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Study of Flow of Herschel-Bulkley Fluid Through an Overlapping Stenosis

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ABSTRACT

In this present paper, we studied about the flow of Herschel-Bulkley fluid through an overlapping stenosis. Most of the people in today's world are much worried about the health-related issues. One among the many diseases that is causing worry to the people is the cardiovascular diseases. The entire human body is controlled by the heart and the blood vessels, which are important in transporting the blood from heart to various parts of body and vice versa, due to the pressure variation developed when the heart extracts and contracts periodically. One of the most common cardiovascular diseases is atherosclerosis that occurs due to the improper flow of blood in human body. In this view, as blood flow is playing an important role in diseases related with heart, many researchers are showing interest in this area. Atherosclerosis is a cardiovascular disease that is caused due to an aberrant growth formed in lumen of an artery. It generally occurs when the human body gets habituated to unhealthy habits such as chewing tobacco, lack of exercises etc. Owing to this the characteristics of blood such as pressure distribution, shear stress on the wall, flow resistance etc. gets affected.

Keywords : MHD, Fluids, Porous Medium, Heat Transfer

I. INTRODUCTION

As stenosis causes severe health related sickness, many researchers have done research on the flow characteristics of blood in the constricted arteries (Lee and Fung [1], Radhakrishnamachary and Srinivas Rao [2]. Blood can be observed as Newtonian or non-Newtonian fluid for mathematical analysis. But Majhi and Nair [3] have taken blood as a non-Newtonian fluid. Single stenosis is taken into consideration in all these investigations, but in general stenosis maybe formed in multiple or irregular or overlapping. The pressure gradient in radial and axial direction of the artery in the presence of overlapping stenosis was analysed by Layek et al., [4]. Srivastava et al., [5] developed mathematical model in the presence of overlapping stenosis by treating blood as a two layered macroscopic two-phase model. With this motivation, the consequences of blood flow through an overlapping stenosis are studied by considering it as steady, incompressible non-Newtonian (Herschel-Bulkley) fluid. Here the arterial wall is rigid and the length of the tube is taken large as compared to the radius, so that the entrance and end wall effects can be discarded.

II. METHODS AND MATERIAL

The geometry of the model for this study is depicted in Figure 1. Here blood isconsidered as steady and incompressible Herschel-Bulkley fluid flowing in circular tubewith an overlapping constriction. The mathematical expression for the geometry is givenby (Layek et al.,[4])

$$h = \frac{R(z)}{Ro} = 1 - 3/2(\frac{\delta}{RoLo^4}[11(z-d)Lo^3 - 47(z-d)^2Lo^2 + 72(z-d)^3Lo - 36(z-d)^4], \qquad d \le z \le d + Lo,$$

$=1$

(1)

Where *(Z), $R_0(Z)$* represent the radius of tube with and without constriction, $r_0$represents radius of plug flow region, L_0represents length of stenosis and *d* represents its location, δ represents the maximum height of the stenosis located at the two positions Z=d+L_0/σ, Z=d+5L_0/σ respectively. $\frac{3\delta}{4}$ is the critical height at Z= d+L_0/2 measured from the origin.

