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Five dimensional Plane Symmetric Bianchi Type-I Cosmological model with Quark and Strange Quark Matter in $f(R,T)$ Theory of Gravity

S. R. Gomkar[1], Jyotsna Jumale[2], R. K. Jumale[3]

[1] Head Department of Mathematics, Janta Mahavidyalaya, Chandrapur, Maharashtra, India

[2]R.S.Bidkar College, Hinganghat , Distt. Wardha, Maharashtra, India

[3]Sant Gadge Maharaj Mahavidylaya Hingna, Distt.Nagpur, Maharashtra, India

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ABSTRACT

In the present paper, we have obtained the solution of five dimensional plane symmetric Bianchi type I model in the presence of quark and strange quark matter in modified $f(R,T)$ theory of gravity. The field equations are solved using special law of variation of Hubble's parameter which is proposed by Berman (1982,83). Also the physical properties have been discussed.

Keywords : Five dimensional plane symmetric Bianchi type-I cosmological model, Quark matter and Strange quark matter, modified $f(R,T)$ theory of gravity.

I. INTRODUCTION

Modification of general relativity are attracting much attention in recent years to explain late time acceleration and dark energy. There had been several modification of general relativity to provide natural gravitational alternative for dark energy. Universe has 4% ordinary matter, 20% dark matter and 76% dark energy. Recently Harko et. al.(2011) proposed the modified $f(R,T)$ theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the energy momentum tensor T_{ij}. Many authors (Pradhan et. al. (2014), Naidu (2013), Yadav (2013), Shamir et. al (2012), have obtained exact solution of Bianchi types I and V models in $f(R,T)$ gravity. Adaho(2012) has investigated LRS Bianchi-type-I cosmological model with perfect fluid in $f(R,T)$ gravity. Hiwarkar et.al. (2017) have study perfect fluid Bianchi type-III cosmological model in $f(R,T)$ gravity. Jumale et. al.(2020) have studied five dimensional specially homogeneous anisotropic Bianchi type-III cosmological model in $f(R,T)$ theory of gravity. Jumale et. al.(2019) studied five dimensional Bianchi-type-V String Cosmology in $f(R,T)$ theory of Gravity. S. R. Gomkar et. al.(2022) have studied the five dimensional Bianchi-type-III String cosmological model in $f(R,T)$ theory of Gravity with Bulk Viscous fluid. Pawar et.al (2015) studied cosmological models filled with a perfect fluid in $f(R,T)$ theory of gravity. Inhomogeneous plane-symmetric model was first studied by Taub (1951,56), and Tomimura (1978), Khadekar and Shelote (2012) have obtained higher dimensional cosmological model in the presence of quark and strange quark matter. Agrawal et. al (2017) have obtained plane symmetric Cosmological Model with Quark and Strange Quark Matter in $f(R,T)$ modified theory of gravity.

The phase transition of the universe took place from quark glucon plasma to hydrogen gas, which is referred as 'quark hadrons phase'. Strange quark matter is developed with an equation of state (EoS) $p = \frac{(\rho - 4B_c)}{3}$ based on the phenomenological bag model of quark matter in which quark constraint is described by an energy term proportional to the volume. Here B_c represent the bag constant, p and ρ are the thermodynamic pressure and energy density of the quark matter respectively. Quarks are considered as degenerate Fermi gas, which exist only in a region of space providing with a vacuum energy density B_c called as the bag constant. In the bag model, it is assumed that quarks are mass-less and non-interacting. With quark pressure $p_q = \frac{\rho_q}{3}$ where ρ_q is the quark energy density. Quark matter studied in general relativity using various assumptions. Thus in this paper, we wish to investigate five dimensional plane symmetric Bianchi type-I cosmological model with quark and strange quark matter in $f(R,T)$ theory of gravity.

The paper is organized as follows : In section-2, we briefly give the gravitational field equation in $f(R,T)$ modified theory of gravity. Section-3 deals with the solutions of the field equations in $f(R,T)$ gravity. Section-4, gives the physical parameter of the model and Section-4a studied quark matter for plane symmetric space-time, Section-4b gives strange quark matter for plane symmetric space-time and the last section gives the conclusion and result.

II. Five dimensional field equations in $f(R,T)$ theory of Gravity

The gravitational field equations of $f(R,T)$ gravity is given by

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_i) f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad (1)$$

where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T = \frac{\partial f(R,T)}{\partial T}$, $\theta_{ij} = -2T_{ij} - pg_{ij}$ and ∇_i denotes the covariant derivate.

The stress energy tensor is given by

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij}, \qquad (i, j = 1,2,3,4,5) \quad (2)$$

satisfying the equation of state

$$p = \in\rho, \; 0 \le \in \le 1. \quad (3)$$

we obtain for the variation of stress-energy of perfect fluid the expression

$$\theta_{ij} = -2T_{ij} - pg_{ij}. \quad (4)$$

The energy momentum tensor for quark matter is given by

$$T_{ij}^{(Quark)} = (p+\rho)u_i u_j - pg_{ij} = diag(-p,-p,-p,-p,\rho) \quad (5)$$

where $\rho = \rho_q + B_c$ is the quark matter total energy density and $p = p_q - B_c$ is the quark matter total pressure and u_i is the five velocity vector such that $u_i u^i = 1$.

