemannian Metric. The two curvature tensors $\dot{\rho}^{h}_{ijk}$ and $\dot{R}^{h}_{ijk}$ of $RR\Gamma$ satisfy equation (3.4) and (3.8) respectively.

Conclusion :

In this paper main result is proposition 2 and Theorem 1.

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