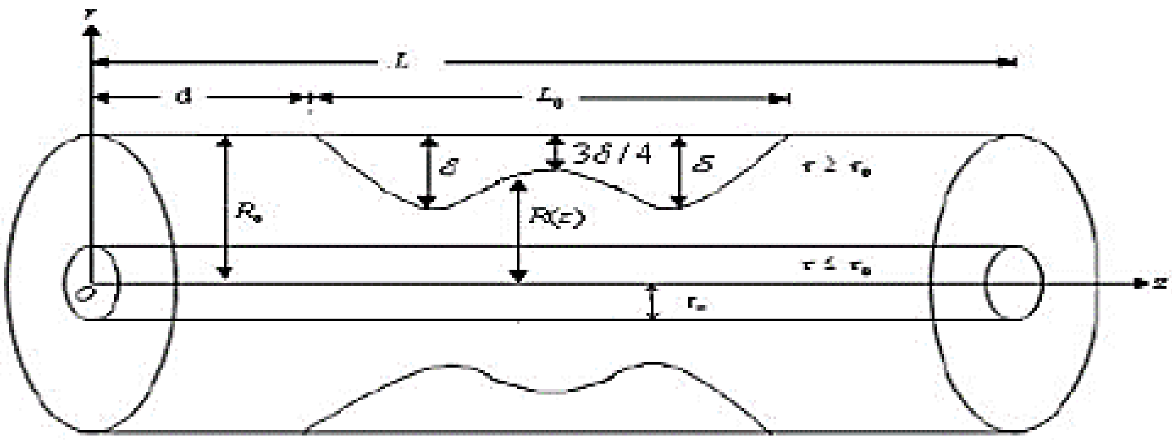

Figure 1: Geometry of a uniform tube with an overlapping stenosis

The equation of momentum is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\frac{\partial p}{\partial z} \qquad (2)$$

Here T_{rz}represents the shear stress and is defined as

$$\tau_{rz} = (\frac{-\partial u}{\partial r})^n + \tau_{0}, \ \text{if} \quad \tau_{rz} \ge \tau_0 \qquad (3)$$

$$\frac{\partial u}{\partial r} = 0, \qquad \text{if } \tau_{rz} < \tau_0 \tag{4}$$

Cylindrical polar co-ordinates *(r, θ, z)* are considered, where zand rare measured along radial and axial directions. 'p' represents the pressure, 'T0' represents the yield stress and 'u' represents the fluid velocity.

The boundary conditions are

(i) $t_{rz}$is finite at r= 0 (5)

(ii) u= 0 , at r= h(z) (6)

Integrating Eq. (2) and applying the boundary conditions (5), (6) the velocity can be calculated as

$$u = \frac{h^{(k+1)}p^k}{2^k(k+1)}\{\left(1-\frac{2\tau_o}{hp}\right)^{k+1} - \left(\frac{r}{h}-\frac{2\tau_o}{hp}\right)^{k+1}\}, \text{ where } r \in [r_0, h] \tag{7}$$

Here, $: P = -\frac{\partial p}{\partial z}, \quad k = \frac{1}{n}$

Making use of the condition (4), in (7)

$$r_0 = 2\tau_0/p \tag{8}$$

Using $\tau_{rz} = \tau_h$ at $r = h,$

$$\frac{r_o}{h} = \frac{\tau_o}{\tau_h} = \tau, 0 < \tau < 1 \tag{9}$$

$$u_p = \frac{h^{(k+1)}p^k}{2^k(k+1)}\left(1-\frac{2r_o}{hp}\right)^{k+1} \qquad \text{Where } r \in [0, r_0] \tag{10}$$

The flow flux $Q$is calculated as

$$Q = 2\left[\int_0^{r_o} u_p r\, dr + \int_{r_0}^{h} urdr\right] \tag{11}$$

After integrating eq. (11), we get

$$Q = [(k+2)(k+3)\left(1-\frac{r}{h}\right)^{k+1} - 2(k+3)\left(1-\frac{r_o}{h}\right)^{k+2} + 2\left(1-\frac{r_o}{h}\right)^{k+3}] \tag{12}$$

where, $A = \frac{h^{(k+3)}p^k}{2^k(k+1)(k+2)(k+3)}$

From eq. (12)

$$\frac{dp}{dz} = -P = \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}} \tag{13}$$

The pressure drop Δp of the constricted artery is obtained by integrating eq. (13), between *z=0 to z=L* as

$$\Delta p = \int_0^L \frac{dp}{dz}\, dz \tag{14}$$

$$\Delta p = \int_0^L \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}}\, dz \tag{15}$$

Making use of the non-dimensional quantities in (15)

$$\bar{z} = \frac{z}{L},\ \bar{\delta} = \frac{\delta}{R_0},\ \bar{R}(z) = \frac{R(z)}{R_0},\ \bar{P} = \frac{P}{\left(\mu UL/R_0^2\right)},\ \bar{\tau_0} = \frac{\tau_0}{\mu\left(\frac{U}{R_0}\right)},\ \bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu\left(\frac{U}{R_0}\right)},\ \bar{Q} = \frac{Q}{\pi R_0^2 U}$$

$$\Delta p = \int_0^1 \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}}\, dz \tag{16}$$

