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Some Special Connection in Finsler Space

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ABSTRACT

In this paper we have studied some special connection in Finsler space: Berworld'sConnection, Cartan's Connection and Rund's Connection are special r-TM Connection.

Keywords : Berworld's Connection, Cartan's Connection and Rund's Connection, r-TM Connection.

I. INTRODUCTION

In Berworld's Connection, Cartan's Connection and Rund's Connection the deflexion tensor and torsion tensor vanishes. H. Yasuda has considered connection on Finsler Space with given deflexion and torsion tensor field [4-6]. Finsler Connection with respect to which metric tensor is h-recurrent or v-recurrent [1-3]. In this paper the connections will be discuss for which following condition are satisfied: The connection is h and v-recurrent and Their deflexion tensors do not vanish also the torsion tensor field do not vanish and They are closely similar to the connection introduced by Prasad B.N. [1].

Let M be n dimensional Finsler Space with fundamental function $L(x, y)$. We will consider one of the most common connections on M. This connection can be represented by

(1.1) $\Gamma = (\Gamma^i_{jk}, \Gamma^i_k, \ \tilde{C}^i_{jk})$

where $\Gamma^i_{jk}, \Gamma^i_k, \ \tilde{C}^i_{jk}$ are positively homogenous of degree zero, in y^i respectively and are called h- connection, non linear connection and v-connection of Γ respectively. The hv torsion tensor,

(1.2) $\tilde{P}^i_{kj} = \ \Gamma^i_{k||j} - \Gamma^i_{jk}$,

where symbol $||$ denote partial differentiation by y^j. We put $Q^i_{jk} \ = -P^i_{kj}$ then h-connection is expressible as,

(1.3) $\Gamma^i_{jk} = \Gamma^i_{k||j} + Q^i_{jk}$

where Q^i_{jk} is positively homogeneous of degree 0. If we denote non linear connection of Cartan's (or Barwald's) by G^i_k than non linear connection Γ^i_k of Γ is expressible as,

(1.4) $\Gamma^i_k = G^i_k + T^i_k$ for T^i_k

is positively homogeneous of degree 1.

Now applying (1.4) after Partial differentiation with respect to y^j on (1.3) we get,

(1.5) $\Gamma^i_{jk} = G^i_{jk} + T^i_{jk} + Q^i_{jk}$

where $T^i_{jk} = T^i_{k||j}$

Then a connection Γ on M is uniquely determined by (1.1), (1.4), (1.5) First we shall give important axiom concerning connection M Γ in Finsler geometry.

F_1: $M\Gamma$ is L recurrent w.r.t. recurrence vector K_K i.e.

(1.6) $L_{|k} = K_K L$

F_2 : The defluxion tensor

(1.7) $D^i_k = y^j\ \Gamma^i_{jk} - \Gamma^i_k$

F_3: MΓ is v-recurrent with respect to recurrence vector b_k and v-symmetric i.e. ,

(1.8) $g_{ij|k} = bkg_{ij}$.

(1.9) $C^i_{jk} = C^i_{kj}$.

F_3': The V-connection of $M\Gamma$ vanishes i.e.

(1.10) $\tilde{C}^i_{jk} = 0$.

F_4: With respect to Γ the absolute differentiation Dy_i of y_i $(g_{ij}\ y^i)$ is given by

(1.11) $Dy^j = g_{ij}Dy^j$.

F_5: Paths w.r.t. to Γ are always geodesics of M.

F_6: MΓ is h recurrent with respect to recurrence vector a_k

(1.12) $g_{ij|k} = a_k\ g_{ij}$.

F_7: Γ is h symmetric that is h torsion tensor

(1.13) $\bar{\Gamma}^i_{jk}$ $(= \Gamma^i_{jk} - \Gamma^i_{kj})$ vanishes.

F_8: The hv-torsion tensor P^i_{kj} $(= -Q^i_{jk})$ of Γ vanishes.

<u>SPECIAL r-TM CONNECTION</u>

We have studied some typical connection r-C Γ, r-H Γ, r-R Γ, r-B Γ are special r-TM (r-TM (0)) connection. First we have studied r-C Γ connection it is given by axiom F_2, F_3, F_6, F_7 from F_3, we get

(2.1) $\bar{C}^k_{ij} = C^k_{ij} - \frac{1}{2}(b_j\ \delta^k_i + b_k\ \delta^i_j - g_{ij}\ b^k)$.

From axiom F_6 , we get

(2.2) $a_k g_{ij} + 2P_{ijk} + \frac{\partial g_{ij}}{\partial y^h} T^h_k + T_{ijk} + T_{jik} + Q_{ijk} + Q_{jik} = 0$.

