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Five-dimensional Plane Symmetric Bianchi Type-I Cosmological model with Quark and Strange Quark Matter in $f(R)$ Theory of Gravity

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ABSTRACT
In the present paper, we have obtained the solution of five-dimensional plane symmetric Bianchi type-I space time in the presence of quark and strange quark matter in modified $f(R)$ theory of gravity. The field equations are solved using special law of variation of Hubble's parameter which is proposed by Berman(1982,83). Also, the physical properties have been discussed.

Keywords : Five-dimensional plane symmetric Bianchi I space-time, Quark matter and strange quark matter, modified $f(R)$ theory of gravity.

I. INTRODUCTION

The $f(R)$ theory of gravity is considered as the most suitable due to cosmologically important $f(R)$ models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Viable $f(R)$ gravity models have been studied by Nojiri S and Odintsov S. D. (2008) and showed the unification of early time inflation and late-time acceleration. The problem of dark matter can also be addressed by using viable $f(R)$ gravity models. The modified $f(R)$ theory of gravity has received considerable attention. It gives an easy unification of early time inflation and late time acceleration. It provides a natural gravitational alternative to dark energy. The explanation of cosmic acceleration is obtained just by introducing the term $1/R$ which is essential at small curvature. It also describes the transition phase of the universe from deceleration to acceleration. Thus $f(R)$ theory of gravity seems attractive and a reasonable amount of work has been done in different contexts.

Many authors have obtained spherical, cylindrical and plane symmetric solutions in $f(R)$ gravity [Hollenstein and Lobo (2008), Azadi et al. (2008), M. Sharif and M. Farasat Shamir (2009), Urkude et. al (2013) etc.]. Also Sotiriou and Faraoni (2010) presented some important aspects of $f(R)$ theory of gravity in Metric, Palatini and Metric-Affine formalisms. M. Sharif and M. Farasat Shamir (2010) have studied the non-vacuum perfect fluid solutions of four dimensional Bianchi type-I and V space-times in the framework of metric $f(R)$ gravity. M. Farasat

Shamir (2011) has studied the plane symmetric vacuum solution of Bianchi type-III space-time in metric $f(R)$ gravity. For this purpose he assumed expansion scalar θ proportional to shear scalar σ. Yilmazet et. al. (2012) studied quark and strange quark matter $f(R)$ gravity for Bianchi type-I and V space-times.

Mak and Harko (2004) have studied charged strange quark matter with the help of spherically symmetric space time using conformal motion. Katore et. al. (2012) have studied FRW cosmological model with strange quark matter attached to the string cloud in general relativity. Khadekar and Shelote (2012) have obtained higher dimensional cosmological model in the presence of quark and strange quark matter. Adhav et. al. (2015) have discussed the Kantowski-Sachs Cosmological model with quark and strange quark matter in $f(R)$ theory gravity. Chirde and Shekh (2018) studied transition between general relativity and quantum gravity using dark and strange quark matter and some Kinematical test. Sen and Aygun (2016) have studied higher dimensional flat Friedmann-Robertson-Walker cosmological models for string cloud with perfect fluid attached to strange quark matter in self creation theory. Tripade et. al. (2021) have studied LRS Bianchi type-I cosmological model with quark and strange quark matter in $f(R)$ gravity. Shamir (2016) has investigated dynamics of LRS Bianchi type-I power law $f(R)$ cosmology.

At early stage when the temperature (?) of the Universe was T = 200Mev, the phase transition of the universe took place from quark glucon plasma to hydrogen gas, which is referred as 'quark hadrons phase'. Strange quark matter is developed with an equation of state (EoS) $p=\frac{(\rho-4B_c)}{3}$ based on the phenomenological bag model of quark matter in which quark constraint is described by an energy term proportional to the volume. Here B_c represent the bag constant, p and ρ are the thermodynamic pressure and energy density of the quark matter respectively. Quarks are considered as degenerate Fermi gas, which exist only in a region of space providing with a vacuum energy density B_c called as the bag constant. In the bag model, it is assumed that quarks are mass-less and non-interacting. With quark pressure $p_q=\frac{\rho_q}{3}$ where ρ_q is the quark energy density. Quark matter studied in general relativity using various assumptions. Thus in this paper, we wish to investigate five dimensional plane symmetric Bianchi type-I cosmological model with quark and strange quark matter in $f(R)$ theory of gravity.

