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Distributed Estimation with Decentralized Control for Quadruple-Tank Process

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ABSTRACT

This paper proposes the design of quadruple-tank process due to the unique multivariable MIMO system under minimum and non-minimum scenario with respect to the valve ratio. This model is then implemented the distributed estimation algorithm with decentralized control. The inputs are set in divergent gains of pumps while the four-tank process is interconnected so that the stability properties are different, making the usage of decentralized control is reasonable. The number of outputs is designed the same as those of inputs which are also that of distributed Luenberger observer with the continuous linearized dynamical system. This distributed comprises local estimates only in certain output, meaning that it would lead to insufficiency so that the neighbouring links under some network topologies are required in the dynamical system. This concept fortunately works in two different characteristic stability of the tank process regarding estimating the states. This success leads to the further research of the more large-scale complex system.

Keywords: Decentralized Control, Distributed Estimation, Quadruple-Tank Process, Sensor Networks

I. INTRODUCTION

The design dealing with complex multivariable dynamical systems have been attracting a lot of interest in the field of control theory, such as quadruple-tank process [1] and [2]. This scaled structure is particularly suitable in measuring the performance limitation according to the identification batch-algorithm model [3] of the complex control system with the non-minimum mentioned [4] as the elaboration of [5]. Since the system is interconnected meaning that one could influence another, this also leads to the importance of guaranteeing the poles in the left-plane. Several control theories have been proposed to handle this with the mathematical model built in from the sliding-mode [6], robust control [7], or the more advanced predictive control as done in [8]. Furthermore, it could be generalized with the structure of decentralized control as stated in [9] and [10] with the capable of linearizing the non-linear dynamics so that the location of the stability could be well-administered. This quadruple tanks refers to what was done by [2] with two divergent scenarios of the stable minimum and the difficult with the non-minimum phase. Notwithstanding, this plant is applied to test based on the estimation concept of the proposed filtering module [11] and distributed estimation based on the classical Luenberger observer as conducted in [12] and [13] for the linear system. This distributed algorithm currently has been widely studied as a new window in the control field to locally predict the states through neighbouring links. The history of distributed is succeeded by the decentralized done in [14] with the interconnected system based on the classical Kalman filtering and its distributed in [15]. Furthermore, the track fusion applying the cross covariance was also initiated by [16] with the evolution of the maximum likelihood (ML) as [17]. The consensus of the distributed is well-defined in [18] whereas the consensus filtering is conducted in [19] with the same Kalman filtering and its pseudo estimates [20] and decoupling control [21], even with the augmented estimates from the fusion itself [22]. The ideas behind the research conducted in [12] are used further in

[23] being inspired by the estimator in the domain of discrete-time applying deep elaboration of the observability connection as stated in [24]. The required conditions are suggested in [24] regarding the necessary and sufficient to build the augmented observer with appearance of the distributed estimation using the concept of detectability [5] that for certain node i paralleling with the output i, it needs the information from the connected node from the topology. The construction of the paper is then initiated with the mathematical modelling along with the decentralized control. The following is the distributed observer and the numerical scenarios to show the proposed ideas under some criteria regarding the limitation ended by the conclusion.

II. MATHEMATICAL DESCRIPTION

The scheme of quadruple-tank process comprising four interconnected tanks being driven by two pumps as depicted in Fig. (1). This tank process includes multivariable-input multivariable-output (MIMO) plant with two inputs and outputs, constituting the input (u) voltages to both pumps (v_1, v_2) influencing the whole tanks and the output (y) voltages from both level measurement devices (y_1, y_2) in tank 1 and 2. Since the measurement devices are located only in the bottom two tanks, the objective is to maintain the level (h_i) of the tanks working in certain design of set-point with inlet flow rates. While the pumps run, they are then divided into two directions using the three-way valve, which each of them operates only to the two diagonal position tanks.