Applying Christoffel process to (2.2), we get

$$(2.3)\ \frac{1}{2}(a_k\delta^i_j + a_j\delta^i_k - a^i g_{jk}) + (C^i_{kr}\ T^r_j + C^i_{jr}T^r_k + g^{im}C_{jkr}\ T^r_m) + P^i_{jk} + T^i_{kj} + Q^i_{kj} = 0.$$

Now contracting (2.3) by y^k , we get

(2.4) $T^i_j = \frac{1}{2}\ (a^i y_j - a_j y^i - a_o\delta^i_j - L^2\ C^i_{jr}\ a^r)$.

From equation (2.3) and (2.4) we get

$$(2.5)\ Q^i_{kj} = -T^i_{kj} - P^i_{kj} - (C^i_{kr}T^r_j + C^i_{jr}T^r_k - g^{im}C_{jkr}T^r_m) - \frac{1}{2}\ (a_k\delta^i_j + a_j\delta^i_k - a^i g_{jk})\ .$$

Therefore r-C Γ is determined by equation (2.1), (2.4) and (2.5).

Next we have studied r-R Γ whose characterizing axiom are F_2, F_3', F_6, F_7 by F_3' , we get

(2.6) $\tilde{C}^i_{jk} = 0$

by axiom F_6 equation (2.4) and (2.5), therefore r-R Γ connection is determined by equation (2.6), (2.4), (2.5).

Now we have studied r-H Γ whose characterizing axiom are F_2, F_3, F_7, F_8 and F_1. From axiom F_8 we get.

(2.6) $Q^i_{jk} = 0$ by axiom F_1 , we get

(2.7) $\partial_i F = (\Gamma_{i}^{o} + K_i L^2)$

where $F = \frac{L^2}{2}$.

Therefore non linear connection G_j^i of Berwald is given by $G_j^i = \partial_i G^i$

(2.8) $G^i = \frac{1}{2} g^{ij} [y^r \partial_j \partial_r F - \partial_j F]$.

Putting (2.6) in (2.7) and using axiom F_2, F_8, we get

(2.9) $G^i = \frac{1}{2} \Gamma_{i}^{o} + \frac{L^2}{2} g^{ij} K_{o||j} - L^2 K^i + K_o y^i$

Differentiating (2.9) w.r. to y^i and using axiom F_2, we get

(2.10) $\Gamma_{j}^{i} = G_j^i + (L^2 K^i + K_o y^i)_{||j} - \frac{1}{2}(L^2 g^{ir} K_{o||r})_{||j}$.

Therefore from (2.4) it follows that

(2.11) $T_j^i = (L^2 K^i + K_o y^i)_{||j} - \frac{1}{2}(L^2 g^{ir} K_{o||r})_{||j}$.

Therefore r-H Γ is determined by equations (2.1), (2.6), (2.11). Now, we consider r-B Γ whose characterizing axioms are F_1, F_2, F_3', F_7, F_8. From axiom F_3' and F_8, follows that

$C_{jk}^i = 0, \; Q_{jk}^i = 0$.

And from axiom F_1, F_2, F_8 the tensor T_j^i is given by equation (2.11) therefore r-B Γ is determined. If M is a Riemannian Space then torsion tensor C_{ijk} vanishes. In this case fundamental function L (x, y) is given by L (x, y) = $(g_{ij}(x)\, y^i\, y^j)^{\frac{1}{2}}$ and Riemannian Connection is given by RN Γ = $[\left\{ {i \atop j\;k} \right\}, y^j \left\{ {i \atop j\;k} \right\}, 0]$, where $\left\{ {i \atop j\;k} \right\}$ is christoffel symbol formed with $g_{ij}(x)$. Now we consider ar-RN Γ whose characterizing axioms are F_2, F_3', F_6, F_7 and F_0 : Fundamental tensor g_{ij} is independent of y^i, $C_{ijk} = 0$ from axioms F_3', F_6, we get

(2.12) $\frac{\partial g_{ij}}{\partial x^k} - g_{hj} \Gamma_{ik}^{h} - g_{ih} \Gamma_{jk}^{h} = a_k g_{ij}$.

Now apply Christoffel process to his equation and using axiom F_7, we get

(2.13) $\Gamma_{ij}^{h} = \left\{ {h \atop i\;j} \right\} - \frac{1}{2}(a_i \delta_j^h + a_j \delta_i^h - a^h g_{ij})$.

From axiom F_2 it follows that $\Gamma_{k}^{i} = \Gamma_{jk}^{i} y^i$ therefore r-RN Γ is determined.

CONCLUSION

In this paper we have obtained Typical r-C Γ, r-H Γ, r-R Γ, r-B Γ are special r-TM (r-M (o)) connection r-C Γ is given by (2.1), (2.4), (2.5), r-R Γ connection is given by (2.6), (2.4), (2.5) r-H Γ connection is given by (2.1), (2.6), (2.11), r-B Γ is given by (2.11) r-RN Γ is given by (2.12), (2.13).

II. REFERENCES

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