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Big Data Analytics Approaches to your Business Enhancement Strategies

Mayur P Deshmukh[1], Manjusha M Deshmukh[2]
Asst. Prof, Department of Computer Science, Vishwakarama University, Survey No 2,3,4 Laxminagar, Kondhwa (Bk.) Pune, India
Researcher, Dr. B. A. M. University, Chatrapati Sambhaji Nagar, (MS) India

ABSTRACT

In this article we will introduce what Big Data is and the applications it can provides to your business. We will provide some technical details on how to adopt it into your business [1]. This study aims at mapping the massive knowledge landscape in Asian country and scrutinizing the challenges and opportunities in it. The blueprint of size, rate of growth of this sector and infrastructural challenges can enhance the data on Indian huge knowledge Landscape. As India's huge knowledge landscape continues to be growing and is facing infrastructural and policy level challenges, issue analysis methodology is ideal for analyzing the state of the art, challenges and opportunities [2].

Keywords : Big Data, Data Protection, Policy, Issue Analysis

I. INTRODUCTION

'Big Data' is the application of specialized techniques and technologies to process very large sets of data. These data sets are often so large and complex that it becomes difficult to process using on-hand database management tools. Examples include web logs, call detail records, medical records, military surveillance, photography archives, video archives and large-scale e-commerce [1]. By 'very large' we're talking about datasets that require at least one terabyte – if not hundreds of petabytes – of storage. (Note that 1 petabyte = 1024 terabytes!). Facebook is estimated to store at least 100 petabytes of pictures and videos alone. Big data is first and foremost, data; an elusive, yet valuable, corporate asset quite unlike any other that needs to be properly managed.

Big Data is defined from the McKinsey Global report from 2011: Big Data is data whose scale, distribution, diversity, and/or timeliness require the use of new technical architectures and analytics to enable insights that unlock new sources of business value [3]. McKinsey's definition of Big Data implies that organizations will need new data architectures and analytic sandboxes, new tools, new analytical methods, and an integration of multiple skills into the new role of the data scientist. Figure 1 highlights several sources of the Big Data deluge.

Fig 1. Highlighting several sources of Big Data deluge

The rate of data creation is accelerating, driven by many of the items in Figure 1. Social media and genetic sequencing are among the fastest-growing sources of Big Data and examples of untraditional sources of data being used for analysis.

Unique characteristics of data:

- It can be copied perfectly at very low cost
- It can easily be combined with other data to uncover valuable insights
- It can be used by more than one person at the same time

By managing big data effectively, businesses are better able to capitalize on its value. Data is everywhere. In fact, the amount of digital data that exists is growing at a rapid rate—in fact, more than 2.7 zetta bytes of data exist in today's digital universe, and that is projected to grow to 180 zetta bytes in 2025[3][4]. All this data—from your photos to the Fortune 500's financials—has only recently begun to be analysed to tease out insights that can help organizations improve their business. That's why more organizations are seeking professionals who can make sense of all the data. The following fig.2 shows network connectivity for data transmission.

II. Big Data Big Money

Big data spending is expected to reach $20 billion by 2016. The reason is simple; big data can dramatically improve productivity.

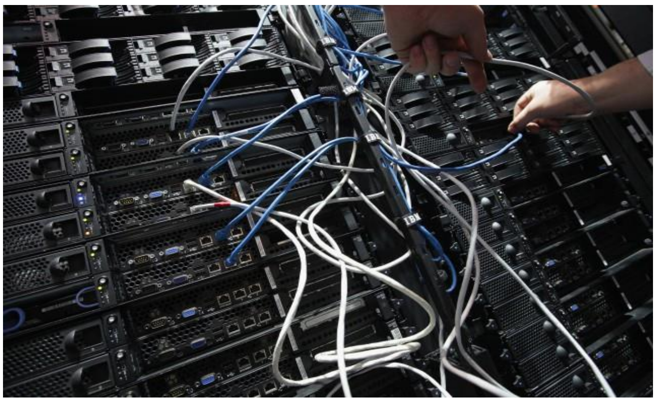

Fig. 2. Network connectivity for data transmission

According to a detailed study conducted by the McKinsey Global Institute, the potential value of big data for the US health sector alone could amount to more than $300 billion annually [5].

In the banking industry, big data is being used to target consumers with “right time” offers. These offers are informed by the large volumes of data the bank already has on each consumer, such as their spending habits.

Citigroup, for example, monitors credit-card transactions and uses this information to send text messages to customers offering them special deals. The Economist reports: “A customer buying clothes around lunchtime, for example, might be offered a discounted meal at a nearby restaurant.”