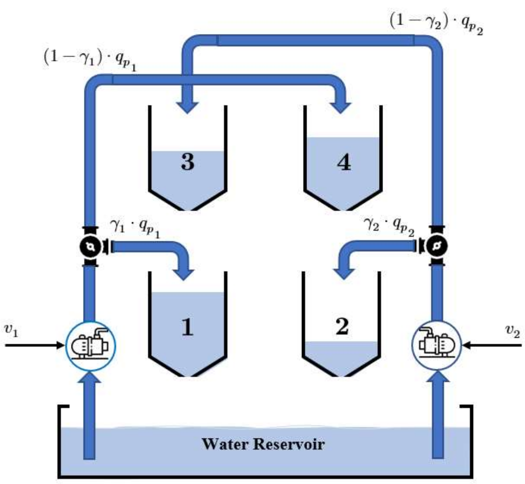

Figure 1: The design of quadruple-tank process

The voltage being implemented to pump n with $n = 1,2$ is v_n and the corresponding outlet flow from n-th pump is $q_p(n)$, which equals to $k_n v_n$ where k is the constant pump and v_n is the velocity rate going through the pump. Another important scenario is the position or behaviour of the valves, affecting the distribution to those four tanks, with the ratio of $(\gamma_1, \gamma_2) \in [0,1]$. This means that if from the i-th pump, the ratio to tank 1 is (γ_1), with flow rate $\gamma_1 k_1 v_1$, then the counterpart ratio of tank 4 is $(1-\gamma_1)$, with flow rate $(1-\gamma_1)k_1 v_1$. Similarly, this concept also administers the rest two tanks with another ratio (γ_2) from another pump. The dynamic of the tanks refers to this paper [2] and the mathematical models are presented as follows. Firstly, it is required to consider the mass balance theorem and the law of Bernoulli's, saying that the accumulation rate of mass in a system (m_T) equals to the difference between mass of inlet flow (m_i) and the outlet (m_o) to the system.

$$\frac{dm_T}{dt} = m_i - m_o \tag{1}$$

and the Eq. (1) could be altered into the non-linear process depending on the fluids, therefore

$$A\frac{dh}{dt} = \rho q_i - \rho q_o \tag{2}$$

Since the fluid is the same with $\rho_1 = \rho_2 = \rho_3 = \rho_4$, Eq. (2) is simplified with Eq. (3)

$$A_i\frac{dh_i}{dt} = (q_i)_i - (q_o)_i \tag{3}$$

where for certain tank $i = 1, \dots, 4$, the variables of A_i, h_i, q_i, and q_o represent the cross-sectional area of certain tank, the fluid level, the inlet and the outlet flow of the tanks respectively. Moreover, the inlet flow for the whole tanks $q_{i_1}, \dots, q_{i_4}$ affected by the ratio of the valve γ_n is described as follows,

$$\begin{aligned} q_{i_1} &= \gamma_1 k_1 v_1; & q_{i_3} &= (1-\gamma_2)k_2 v_2; \\ q_{i_2} &= \gamma_2 k_2 v_2 & q_{i_4} &= (1-\gamma_1)k_1 v_1 \end{aligned} \tag{4}$$

whereas the outlet flow from a tank $q_o(i)$ is denoted in Eq. (5) with a_i and g are the open cross-section of the bottom-outlet flow and the gravitational acceleration in turn

$$q_{o_i} = a_i\sqrt{2gh_i} \tag{5}$$

Taking the whole dynamics (inlet-outlet) of the tanks, the non-linear dynamic of the quadruple-tank from Fig. (1) is shown below. Keep in mind that there exist two inputs from lower tanks, flowing from the pipe and the upper outlet tanks which are affected by the diagonal-term of the corresponding pump

$$\begin{aligned} A_1\frac{dh_1}{dt} &= q_{i_1} + q_{o_3} - q_{o_1} \\ &= \gamma_1 k_1 v_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1} \\ A_2\frac{dh_2}{dt} &= q_{i_2} + q_{o_4} - q_{o_2} \\ &= \gamma_2 k_2 v_2 + a_4\sqrt{2gh_4} - a_2\sqrt{2gh_2} \\ A_3\frac{dh_3}{dt} &= q_{i_3} - q_{o_3} \\ &= (1-\gamma_2)k_2 v_2 - a_3\sqrt{2gh_3} \\ A_4\frac{dh_4}{dt} &= q_{i_4} - q_{o_4} \\ &= (1-\gamma_1)k_1 v_1 - a_4\sqrt{2gh_4} \end{aligned} \tag{6}$$