The field equations depends on the physical nature of the matter field(through the tensor θ_{ij}) Therefore, we obtain several theoretical models for different choice of $f(R,T)$ depending on the nature of the matter source.

Recently Reddy et.al.(2012a,2012b) and Naidu et. al. (2013) have studied the cosmological models, assuming $f(R,T) = R + 2f(T)$. Ahmad & Pradhan studied Bianchi type-V cosmological model in $f(R,T)$ gravity by considering $f(R,T) = f_1(R) + f_2(T)$.They have assumed perfect fluid as a source of matter to describe the physical consequences of early universe. In this paper, we choose $f(T) = \mu T$ where μ is constant.

The field equation (1) can be written as

$$R_{ij} - \frac{1}{2} g_{ij} R = 8\pi T_{ij} + 2f'(T)T_{ij} + [f(T) + 2pf'(T)]g_{ij} . \tag{6}$$

III. Solutions of Bianchi type-I cosmological model in V_5

The plane symmetric Bianchi type-I space-time in V_5 is given by

$$ds^2 = dt^2 - A^2(t)(dx^2) - B^2(t)(dy^2 + dz^2 + du^2) \tag{7}$$

where A and B are cosmic scale factors. The corresponding Ricci scalar is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{3\ddot{B}}{B} + \frac{3\dot{A}\dot{B}}{AB} + \frac{3\dot{B}^2}{B^2}\right]. \tag{8}$$

The spatial volume of this model is given

$$V = (AB^3). \tag{9}$$

The average scale factor of the anisotropic Bianchi type-I space-time in V_5

$$a = (AB^3)^{1/4} \tag{10}$$

The Hubble's parameter is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left(\frac{\dot{A}}{A} + \frac{3\dot{B}}{B}\right) \tag{11}$$

where dot denotes derivative with respect to t. Also

$$H = \frac{1}{4}(H_1 + H_2 + H_3 + H_4) \tag{12}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = H_4 = \frac{\dot{B}}{B}$ are directional hubble's parameter in the direction of x,y,z and u direction respectively.

The expansion scalar θ and shear scalar σ are defined as follows

$$\theta = u^i_{;i} = 4H = \frac{\dot{A}}{A} + \frac{3\dot{B}}{B}, \tag{13}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{A}}{A} - \frac{B}{B}\right)^2. \tag{14}$$

The mean anisotropic parameter is

$$\bar{A} = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right) = \frac{\sigma^2}{2H^2}. \tag{15}$$

The energy momentum tensor for quark matter is taken as

In a co-moving coordinate system, the field equations (6) for the anisotropic Bianchi type-I space time (7) is given by

$$\frac{3\ddot{B}}{B} + \frac{3\dot{B}^2}{B^2} = -p_q + B_c + \lambda(\rho_q + 5B_c - 4p_q) \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = -p_q + B_c + \lambda(\rho_q + 5B_c - 4p_q) \tag{17}$$

$$\frac{3\dot{A}\dot{B}}{AB} + \frac{3\dot{B}^2}{B} = \rho_q + B_c + \lambda(\rho_q + 9B_c) \tag{18}$$

To solve these field equations, law of variation of Hubble parameter proposed by Berman (1983,92), which gives the constant value of deceleration parameter is given by

$$q = \frac{-a\ddot{a}}{\dot{a}^2}. \tag{19}$$

The average scale factor and average Hubble parameter of Bianchi type-I model becomes

$$H = la^{-n} = l(AB^3)^{-\frac{n}{4}}, \tag{20}$$

where l and n are constants. Using equation (12)

$$\dot{a} = la^{1-n} \text{ and } \ddot{a} = l^2(1-n)a^{1-2n}.$$

Now substituting the values of $\dot{a}$ and $\ddot{a}$ in equation (19), we get

$$q = n - 1. \tag{21}$$

The positive value of q gives standard deceleration model while negative value gives inflection or the accelerating universe of the model.
The equation (19) gives

$$a = (k_1 t + k_2)^{\frac{1}{1+q}}, \quad q \neq -1 \tag{22}$$

where $k_1 \neq 0$ and k_2 are constant of integration.