The flow resistance in the constricted artery ƛ, is defined by

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q}\int_0^1 \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}}\, dz \tag{17}$$

The pressure drop in the normal artery Δp_N, is calculated from eq. (16)

$$\Delta p_N = \int_0^1 \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}}\, dz \tag{18}$$

The flow resistance in the normal artery $ƛ_N$ is calculated with the help of eq. (18)

$$\lambda_N = \frac{\Delta p_N}{Q} = \frac{1}{Q}\int_0^1 \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{\{(k+2)(k+3)(1-\tau)^{k+1}-2(1-\tau)^{k+2}(k+2+\tau)\}^{\frac{1}{k}}}\, dz \tag{19}$$

The resistance to the flow

$$\bar{\lambda} = \lambda/\lambda_N \tag{20}$$

Shear stress on the stenotic wall is defined as

$$\tau_h = -\frac{h}{2}\frac{dp}{dz} \tag{21}$$

III. RESULTS AND DISCUSSIONS

In order to explore the effects of various parameters on flow resistance ($\bar{\lambda}$) and wall shear stress (τ_h), these quantities are calculated and given in the equations (20) and (21). For the purpose of numerical computation of the quantities of interest, a through quantitative analysis is performed by taking the following values of the different parameters involved in the present study. The values are d= 0.2, L= 1, L_0=0.2,0.25,0.3; q= 0.1, δ= 0,0.02,0.04, 0.06, 0.08, 0.1 ([5]). Figure 2–11 show the effects of various parameters on resistance to the flow. It can be observed that the resistance to the flow increases with the height of the stenosis (δ).

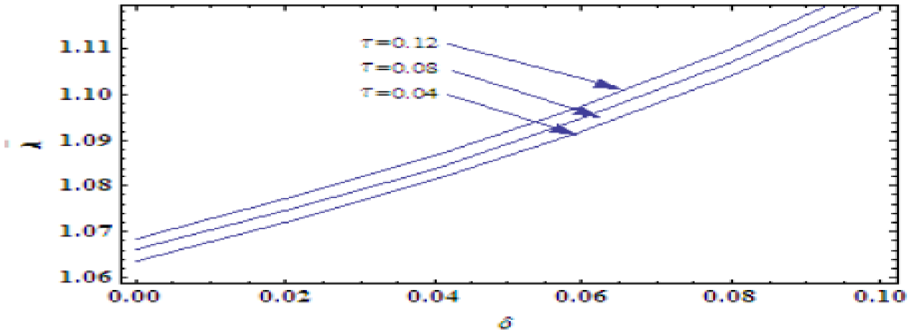

Figure 2: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of Ʈ (d= 0.2, L= 1, L_0= 0.2, q= 0.1, k= 2)

The variations in the flow resistance with stenosis height for various values of stress ratio parameter(T) are depicted in Figure 2 & 3. It is noticed that the flow resistance increases with stress ratio parameter (i.e the ratio between the yield stress and wall shear stress ($\frac{\tau_0}{\tau_h} = \tau$). It is also observed that, the resistance to the flow increases with the height of stenosis.

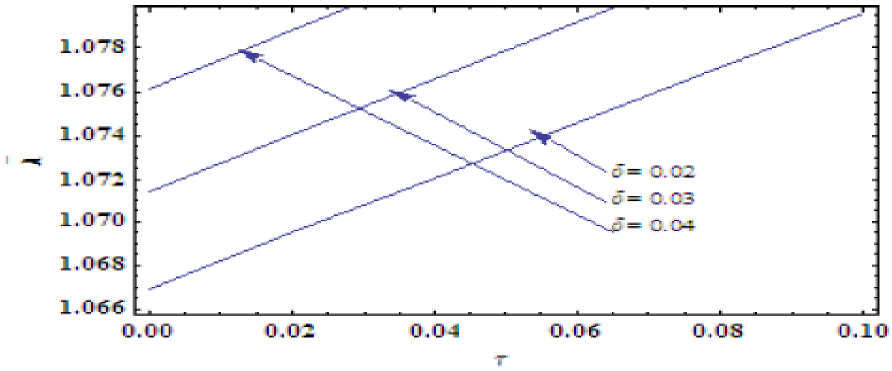

Figure 3: Resistance to the flow ($\bar{\lambda}$) verses stress ratio parameter Ʈ for various values of δ (d= 0.2, L= 1, L_0= 0.2, q= 0.1, k= 2)