The Bernoulli's law in Eq. (6) could be then reconstructed in Eq. (7) for the sake of the state-space representation, therefore

$$\begin{aligned}\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_3}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1\end{aligned} \tag{7}$$

Eq. (7) could be also simplified for the so-called conductance K_i. The variables being used in the laboratory-scale process are written in Table (I) regarding the upper (A_i) and lower (a_i) open cross-sectional for each tank (i) along with the ratio of the measured gain signals (k_c) whilst Table (II) asserts the condition of the couple operating points of the quadruple-tank process comprising the initial values of level (h_i^0) and velocity (v_i^0). Furthermore, those are defined as $P\_$ and P_+ declaring the minimum-phase and the counterpart of non-minimum-phase scenario in turn

$$K_i = \frac{a_i}{A_i}\sqrt{2g} \tag{8}$$

TABLE I

Parameter of the laboratory-scale quadruple-tank

Variable	Unit	Values
A_1, A_3	cm²	28
A_2, A_4	cm²	32
a_1, a_3	cm²	0.071
a_2, a_4	cm²	0.057
k_c	V/cm	0.5
g	cm/s²	981

TABLE II

Operating points of the minimum $P\_$ and non-minimum phase P_+ of the quadruple-tank process

Variable	Unit	P_-	P_+
$(h_1^0), (h_2^0)$	cm	(12.4), (12.7)	(12.6), (13.0)
$(h_3^0), (h_4^0)$	cm	(1.8), (1.4)	(4.8), (4.9)
$(v_1^0), (v_2^0)$	V	(3.00), (3.00)	(3.15), (3.15)
$(k_1), (k_2)$	cm³/Vs	(3.33), (3.35)	(3.14), (3.29)
$(\gamma_1), (\gamma_2)$		(0.70), (0.60)	(0.43), (0.34)

The non-linear model in Eq. (7) could be changed into the linear approximation by proposing the following variables $x_i = h_i - h_i^0, u_i = v_i - v_i^0$ from Table (II). With the standard state-space design of $\dot{x} = Ax + Bu$ and the output $y = Cx$, the complete equation is presented in Eq. (9). Moreover, the certain values of time-constant for each tank T_i is influenced by the initial level h_i^0 and the static variables, as shown in Eq. (10), such that

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}} \tag{10}$$

From Eq. (10) and Table (II), the time-constant for each operating-point is shown in Table (III) which is used in the state-space matrices Eq. (9) and the transfer functions Eq. (11),

TABLE IIII

Time-constant for the operating points $P\_$ and P_+

Variable	P_-	P_+
(T_1, T_2)	(62,90)	(63,91)
(T_3, T_4)	(23,30)	(39,56)

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_2}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_1 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_2} \\ \frac{(1-\gamma_1)k_1}{A_1} & 0 \end{bmatrix} u; \qquad y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x \tag{9}$$

$$\begin{aligned} G(s) &= C(sI - A)^{-1}B + D \\ &= \begin{bmatrix} \frac{\gamma_1 c_1}{1 + sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \rightarrow c_n = \frac{T_n k_n k_c}{A_n} \end{aligned} \tag{11}$$

$$G_-(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.8}{1+90s} \end{bmatrix} \qquad G_+(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix} \tag{12}$$

$$\det G(s) = \frac{c_1 c_2}{\gamma_1\gamma_2 \prod_{i=1}^{4}(1+sT_i)} \times \left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2}\right] \tag{13}$$

For the particular h_i^0, the transfer function in Eq. (11) is utilized to yield the stationary control signal from Eq. (9) with specific (c_i). Keep in mind that the valve ratio γ_n for the non-minimum P_+ and minimum $P\_$ is set with $0 < (\gamma_1 + \gamma_2) < 1$ and $1 < (\gamma_1 + \gamma_2) < 2$ as written in Table (2).