The paper is organized as follows: In section-2, we briefly give the five dimensional field equations in metric $f(R)$ theory of gravity. In the section-3, we can find solution of the field equations for Bianchi type-I space-time in V_5 and in the section-4 some physical properties have been discussed. In the last section, we summarize and conclude the results.

II. Five dimensional field equations in metric $f(R)$ theory gravity

The five dimensional field equations in $f(R)$ theory of gravity are given by:

$$F(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\nabla_i\nabla_j F(R)+g_{ij}\Box F(R)=T_{ij}^{M}, \quad (i,j=1,2,3,4,5) \tag{1}$$

where $F(R)\equiv\frac{df(R)}{dR}$, $\Box\equiv\nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

The energy momentum tensor for quark matter is given by

$$T_{ij}^{(Quark)} = (\rho + p)u_i u_j - pg_{ij} = dig(\rho, -p, p, -p, -p) \quad (2)$$

Where $\rho = \rho_q + B_c$ is the quark matter total energy density and $p = p_q - B_c$ is the quark matter total pressure and u_i is the five velocity vector such that $u_i u^i = 1$

EOS parameter for quark matter is defined as

$$p_q = \in \rho_q, \quad 0 \le \in \le 1 \quad (3)$$

For strange quark matter, the linear equation of state is given by

$$p = \in (\rho - \rho_0) \quad (4)$$

where $\in$ is a constant and ρ_0 is the energy density at zero pressure

where B_c is the bag constant.

When $\in = \frac{1}{3}$ and $\rho_0 = 4B_c$ the above linear equation of state is reduced to the following EoS for strange quark matter in the bag model.

$$\rho = \frac{\rho - 4B_c}{3}. \quad (5)$$

III. Solutions of plane symmetric Bianchi type-I cosmological model in V_5

The plane symmetric Bianchi type-I space-time in V_5 is given by

$$ds^2 = dt^2 - A^2(t)(dx^2) - B^2(t)(dy^2 + dz^2 + du^2) \quad (6)$$

where A and B are cosmic scale factors. The corresponding Ricci scalar is

$$F\left(-\frac{2\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C}\right) - \frac{1}{2} f(R) + \left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{F} = (\rho_q + B_c). \quad (7)$$

The spatial volume of this model is given

$$V = (AB^3) \quad (8)$$

The average scale factor of the Bianchi type-I space-time in V_5 is

$$a = (AB^3)^{1/4} \quad (9)$$

The Hubble's parameter is given by

$$H = \frac{\dot{V}}{V} = \frac{1}{4}\left(\frac{\dot{A}}{A} + \frac{3\dot{B}}{B}\right) \quad (10)$$

Where dot denotes derivative with respect to t. Also

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4) \quad (11)$$

Where $H_1 = \frac{\dot{A}}{A}$, $\quad H_2 = H_3 = H_4 = \frac{\dot{B}}{B}$ are directional Hubble's parameter in the direction of x, y, z and u direction respectively.

The expansion scalar θ and shear scalar σ are defined as follows

$$\theta = u^i_{;i} = 4H = \frac{\dot{A}}{A} + \frac{3\dot{B}}{B}, \tag{12}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{A}}{A} - \frac{B}{B}\right)^2. \tag{13}$$

The mean anisotropic parameter is

$$\bar{A} = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right) = \frac{\sigma^2}{2H^2}. \tag{14}$$

In a co-moving coordinate system, the field equations (1) for the anisotropic Bianchi type-I space time (6) is given by

$$F\left(\frac{\ddot{A}}{A} + \frac{3\dot{A}\dot{B}}{AB}\right) + \frac{1}{2}f(R) - \frac{3\dot{B}\dot{F}}{B} - \ddot{F} = (p_q - B_c), \tag{15}$$

$$F\left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2}\right) + \frac{1}{2}f(R) - \frac{2\dot{B}\dot{F}}{B} - \frac{\dot{A}\dot{F}}{A} - \ddot{F} = (p_q - B_c), \tag{16}$$

$$F\left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2}\right) + \frac{1}{2}f(R) - \frac{2\dot{B}\dot{F}}{B} - \frac{\dot{A}\dot{F}}{A} - \ddot{F} = (p_q - B_c), \tag{17}$$

$$F\left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2}\right) + \frac{1}{2}f(R) - \frac{2\dot{B}\dot{F}}{B} - \frac{\dot{A}\dot{F}}{A} - \ddot{F} = (p_q - B_c), \tag{18}$$

$$F\left(\frac{\ddot{A}}{A} + \frac{3\ddot{B}}{B}\right) + \frac{1}{2}f(R) - \frac{\dot{A}\dot{F}}{A} - \frac{3\dot{B}\dot{F}}{B} = -\rho. \tag{19}$$