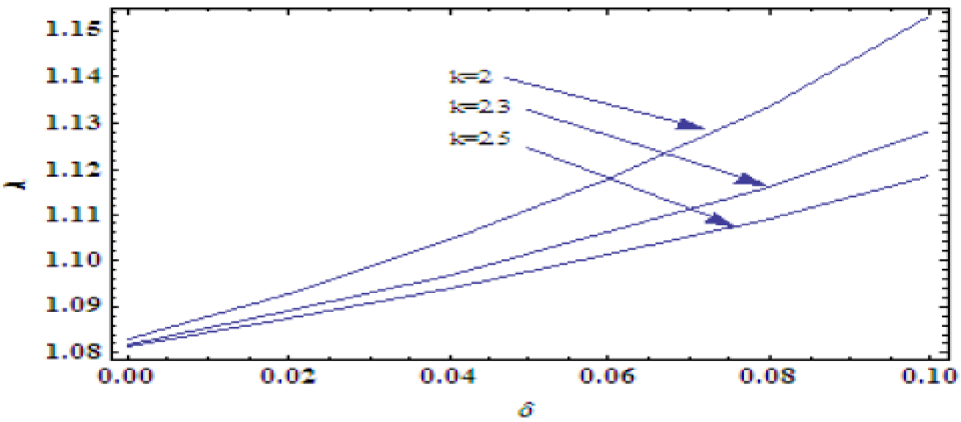

Figure 4: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of k (d= 0.2, L= 1, L_0= 0.2, q= 0.1, Ʈ0= 0.2)

Figure 4 shows that the resistance to the flow increases with power law index (n=1/k). From figure 5 it can be seen that, the resistance to the flow increases with the length of the stenosis (L_0).

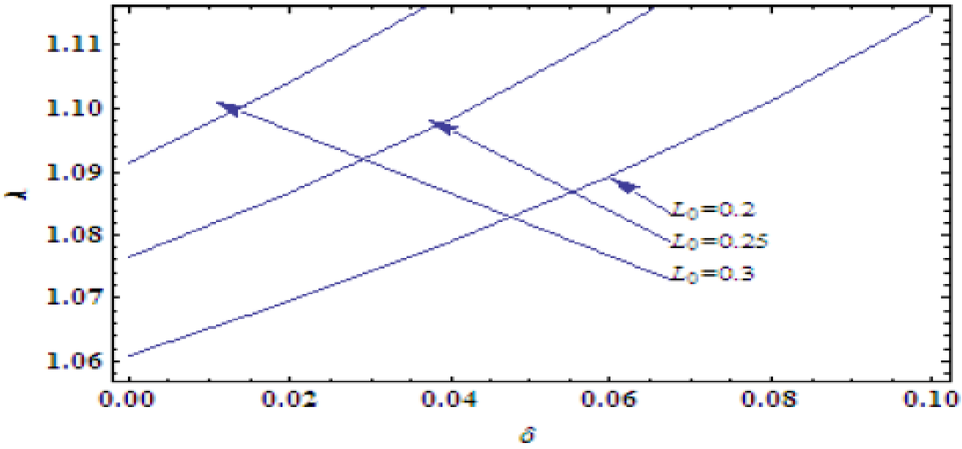

Figure 5: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of L_0 (d= 0.2, L= 1, q= 0.1, Ʈ= 0.02)