The transfer function in Eq. (11) is affected by the variables working in two different operating points. Moreover, this means the transfer function results in two divergent physical modelling as reported in Eq. (12). More specifically, $G\_(s)$ represent the minimum phase whereas the $G_+(s)$ constitutes the non-minimum scenario. Transfer functions in Eq. (11) and (12) have zero locations leading to the physical representation of the system with respect to certain ratio of γ_n. The zeros in Eq. (11) are then supposed to be the numerator of the following characteristic rational formula as written in Eq. (13). These zero results furthermore in the analysis of either left- or right-half plane. From Eq. (14), it can be inferred that the analysis of determining the scale of γ_1 and γ_1 is if the $\eta \to 0$, the two zeros are approaching the negative of either a T_3 or T_4 while as $\eta \to \infty$, those would be then in the extremely asymptotically $(-/+)$ of ∞, such that

$$\eta := \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1\gamma_2} \tag{14}$$

Recalling the parameters of minimum and non-minimum, the first accounts for $(\gamma_1 + \gamma_2) = 1.30 > 1$ which means that the flow going to the two bottom tanks is greater than that of the two top tanks and by contrast, (P_+), the flow to the lower tanks would be smaller compared to the upper. This also indicates that controlling the two bottom tanks is much easier than the left (1 & 3) or the right (2 & 4) tanks. Beyond that, the zeros location is not the only consideration, rather the direction. Likewise, the transfer function (G) is having the zero direction by the following equation Eq. (15) and (16).

Another concept is what was proposed by [25] regarding the relative gain array (RGA) denoting how the MIMO control system is measured. This is defined as $\Upsilon = (G)_0 * (G^{-\dagger})_0$ where the symbol of $(*)$ describes the multiplication by element $(-\dagger)$ with the inverse transpose of matrix. The RGA of this system is given as follow depending solely on the valve ratio, therefore

$$\lambda = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \qquad \tilde{\lambda} = \frac{(1-\gamma_1)(1-\gamma_2)}{1-\gamma_1-\gamma_2} \tag{17}$$

For the decentralized scenario with the non-minimum, the RGA then is designed as Eq. (17) indicating that $\tilde{\lambda} > 0$ and this is preferable. Moreover, the stability property is also considered for the input gain flow v_n^0 if Eq. (18)

$$\begin{bmatrix} \gamma_1 k_1 & (1-\gamma_2)k_2 \\ (1-\gamma_1)k_1 & \gamma_2 k_2 \end{bmatrix} \tag{18}$$

is a non-singular matrix with $\gamma_1 + \gamma_2 \neq 1$

III. DECENTRALIZED CONTROL

Since the quadruple-tank is the multivariable control system, the decentralized control is proposed with $u = \text{diag}[C_1 \quad C_2]\, e$ as depicted in Fig. (2) for the specific proportional-integral (PI) control law as written in Eq. (20). Decentralized control requires the parallel dimension of input-output system and the positive diagonal element of RGA $G(0)$ with this decentralized makes it easy to be controlled, otherwise, with negative diagonal element, it leads to be the unstablility

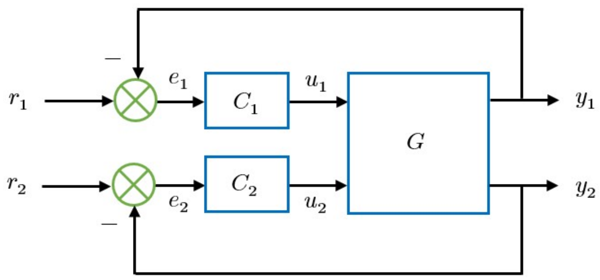

Figure 2: Decentralized control design with the coupling of C_1 and C_2

Inside the $G(s)$, the transfer function is designed as in Eq. (19) and the control gain parameters being used in the simulation are obtained from the root-locus calculation. Those gain K_p, K_i values for certain pump n are different and this research focuses on the minimum phase $P\_$ only to be implemented using the distributed estimation explained in the next chapter.