After subtracting equation (19) from (15) and (19) from (16) then

$$F\left(-\frac{3\ddot{B}}{B} + \frac{3\dot{A}\dot{B}}{AB}\right) + \frac{\dot{A}\dot{F}}{A} - \ddot{F} = (p_q + \rho_q), \tag{20}$$

$$F\left(-\frac{\ddot{A}}{A} - \frac{2\ddot{B}}{B} + \frac{2\dot{B}^2}{B^2} + \frac{\dot{A}\dot{B}}{AB}\right) + \frac{\dot{B}\dot{F}}{B} - \ddot{F} = (p_q + \rho_q). \tag{21}$$

To obtain the solution, use power law relation between scale factor a and scale field F is

$$F = l\,a^m = l(AB^3)^{m/4} \tag{22}$$

where l is constant of proportionality.

Now assume that shear scalar is proportional to the expansion scale, it gives an anisotropic relation between the scale factors A and B

$$A = B^n \tag{23}$$

where n is arbitrary constant, $n \neq 1$.

From above the equation (20) and (21), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{2\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{F}}{AF} - \frac{\dot{B}\dot{F}}{BF} = 0. \tag{24}$$

From equation (22), (23) and (24), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2}\left[(n+2) + \frac{m(n+3)}{4}\right] = 0, \tag{25}$$

$$\frac{\ddot{B}}{B}+\alpha\frac{\dot{B}^2}{B^2}=0 \tag{26}$$

where $\alpha=[(n+2)+\dfrac{m(n+3)}{4}]$

After integration (26) equation, we get

$$B=(\alpha+1)^{\frac{1}{\alpha+1}}(k_1t+k_2)^{\frac{1}{\alpha+1}} \tag{27}$$

where $\beta=(\alpha+1)^{\frac{1}{\alpha+1}}$ and $k_1,\ k_2$ are constant of integration.

$$B=\beta(k_1t+k_2)^{\frac{1}{\alpha+1}}. \tag{28}$$

From the equation (23), we get

$$A=B^n=\beta^n(k_1t+k_2)^{\frac{n}{\alpha+1}}. \tag{29}$$

The average scale factor of the Bianchi type-I space-time in V_5 is

$$a=(AB^3)^{1/4}=\beta^{\frac{n+3}{4}}(k_1t+k_2)^{\frac{n+3}{4(\alpha+1)}}. \tag{30}$$

The value of q (deceleration parameter) is defined as

$$q=\frac{d}{dt}(\frac{1}{H})-1=\frac{4(\alpha+1)}{(n+3)}-1,$$

$$(\alpha+1)=\frac{(q+1)(n+3)}{4}. \tag{31}$$

The sign of q indicates whether the model inflates or not. Positive value of q gives the standard deceleration model while negative values give inflation or accelerating universe.

Therefore the metric (6) becomes

$$ds^2=dt^2-\beta^{2n}(k_1t+k_2)^{\frac{8n}{(q+1)(n+3)}}dx^2-\beta^2(k_1t+k_2)^{\frac{8}{(q+1)(n+3)}}(dy^2+dz^2+du^2). \tag{32}$$

IV. Physical parameters

The spatial volume v of the universe is given by

$$V=\beta^{3(n+3)}(k_1t+k_2)^{\frac{4}{(q+1)}}. \tag{33}$$

Using equation (29) the directional Hubble parameter are given by

$$H_1=\left(\frac{4n}{(q+1)(n+3)}\right)\left(\frac{k_1}{k_1t+k_2}\right),\quad H_2=H_3=H_4=\left(\frac{4}{(q+1)(n+3)}\right)\left(\frac{k_1}{k_1t+k_2}\right). \tag{34}$$