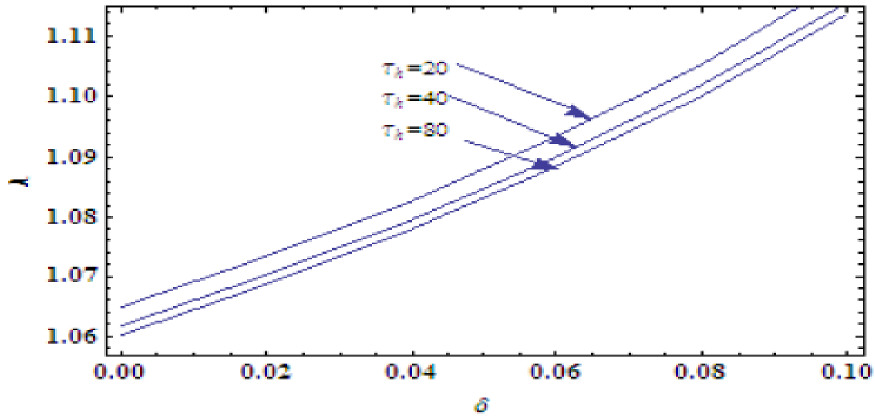

Figure 6: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of $Ʈ_h$ (d= 0.2, L= 1, L0= 0.2, Q= 0.1, Ʈ0= 1, k=2)

Figure 6 depicts the influence of wall shear stress on flow resistance with respect to the height of stenosis. It is noticed that the resistance to the flow decreases with wall shear stress.

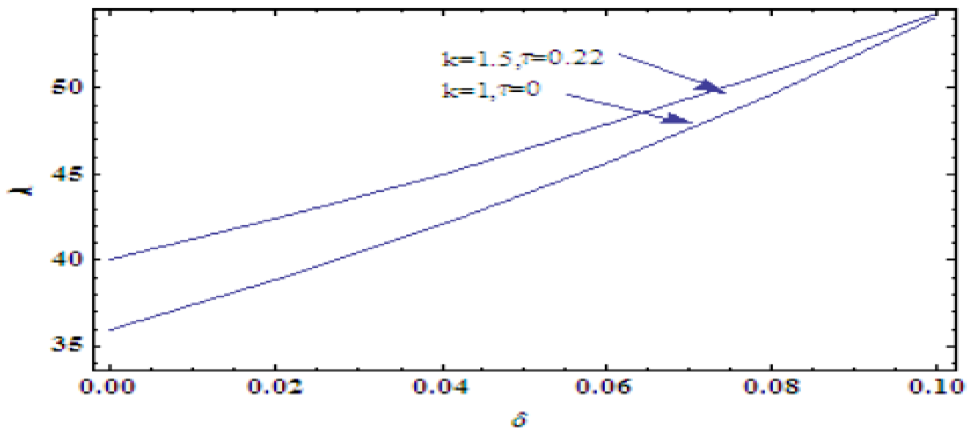

Figure 7: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of Ʈ, k (d= 0.2, L= 1, L_0= 0.2, Q= 0.1)

The yield stress effect on flow resistance is shown in Figure4.8. It is observed that the resistance to the flow increases with the yield stress (τ_0). These results agree with the previous results obtained by Shukla et al. [6], Chaturani and Ponalagusamy [7]. A comparative study of flow resistance for Newtonian, power law and Herschel-Bulkley fluids is represented graphically in Figures 7& 9.

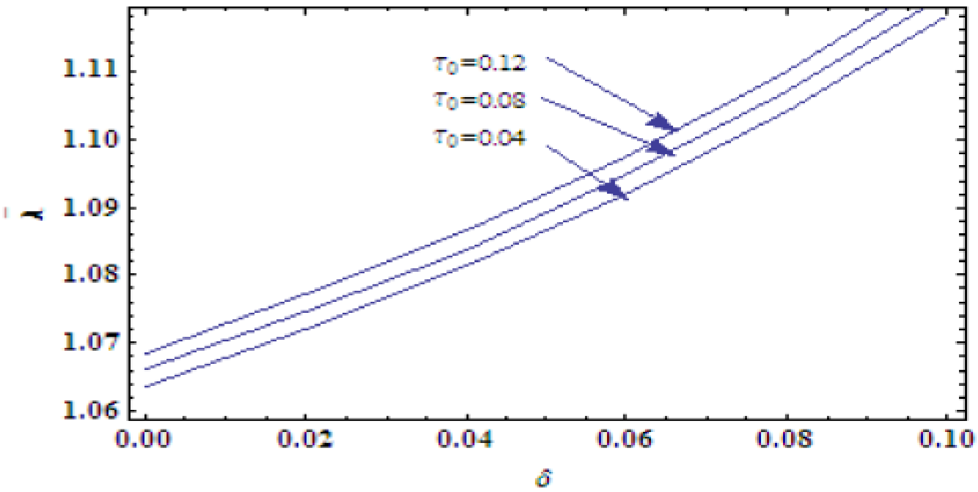

Figure 8: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for various values of τ_0 (d= 0.2, L= 1, L_0= 0.2, Q= 0.1, k=2)