$$\begin{aligned} G(s) &= \begin{bmatrix} G_1 & G_2 \\ G_3 & G_4 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{\Phi_1}{\varphi_1 s + \xi_1} & \dfrac{\Phi_2}{\vartheta_2 s^2 + \varphi_2 s + \xi_2} \\ \dfrac{\Phi_3}{\vartheta_3 s^2 + \varphi_3 s + \xi_3} & \dfrac{\Phi_4}{\varphi_4 s + \xi_4} \end{bmatrix} \end{aligned} \tag{19}$$

$$C_n = K\left(1 + \frac{1}{(T_i)_n s}\right) \to n = 1{,}2 \tag{20}$$

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}^T \begin{bmatrix} \dfrac{\gamma_1 c_1}{1+zT_1} & \dfrac{(1-\gamma_2)c_1}{(1+zT_3)(1+zT_1)} \\ \dfrac{(1-\gamma_1)c_2}{(1+zT_4)(1+zT_2)} & \dfrac{\gamma_2 c_2}{1+zT_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T \tag{15}$$

$$\text{RGA } \Upsilon = \begin{bmatrix} \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} & \dfrac{-(1-\gamma_1)(1-\gamma_2)}{\gamma_1+\gamma_2-1} \\ \dfrac{-(1-\gamma_1)(1-\gamma_2)}{\gamma_1+\gamma_2-1} & \dfrac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1} \end{bmatrix} \to \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix} \tag{16}$$

IV. DISTRIBUTED ESTIMATION

Since the area of control systems have been increasing upon the demand of the more complex systems, one is to estimate the state from networked system. To deal with this, the usage of distributed estimation with switching its localization into key neighbourhood communication [12] and [13] attracts the most as portrayed in Fig. (3)

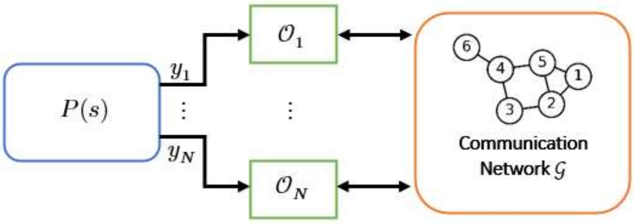

Figure 3: Scheme of distributed estimation over N output y_N and observer $\mathcal{O}_N$

Suppose the following time-domain linear system as in Eq. (21), where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$ are the state and measurement in turn. This distributed enables the y as $\mathrm{col}(y_1, \dots, y_N)$ and H_i as $\mathrm{col}(H_1, \dots, H_N)$ with N is assigned the number of nodes in the network $\mathcal{G}$, where $\sum_{i=1}^{N} p_i = p$ and $y_i \in \mathbb{R}^{p_i}$. This y_i is then assumed as the solely key data being obtained by certain local node (i) to estimate the states by using the neighbouring links to cope with the lack of insufficient data with the constraint of the designed network topology,

$$\frac{dx}{dt} = Ax, \qquad y = Hx = \begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix} x = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \tag{21}$$