To obtain the solution, now assume that shear scalar σ is proportional to the expansion scalar θ, it gives relation between the scale factors A and B

$$A = B^n \tag{23}$$

where n is arbitrary constant, $n \neq 1$. If $n = 1$, the model becomes isotropic model otherwise it becomes anisotropic.

$$A = (k_1 t + k_2)^{\frac{4}{(n+3)(q+1)}} \quad \text{and} \quad B = (k_1 t + k_2)^{\frac{4n}{(n+3)(q+1)}}, \quad q \neq -1 \quad \& \quad n \neq -3 \quad . \tag{24}$$

Thus the metric (7) becomes

$$ds^2 = dt^2 - (k_1 t + k_2)^{\frac{8}{(n+3)(q+1)}}(dx^2) - (k_1 t + k_2)^{\frac{8n}{(n+3)(q+1)}}(dy^2 + dz^2 + du^2). \tag{25}$$

IV. Physical parameters for Bianchi type-I cosmological model in V_5

The spatial volume v of the Bianchi type-I model is given by

$$V = (k_1 t + k_2)^{\frac{4}{(q+1)}} \tag{26}$$

Using equation (24), the directional Hubble parameter are given by

$$H_1 = \left(\frac{4n}{(q+1)(n+3)}\right)\left(\frac{k_1}{k_1 t + k_2}\right), \quad H_2 = H_3 = H_4 = \left(\frac{4}{(q+1)(n+3)}\right)\left(\frac{k_1}{k_1 t + k_2}\right). \tag{27}$$

The generalized Hubble's parameter H is given by

$$H = \left(\frac{4}{(q+1)}\right)\left(\frac{k_1}{k_1 t + k_2}\right). \tag{28}$$

The expansion scalar θ, shear scalar σ and mean anisotropic parameter $\overline{A}$ of the Bianchi type-I model is given by

$$\theta = 4H = \left(\frac{16}{(q+1)}\right)\left(\frac{k_1}{k_1 t + k_2}\right), \tag{29}$$

$$\sigma^2 = \left(\frac{6(n-1)^2}{(k_1t+k_2)^2} \frac{c_1^2}{(q+1)^2(n+3)^2} \right), \tag{30}$$

$$\bar{A} = \frac{3}{16} \frac{(n-1)^2}{(n+3)^2}. \tag{31}$$

4a. Quark matter for plane symmetric Bianchi type-I model in V_5

With the help of EoS parameter (for $\in = \frac{1}{3}, p_q = \in \rho_q = \frac{\rho_q}{3}, \quad \rho_q = 3p_q$) and using equation (24), then we get the pressure and energy density of the strange quark matter as

$$p_q = \frac{1}{4(1+\lambda)} \{ \frac{4\eta}{(c_1t+c_2)^2} [(4n(1-n) + (n+2)(n+3)(1+q)] - 4\lambda B_c \} \tag{32}$$

where $\eta = \frac{c_1^2}{(n+3)^2(1+q)^2}$

$$\rho_q = \frac{3}{4(1+\lambda)} \{ \frac{4\eta}{(c_1t+c_2)^2} [(4n(1-n) + (n+2)(n+3)(1+q)] - 4\lambda B_c \}, \tag{33}$$

Using equation (24) in (8), we get Ricci scalar

$$R = \{ \frac{-8\eta}{(c_1t+c_2)^2} [(4(n^2+3n+6) - (n+3)^2(1+q)] \}. \tag{34}$$

4b. Strange Quark matter for plane symmetric Bianchi type-I model in V_5

With the help of EoS parameter and using equation (24), then we get the pressure and energy density of the strange quark matter as follows

$$p = \frac{1}{4(1+\lambda)} \{ \frac{4\eta}{(c_1t+c_2)^2} [(4n(1-n) + (n+2)(n+3)(1+q)] - 4\lambda B_c \} - B_c \tag{35}$$

and $$\rho = \frac{3}{4(1+\lambda)} \{ \frac{4\eta}{(c_1t+c_2)^2} [(4n(1-n) + (n+2)(n+3)(1+q)] - 4\lambda B_c \} + B_c \tag{36}$$

V. CONCLUSION

The five dimensional modified Einstein's field equations in $f(R,T)$ theory of gravity are very difficult to find their solutions without taking any assumptions. The assumption is that the expansion scalar θ is proportional shear scalar σ (i.e $A = B^n$, where A and B are metric coefficients and n is any arbitrary constant) and second assumption is the power law relation between F and a is used to find the solution. Some physical quantities such as expansion scalar θ , shear scalar σ^2 are obtained.

The physical parameters $H_1, H_2, H_3, H_4, \theta$ and σ are the functions of time. As cosmic time tends to infinity (i.e, $t \to \infty$) then these parameters tends to zero but cosmic time $t = -\frac{c_2}{c_1}$ then these parameters tends to ∞ except the spatial volume.

The spatial volume of this model is zero when $t = -\frac{c_2}{c_1}$.

When $q < -1$ then the model is expanding with big bang singularity at $t = -\frac{c_2}{c_1}$.

The pressure p_q and density ρ_q for quark matter are finite at $t = 0$ and both become zero at $t \to \infty$.

The pressure p and density ρ for string quark matter are same as quark matter.

When $q > -1$ at $t = 0$ the model has constant volume and increases with increase in time and become infinite at $t \to \infty$.

The pressure p_q and density ρ_q for quark matter are finite at $t = 0$ and become zero at $t \to \infty$.

The pressure p and density ρ for string quark matter are same as that of quark matter.

The mean anisotropy parameter $\overline{A}$ is zero for $n = 1$ then model becomes isotropic. The mean isotropic parameter is constant throughout the evolution of universe as it does not depend on the cosmic time.

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