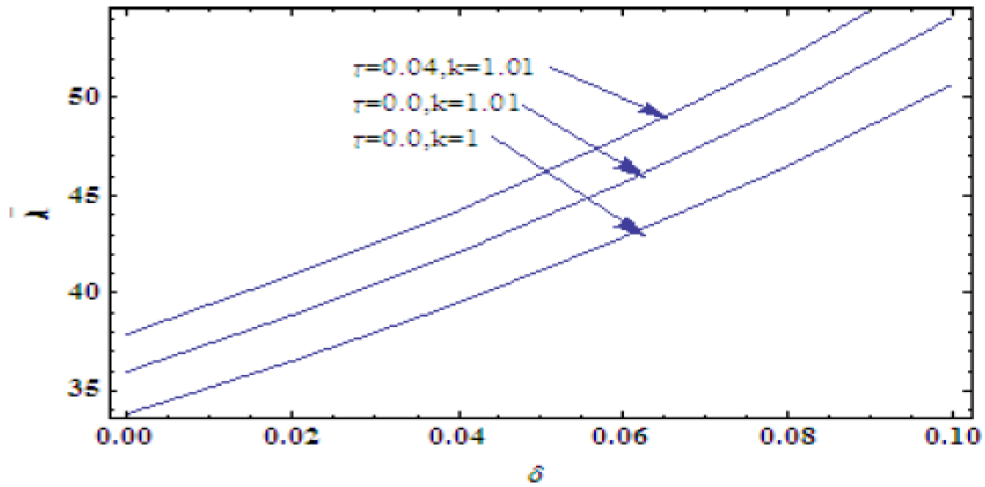

Figure 9: Resistance to the flow ($\bar{\lambda}$) verses height of stenosis δ for different fluids (d= 0.2, L= 1, L0= 1, Q= 0.1)

The resistance to the flow increases with stenosis height, the increase is more in Herschel-Bulkley fluid compared to power law and Newtonian fluids [8-13]. The variation of flow resistance with stenosis height for different values ofradius of plug core region is presented in Figure 10. It is seen that the resistance to the flow increases with the radius of the core region (r_0) [14-21].

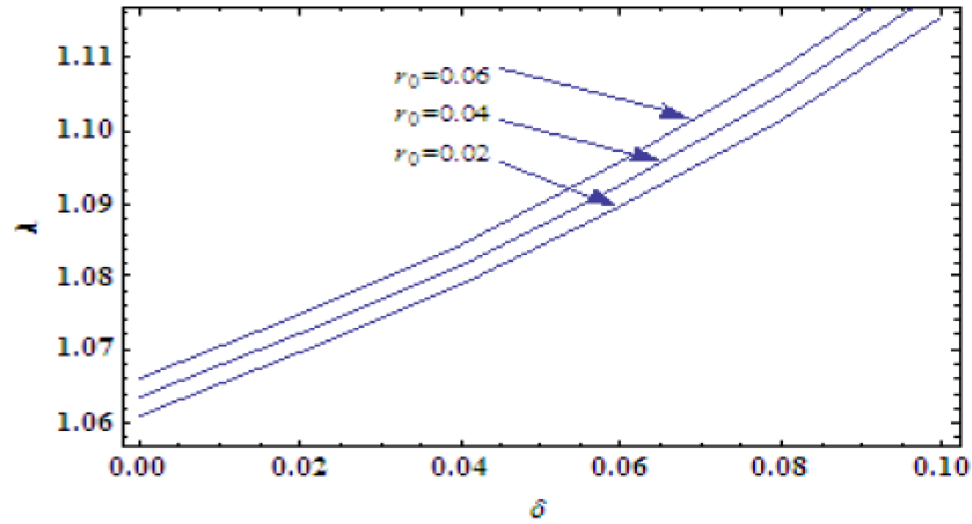

Figure 10: Resistance to the flow ($\bar{\lambda}$)verses height of stenosis δfor various values of r_o (d= 0.2, L= 1, L0= 0.2, Q= 0.1, k=2)