Furthermore, this research considers the distributed estimation with Luenberger structure containing N local output and observers having the following dynamics for each node i as stated in Eq. (21), where $\hat{x}_i \in \mathbb{R}^n$, $L_i \in \mathbb{R}^{n \times p_i}$ and $M_i \in \mathbb{R}^{n \times n}$ are the estimated state, the injection and weighting gain matrices to be designed for node i. The values of γ and $\mathcal{N}_i$ is a scalar coupling gain and the communicated nodes of certain node i. More specifically, the matrices of L_i and M_i comprises

$$L_i := T_i \begin{bmatrix} L_{\mathrm{id}} \\ 0 \end{bmatrix}, \qquad M_i(k_i) := T_i \begin{bmatrix} k_i M_{\mathrm{id}} & 0 \\ 0 & I_{\sigma_i} \end{bmatrix} \tag{23}$$

with the weighted values of $k_i \geq 1$ along with the identity matrix of I_{σ_i}. σ_i shows the size of the matrix from the size information from the undetectable subspace U_i of the couple (A, H_i). Moreover, the orthonormal matrix T_i should be designed from the following criteria to obtain the desired value, such that

$$\begin{aligned} H_i T_i &= [H_{\mathrm{id}} \quad 0], \\ T_i^T A T_i &= \begin{bmatrix} A_{\mathrm{id}} & 0 \\ A_{\mathrm{ir}} & A_{i\mathrm{u}} \end{bmatrix} \end{aligned} \tag{24}$$

where the couple $(A_{\mathrm{id}}, H_{\mathrm{id}})$ suppose to be detectable $U_i = 0$. Keep in mind that the definition if detectable if slightly lower than observability saying that the states condition of the unobservable is stable. The next is to find the matrices of L_{id} and M_{id} which could be opted from solving $A_{id} - L_{id} H_{id}$ as the Hurwitz and the following equation in Eq. (25) in turn and the complete equation for certain local i in the observer is denoted in Eq. (26). Bear in mind that the initial states designed from Eq. (21) and (22) are maintained to be converged the true states as in Eq. (27) with the assumption that the network topology is then connected yet undirected along with the detectability of the couple (A, H)

$$\lim_{t \to \infty} \|\hat{x}_i(t) - x(t)\| \tag{27}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + L_i\big(y_i - H_i\hat{x}_i\big) + \gamma M_i^{-1}(k_i) \sum_{j \in \mathcal{N}_i} (\hat{x}_j - \hat{x}_i) \tag{22}$$

$$(A_{id} - L_{id} H_{id})^T M_{id} + M_{id}(A_{id} - L_{id} H_{id}) = -I_{n-\sigma_i} \tag{25}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + L_i\big(y_i - H_i\hat{x}_i\big) + \gamma M_i^{-1}(k_i) \sum_{j=1}^{N} \alpha_{ij}(\hat{x}_j - \hat{x}_i) \tag{26}$$

$$\left(k_i - \frac{\beta}{\theta(\bar{\epsilon})}\right)\left(\gamma - \frac{\bar{\beta}}{2\lambda_2}\right) > \frac{\bar{\beta}^2 N^2}{2\lambda_2 \theta(\bar{\epsilon})}; \quad \rightarrow \quad \forall i \in \mathcal{N}; \quad k_i \geq 1; \quad \gamma > \frac{\bar{\beta}}{2\lambda_2}; \quad \theta(\bar{\epsilon}) = \frac{1}{2}\left(1 - \left(1 - \frac{\bar{\epsilon}^2}{2}\right)^2\right) \tag{28}$$

$$M_i(k_i)(A - L_i H_i) = T_i \begin{bmatrix} k_i M_{\mathrm{id}} & 0 \\ 0 & I_{\sigma_i} \end{bmatrix} T_i^T (A - L_i H_i) T_i T_i^T = T_i \begin{bmatrix} k_i M_{\mathrm{id}} & 0 \\ 0 & I_{\sigma_i} \end{bmatrix} \begin{bmatrix} A_{\mathrm{id}} - L_{\mathrm{id}} H_{\mathrm{id}} & 0 \\ A_{\mathrm{ir}} & A_{i\mathrm{u}} \end{bmatrix} T_i^T \tag{29}$$

$$\begin{aligned} e_i = \hat{x}_i - x \quad \rightarrow \quad \frac{de_i}{dt} &= (A - L_i H_i) e_i + \gamma M_i \sum_{j=1}^{N} \alpha_{ij}\big(e_j - e_i\big) \\ &= \Lambda e - \gamma \bar{M}(\mathcal{L} \circ I_n) e \quad \rightarrow \quad \begin{cases} \Lambda = \mathrm{diag}\{A - L_1 H_1 \quad \cdots \quad A - L_N H_N\} \\ M = \mathrm{diag}\{M_1 \quad \cdots \quad M_N\} \end{cases} \end{aligned} \tag{30}$$