III.Privacy and BIG DATA

The successful adoption of big data into an organization is by no means simple and can be a steep learning curve. It also presents some obvious challenges around privacy. By capturing and combining big data sets, an organization is able to create an extremely detailed profile of an individual (e.g. sites visited, links clicked, searches made, location of cell phone calls). Regulations that balance productivity gains with privacy protection are yet to be developed.

From a commercial or competitive perspective, organizations also face the challenge of sharing highly confidential big data; things like financial statements, patents, trade secrets and intellectual property. This sensitive data must be securely stored and made available to those who need it.

New innovations, like virtual data rooms, are making this task easier. Virtual data rooms allow authorized individuals to review confidential data and documents in a secure online space. They are essential for data sharing in matters of due diligence, and are widely used for mergers and acquisitions, litigation, bankruptcies, and fundraising.

Companies that implement a comprehensive data security system and have good privacy policies in place, which protect against the unauthorized disclosure of sensitive information, will be in the best position to adopt big data.

IV.Reasons to Accept BIG DATA

By adopting big data techniques into their business operations, organizations are able to:

1. Stop wasting data consummation

An organization’s ability to produce data greatly exceeds its ability to store and manage it. Think of all the data streaming from sensors embedded in devices, from items that contain RFID tags, or from employees with smartphones generating location-based data, videos, and e-mail. Rather than wasting this ‘data consummation,’ big data enables the best of this information to be collective with other data, often producing unexpected new value for the organization.

2. Save time and money

Even when data is captured and saved, it is typically stored in disconnected silos. This increases the amount of time it takes employees to find information. By accepting big data procedures and technologies that can connect data sets together, employees can usually find what they are looking for instantaneously.

3. Improve performance :

Big data enables information from both inside and outside the organization to be combined, so that key performance indicators can be developed and acted upon. The Startup Gnome project [6], for example, collects information about startups by offering them a free data-driven benchmarking tool. Investors then use this data to calculate risk and investment feasibility.

4. Improve Product Offerings

Data from a variety of sources can be combined to improve existing products. For example, BusinessWeek recently reported how manufacturer John Deere is combining GPS data with sensor data from onboard tractors. These tractors can be operated remotely, and can monitor crop yields while they work. Crop harvest data is later used to determine precise amounts of fertilizer to deliver, according to the location in the field.

5. Segment Groups within Larger Populations

Big data techniques can combine and analyse data from a number of sources and segment specific sub-groups within larger populations. For example, Big Data analytics provider Medio Systems [7], offers a module appropriately named "Customers", which "provides automated audience segmentation to help increase customer engagement and monetization capabilities."

6. Improve decision making

Big data is a powerful means for making informed decisions. However, as big data technologies continually improve, there remains a shortage of skilled professionals who can take full advantage of these technologies. In an article titled "Improving Decision Making in the World of Big Data", Forbes magazine reports that for every one manager with big data skills, there will be ten positions left vacant in 2013[8].

7. Renovate

Big Data diminishes the need to rely on preconceived ideas or assumptions, by enabling innovators to analyse and experiment using real data, in real time 2018[10]. Ford Motors, for example, is emerging as a leading data-driven company, using big data techniques to better understand how their customers are using their vehicles.

"Our manufacturing sites are all very well instrumented. Our vehicles are very well instrumented. They're closed loop control systems. There are many, many sensors in each vehicle until now, most of that information was just in the vehicle, but we think there's an opportunity to grab that data and understand better how the car operates and how consumers use the vehicles. We can then feed that information back into our design process and help optimize the user's experience in the future as well", said John Grinder from Ford Research.

V. CONCLUSION

A smart phone these days has additional computing power than all of independent agency once it place a person on the moon in 1969. There square measure 5 billion telephone users and 4000 tweets generated per second. There square measure of over 900 million Facebook users either sharing knowledge, guessing alternative knowledge of People's. This unstructured knowledge is not any longer controlled by organizations; it's in people's hands. The problem is the way to mine, store and transfer, refine, synthesize, and build secure systems that use this knowledge. Big knowledge presents a gorgeous nevertheless alarming image of the longer term for world. The gorgeous side is that it provides United States of America a wider perspective and newer discoveries that successively can modification our life, work, and mental attitude. There is conjointly associate in nursing absence of a legal framework on problems related to knowledge manipulation at the side of the growing fields of information mental image, prophetical and link analyses, and assortment methods. The iniquitousness of huge knowledge needs the proper combination of individuals, tools and technology to leverage its unlimited potential. In these circumstances Asian country conjointly desires info Security Policy for enjoyment of massive knowledge., i.e. both left-justified and right-justified.