The generalized Hubble's parameter H is given by

$$H=\left(\frac{1}{(q+1)}\right)\left(\frac{k_1}{k_1t+k_2}\right). \tag{35}$$

The expansion scalar θ, shear scalar σ and mean anisotropic parameter $\overline{A}$ of the Bianchi type-I model is given by

$$\theta = \left(\frac{4}{(q+1)}\right)\left(\frac{k_1}{k_1 t + k_2}\right), \tag{36}$$

$$\sigma^2 = \left(\frac{6(n-1)^2 \eta}{(k_1 t + k_2)^2}\right), \tag{37}$$

$$\overline{A} = \frac{3(n-1)^2}{(n+3)^2}. \tag{38}$$

Quark matter for Bianchi type-I model

With the help of EoS parameter (for $\in = \frac{1}{3}$, $p_q = \in \rho_q = \frac{\rho_q}{3}$, $\rho_q = 3p_q$) and using equation (32) in (20), then we get the pressure and energy density of the strange quark matter as

$$p_q = \frac{\eta l \beta^{\frac{(q-3)(n+3)}{4}}}{4(k_1 t + k_2)^{\frac{(q+5)}{(q+1)}}}\{[(qn+3q-3n+3)(qn+3q+5n+3)]-[48+(q-3)^2(n+3)^2\}, \tag{39}$$

where $\eta = \frac{k_1^2}{(n+3)^2(1+q)^2}$

$$\rho_q = \frac{3\eta l \beta^{\frac{(q-3)(n+3)}{4}}}{4(k_1 t + k_2)^{\frac{(q+5)}{(q+1)}}}\{[(qn+3q-3n+3)(qn+3q+5n+3)]-[48+(q-3)^2(n+3)^2\}. \tag{40}$$

Using equation (29) in (7), we get Ricci scalar

$$R = \frac{8\eta}{(k_1 t + k_2)^2}[n^2 q + 6nq + 9q - 9n - 3n^2 + 15]. \tag{41}$$

And the $f(R)$ is given by

$$f(R) = \frac{4\eta l \beta^{\frac{(q-3)(n+3)}{4}}}{(k_1 t + k_2)^{\frac{(q+5)}{(q+1)}}}[n^2 q + 6nq + 9q - 6n - 3n^2 - 15] - 2Bc. \tag{42}$$

Strange Quark matter for Bianchi type-I model

Using equation (29) in the equation (20) with the help of EoS parameter, then we get the pressure and energy density of the strange quark matter as follows

$$p = \frac{\eta l \beta^{\frac{(q-3)(n+3)}{4}}}{4(k_1 t + k_2)^{\frac{(q+5)}{(q+1)}}}\{[(qn+3q-3n+3)(qn+3q+5n+3]-[48+(q-3)^2(n+3)^2]\} - B_c \tag{43}$$

and $$\rho = \frac{3\eta l \beta^{\frac{(q-3)(n+3)}{4}}}{4(k_1 t + k_2)^{\frac{(q+5)}{(q+1)}}}\{[qn+3q-3n+3)(qn+3q+5n+3]-[48+(q-3)^2(n+3)^2]\} + B_c. \tag{44}$$

V. CONCLUDING REMARK

The five dimensional modified Einstein's field equations in $f(R)$ theory of gravity are very difficult to find their solutions without taking any assumptions. The assumption is that the expansion scalar θ is proportional shear scalar σ (i.e. $A = B^n$, where A and B are metric coefficients and n is any arbitrary constant) and second assumption is the power law relation between F and a is used to find the solution. Some physical quantities such as expansion scalar θ, shear scalar σ^2 are obtained. Also $f(R)$ is evaluated.

The physical parameters $H_1, H_2, H_3, H_4, \theta$ and σ are the functions of time. As cosmic time tends to infinity (i.e. $t \to \infty$) then these parameters tends to zero but cosmic time $t = -\frac{k_2}{k_1}$ then these parameters tends to ∞ except the spatial volume.

The spatial volume of this model is zero when $t = -\frac{k_2}{k_1}$.

When $q < -1$ then the model is expanding with big bang singularity at $t = -\frac{k_2}{k_1}$.

The pressure p_q and density ρ_q for quark matter are finite at $t = 0$ and become zero at $t \to \infty$.