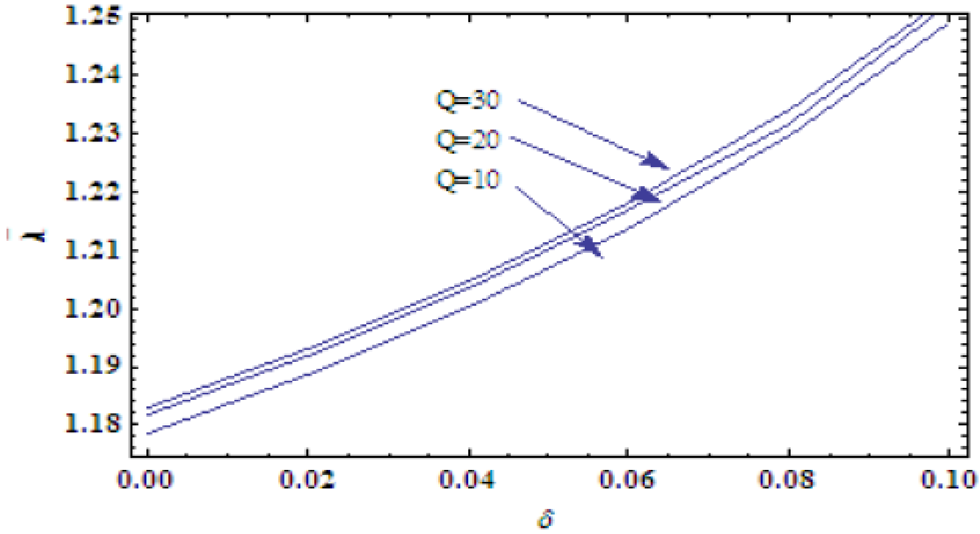

Figure 11: Resistance to the flow ($\bar{\lambda}$) verses δ for distinct Q (d= 0.2, L= 1, L0= 0.2, r_o= 0.2, k=2)

From Figure 11, it is observed that, the resistance to the flow decreases with volumetric flow rate (Q). The Herschel-Bulkely fluid behaves as Newtonian fluid when power law index n= 1 and τ_0= 0 and it acts as a power-law fluid when n ≠ 1 and yield stress τ_0= 0.

IV. CONCLUSION

The conclusions for the Herschel-Bulkley fluid through an overlapping stenosis and also for an inclined tube having uniform cross section are given as: The flow resistance (λ) increases with respect to stenosis height (δ), length of the stenosis, stress ratio parameter(τ), i.e the ratio between the yield stress and wall shear stress (τ0/ τh = τ), power law index, yield stress (τ0) but decreases with the wall shear stress (τh). The resistance of the Herschel-Bulkley fluid is higher than the Newtonian fluid because of yield stress. The flow resistance decreases with inclination (α). The conclusions, by considering the micropolar fluid through a tube with an overlapping stenosis are listed as: The flow resistance increases with stenosis height, coupling number but decreases with the micropolar fluid parameter and length of the tube. The shear stress at the walls increases with stenosis height and coupling number but decreases with the micropolar fluid parameter. The resistance of the flow is more for the non-Newtonian fluid than the Newtonian fluid. Stenosis and post-stenotic dilatation effects on Herschel-Bulkley fluid, micropolar fluid and Jeffrey fluid were studied in chapter 4 and made the following conclusions. The conclusions for Herschel-Bulkley fluid are: The resistance in flow increases with the height, length of the stenosis, yield stress and power law index (n= 1/k), and reduces with the height of the stenotic dilatation. The resistance in flow increases along with stenosis height, and reduces with wallshear stress and stenotic dilatation.

V. REFERENCES

[1]. Lee, J. S. and Fung, Y. C. (1970): Flow in locally constricted tubes at low Reynolds numbers, J. of Appl. Mehanics, 9-16.