If the parameters of k_i and γ are opted rewarding the conditions in Eq. (28) with $\beta_i := 2\|A_{ir}\|^2 + \|A_{iu}^T + A_{iu}\|$ and $\bar{\beta} := \max(i \in \mathcal{N})\, \beta_i$ where β is the sum of β_i from 1 to N. The idea of the characteristic of T_i is that it is the orthonormal matrix so that it satisfies following the Eq. (29) with the error of the local node i in Eq. (30) as the combination of the two equations, Eq. (21) and (22).

V. NUMERICAL SCENARIOS

This chapter is used to elaborate the concept with some simulation. The dynamic of the system is presented in Eq. (9) with the suitable parameter as in Table (1), (2), and (3) for certain phase-conditions either minimum $P_{\_}$ or non-minimum P_{+}. Since the nodes are only two $N = 2$, the communication occurs between them with $y_i = H_i x$,

$$\begin{aligned} H_1 &= [k_c \quad 0 \quad 0 \quad 0] \\ H_2 &= [0 \quad k_c \quad 0 \quad 0] \end{aligned} \tag{17}$$

The detail parameters being used in the simulation are $\gamma = 6$, $k_1 = 3$, and $k_2 = 4.5$ with initial condition of $x_0 = [8 \quad 5 \quad -2 \quad 1]$ along with decentralized control parameters of $(K_1 T_{i1})_1 = (3{,}30)$ and $(K_2 T_{i2})_1 = (2.7{,}40)$ for the minimum phase $P_{\_}$ and for the non-minimum phase P_{+} of $(K_1 T_{i1})_2 = (1.5{,}110)$ and $(K_2 T_{i2})_2 = (-0.12{,}220)$ with ten times settling time longer than that of the minimum-phase. The parameters for distributed estimation are presented in the following details, such that,

$$T_1 = \begin{bmatrix} I_2 & \boldsymbol{O} \\ \boldsymbol{O} & I_2 \end{bmatrix} \qquad T_2 = \begin{bmatrix} I_2 & \boldsymbol{O} \\ \boldsymbol{O} & I_2 \end{bmatrix}$$

$$L_{1\mathrm{d}} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad L_{2\mathrm{d}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$M_{1\mathrm{d}} = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 1 \end{bmatrix} \qquad M_{2\mathrm{d}} = \begin{bmatrix} 0.286 & -0.25 \\ -0.25 & 0.387 \end{bmatrix}$$

The numerical scenario for minimum phase shows that the system could deal with the interconnected tanks system with the proposed parameters as being depicted in Fig. (4c) with the following error in Fig. (4a). Keep in mind that the peak errors happened are due to the changes of set-points as shown in the time of 100, 200, 300 and 350 from the two voltages since the four tanks are the interconnected system which affects one from others. However, the dynamics MIMO system is then stabilizing with the very fast time. By contrast, the non-minimum phase is much more difficult to be controlled and it needs ten times setting time than that of their counterparts as presented in Fig. (4b) for the error and Fig. (4d) for the output dynamics. Likewise, the peaks occurred are made of the changes of set-points. Furthermore, regarding the distributed estimation, both true states (x) response with the black-dashed lines could be followed by the estimates of $(\widehat{x}_1)$ and $(\widehat{x}_2)$. Regarding Fig. (4e)-Fig. (4f), the performance of the estimation is depicted and shows the ability of tracking.