The pressure p and density ρ for string quark matter are same as quark matter.

When $q > -1$ at $t = 0$ the model has constant volume and increases with increase in time and become infinite at $t \to \infty$.

The pressure p_q and density ρ_q for quark matter are finite at $t = 0$ and become zero at $t \to \infty$.

The pressure p and density ρ for string quark matter are same as quark matter.

The mean anisotropy parameter $\bar{A}$ is zero for $n = 1$ then model becomes isotropic. The mean isotropic parameter is constant throughout the evolution of universe as it does not depend on the cosmic time.

VI. REFERENCES

[1]. Adhav et. al. (2015): Kantowski-Sachs Cosmological model with quark and strange quark matter in f(R) theory of gravity. Open Physics, Vol.13, Issue1.

[2]. Adhav et. al. (2015): Anisotropic Cosmological Models in f (R) gravity with Quark and Strange Quark Matter and special form of deceleration parameter. The African Review of Physics, Vol.9.

[3]. Azadi, A., Momeni, D., Nouri-Zonoz, M. (2008): Cylindrical solutions in metric f (R) gravity. Phys. Lett. B 670, 210.

[4]. Chirde and Shekh(2018): Transition between general relativity and quantum gravity using quark and strange quark matter with some kinematical test. Journal of Astrophysics and Astronomy 39.5:56.

[5]. Khadekar and Shelote (2012): Higher Dimensional Cosmological Model with Quark and Strange Quark Matter. International Journal of Theoretical Physics, 51(5), 1442-1447.

[6]. Katore et. al. (2012): Strange quark matter attached to string cosmology in FRW space-time. International Journal of Theoretical Physics 51.1:83-89.

[7]. Mak and Harko(2004):Quark Start Admiting a One-Parameter Group of Conformal Motion. International Journal of Modern Physics D.Vol.13, 149-156.

[8]. G. P. Urkude el. al. (2013): Five dimensional static plane symmetric vacuum solutions in f (R) Gravity. Tensor N. S. Vol. 74.

[9]. Hollenstein, L., Lobo, F.S.N. (2008): Exact solutions of f (R) gravity coupled to nonlinear electrodynamics. Phys. Rev. D Vol.78, Issue 12,124007.

[10].Nojiri S. and Odintsov S. D. (2008): Dark energy, inflation and dark matter from modified F(R) gravity. Problems of modern Theoretical Physics A Volume in honour of Prof. Buchbinder, L. L. in the occasion of his 60th birthday, PP. 266-285(TSPU Publishing, Tomsk).

[11].Shamir, M. F. (2011): Plane symmetric Vacuum Bianchi type-III Cosmology in f (R).Int J Theor. Phys 50: 637-643.

[12].Sotiriou T. P., Faraoni V (2010): f (R) Theories of Gravity. Rev. Mod. Phys82:451-497.

[13].Sharif, M. and Shamir, M. F. (2010): Plane Symmetric Solutions in f (R) Gravity, Mod. Phys. Lett.A25:1281-1288.

[14].Sharif, M., Shamir, M. F. (2009): Exact solution of Bianchi type-I and V space times in f (R) theory of gravity. Class. Quantum Gravity Vol.26, No. 23.

[15].Sharif, M., Shamir, M. F. (2010): Non-Vacuum Bianchi Types I and V in f(R) Gravity. Gen. Relativity Gravit. 42, 2643–2655.

[16].Sotiriou, T. P. and Faraoni, V. (2010): f (R) theories of gravity. Rev. Mod. Phys. 82, 451.

[17].Sen and Aygun (2016): Higher dimensional strange quark matter solutions in self creation cosmology. AIP conference Proceedings. Vol. 1722 No.1 AIP Publishing.

[18].Shamir (2016): Exploring plane-symmetric solutions in f (R) gravity. Journal of Experimental and Theoretical Physics 1222(2016):331-337.

[19].Tripade et.al. (2021): LRS Bianchi type-I cosmological model with quark and strange quark matter in f (R) gravity. Inte. Research Journal of Modernization in Engg. Tec. And Science, Vol.03, Issue 05 May.

[20].TrYilmazet et.al.(2012): Quark and Strange quark matter if f (R) gravity for Bianchi type I and V space times. General Relativity and Gravitation 44.9:2313-2328.

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