[2]. Sankar, D. S.; Ismail, A.I.M. (2009). Two-Fluid Mathematical Models for Blood Flow in Stenosed Arteries: A Comparative Study. Boundary Value Problems, 2009

[3]. G. Radhakrishnamacharya and P. Srinavas Rao, "Flow of a magnetic fluid through a non-uniform wavy tube", Proc. Nat. Acad. Sci. India. 76(A) (2007).

[4]. S.N. Majhi, and V.R. Nair, "Pulsatile flow of third grade fluids under body acceleration-modelling blood flow", Int. J. Engg. Sci., 32 (1994) 839-846.

[5]. Srivastava, V. P., Mishra, S., and Rastogi, R. (2010): Non-Newtonian arterial blood flow through an overlapping stenosis, Applications and Applied Mathematics, 5(1), 225-238.

[6]. G.C. Layek, S. Mukhopadhya and R.S.D. Glora, "Unsteady viscous flow with various viscosity in a vascular tube with an overlapping constriction", Int. J. Engn. Sci., 47(2009) 649-659.

[7]. J.B. Shukla, R.S. Parihar, B.R.P. Rao, "Effects of stenosis on non-Newtonian flow through an artery with mild stenosis", Bull.Math.Biol., 42 (1980) 283-294.

[8]. P. Chaturani, R. Ponnalagar Samy, "Study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases", Biorheol, 22 (1985) 521-531.

[9]. K. Maruthi Prasad and G. Radhakrishnamacharya, "Flow of Herschel-Bulkley fluid through an inclined tube of non-uniform cross-section with multiple stenoses", Arch.Mech., 60(2) (2008) 161-172.

[10]. Devajyoti Biswas and Moumita Paul, "Study of blood flow inside an inclined nonuniform stenosed artery", Int. Journal of Mathematical Archive, 4(5) (2013) 33-42.

[11]. R. Devanathan and S. Parvathamma, "Flow of micropolar fluid through a tube with stenosis", Medical and Biological Engineering and computing, 21(1983) 438-445.

[12]. Kh. S. Mekhemier and M.A. El Kot, "The micropolar fluid model for blood flow through a tapered artery with stenosis", Acta Mechanica Sinica, 24 (2008) 637-644.

[13]. R. Ellahi S.U.Rahman, M. Mudassar Gulzar, S. Nadeem and K. Vafai, "A Mathematical Study of Non-Newtonian Micropolar Fluid in Arterial Blood Flow through Composite Stenosis", Appl. Math.Inf.Sci., 8 (2014) 1567-1573.

[14]. A.C. Eringen, "Theory of micropolar fluids", J. Math. Mech., 16 (1966)1-18.

[15]. J.P. Mynard and P. Nithiarasu. A 1d arterial blood ow model incorporating ventricular pressure, aortic valve and regional coronary ow using the locally conservative galerkin (lcg) method. Communications in Numerical Methods in Engineering, 24:367{417, 2008.

[16]. V.L. Streeter, W.F. Keitzer, and D.F. Bohr. Pulsatile pressure and ow through distensible vessels. Circulation Research, 13(1):3{20, 1963.

[17]. W. Ruan, M.E. Clark, M. Zhao, and A. Curcio. Global solution to a hyperbolic problem arising in the modelling of blood ow in circulatory

systems. Journal of Mathematical Analysis and Applications, 331:1068{1092, 2007.

[18].J.K. Raines, M.Y. Jaffrin, and A.H. Shapiro. A computer simulation of arterial dynamics in the human leg. Journal of Biomechanics, 7(1):77{91, 1974.

[19].B.W. Schaaf and P.H. Abbrecht. Digital computer simulation of human systemic arterial pulse wave transmission: A nonlinear model. Journal of Biomechanics, 5(4):345{364, 1972.

[20].G. Porenta, D.F. Young, and T.R. Rogge. A finite-element model of blood ow in arteries including taper, branches, and obstructions. Journal of Biomechanical Engineering, 108(2):161-7, 1986.

[21].D.F. Young and F.Y. Tsai. Flow characteristics in models of arterial stenoses { ii. Unsteady ow. Journal of Biomechanics, 6(5):547-559, 1973.

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