VI. CONCLUSION

The mathematical dynamics of the quadruple-tank have been written along with some key parameters, such that the valve gains dividing the flow with $\gamma_1 + \gamma_2 < 1$ would be non-minimum and otherwise is the minimum. The constructed

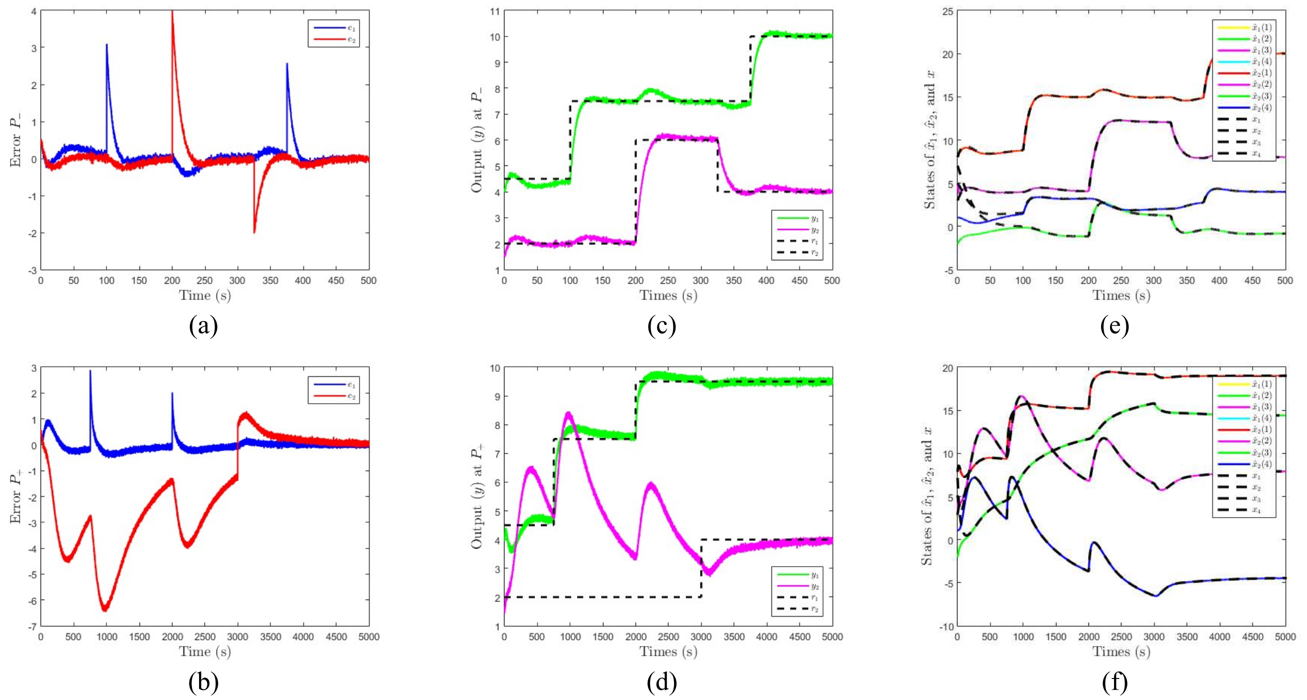

Figure 4: The error of the two parameters from the minimum-phase $P_{\_}$ as (a) and the non-minimum P_{+} as (b) using decentralized PI control; The two responses of the true output (y) from $P_{\_}$ as (c) with 500s and P_{+} as (d) with ten times longer settling time by 5000s with the same gains of control as designed; The response states of the distributed estimation of the $P_{\_}$ (e) and P_{+} (f) with the same initial conditions.

decentralized PI control also show the adverse of maintaining the scenario of non-minimum compared to their counterpart. With respect to the distributed estimation, it has been designed using local communication as much as the number of outputs. This local Luenberger observer design could deal with the dynamics of the quadruple-tank process while it is erratic in the early stages of iterations. Our future work would be the changes of distributed estimation along with some distributed fault detection and fault-tolerant control.

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