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Understanding Fractional Calculus : Theory, Applications, and Advances

Syeda Farra Fatima Nahri
Department of Mathematics, Milliya Arts Science and Management Science College, Beed, India

ABSTRACT

Fractional calculus, a branch of mathematical analysis extending the concepts of differentiation and integration to non-integer orders, has emerged as a powerful tool with diverse applications across various scientific and engineering disciplines. This paper aims to provide a comprehensive understanding of fractional calculus, encompassing its theoretical foundations, practical applications, and recent advances in the field. The introduction sets the stage by tracing the historical development of fractional calculus, highlighting its relevance in addressing complex problems and motivating its study. We delve into the fundamentals of fractional calculus, elucidating the intricacies of fractional derivatives and integrals, while emphasizing the significance of fractional order in the mathematical framework. Mathematical foundations, including the Riemann-Liouville and Caputo derivatives, are explored along with an examination of fractional integral transforms such as Laplace and Fourier transforms. The paper meticulously examines the wide array of applications of fractional calculus in science and engineering. From physics to biology, fractional calculus proves invaluable in modelling and solving real-world problems, offering a unique perspective that conventional calculus falls short of capturing. Moreover, we investigate numerical methods and computational techniques tailored for solving fractional differential equations, addressing the challenges associated with these complex problems.
Keywords: Fractional calculus, Differentiation, Integration, Fourier transforms.

I. INTRODUCTION

Fractional calculus is an extension of traditional calculus that deals with derivatives and integrals of non-integer orders. While conventional calculus primarily involves operations on functions with integer-order derivatives and integrals, fractional calculus broadens this scope to include fractional-order derivatives and integrals, allowing for a more nuanced understanding of dynamic systems and processes. The roots of fractional calculus can be traced back to the work of mathematicians such as Leibniz, Liouville, and Riemann. The concept of non-integer differentiation and integration began to take shape in the 17th and 18th centuries, but it wasn't until the 19th century that significant progress was made. The German mathematician Gottfried Wilhelm Leibniz, one of the co-founders of calculus, hinted at the possibility of fractional derivatives in his correspondence with L'Hôpital. However, it was the French mathematician Joseph Liouville in the mid-19th century who made substantial contributions to the field, defining what is now known as the Riemann-Liouville fractional integral. The German mathematician Bernhard Riemann introduced the Riemann-Liouville fractional integral in the context of his work on number theory. This formulation laid the groundwork for the development of fractional calculus as a distinct branch of mathematics. Although fractional calculus had periods of relative obscurity, it experienced a resurgence in the latter half of the 20th century. This renewed interest was fuelled by its applications in diverse scientific and engineering disciplines, ranging from physics and biology to control theory and signal processing. Today, fractional calculus stands as a mature and vibrant field, continuously evolving with advancements in theoretical foundations, computational techniques, and applications. This paper aims to provide a comprehensive overview of fractional calculus, exploring its theoretical underpinnings, practical applications, and recent advances.

Fractional calculus provides a robust framework for modelling complex physical phenomena, such as anomalous diffusion, viscoelastic materials, and fractal structures. Its application allows for a more accurate representation of non-local and memory-dependent processes. In the study of wave propagation phenomena, fractional calculus offers enhanced modelling capabilities, enabling a more nuanced description of wave behaviour in heterogeneous media, as seen in seismology and acoustics. Fractional calculus plays a pivotal role in modelling biological systems, where anomalous diffusion and non-local interactions are prevalent. Applications include the study of drug diffusion in tissues, neural dynamics, and bio-transport phenomena. Fractional calculus aids in the enhancement of medical imaging techniques by providing improved algorithms for image reconstruction, noise reduction, and feature extraction, contributing to advancements in diagnostic imaging. Fractional-order controllers offer advantages over their integer-order counterparts in terms of increased flexibility and improved performance. They find applications in fields such as robotics, aerospace, and process control. Fractional calculus techniques enhance signal processing algorithms, facilitating the analysis and processing of non-stationary signals, as well as the extraction of hidden patterns in complex data sets. The modelling of mechanical systems involving viscoelastic materials and damping effects benefits significantly from fractional calculus, providing a more. Fractional calculus proves valuable in modelling stochastic processes in finance, offering a more realistic representation of asset price dynamics and improving the accuracy of financial derivatives pricing models.

The application of fractional calculus in risk assessment and management provides a more comprehensive understanding of non-linear dynamics in financial markets, contributing to more robust risk mitigation strategies. Fractional calculus aids in modelling transport phenomena in porous media, groundwater flow, and pollutant dispersion, offering insights into environmental processes with long-range dependencies. In climate science, fractional calculus contributes to the development of models that capture the intricate dynamics of climate systems, particularly in the representation of memory effects and feedback mechanisms. The interdisciplinary nature of fractional calculus underscores its importance as a versatile mathematical tool, facilitating a deeper understanding of complex phenomena across diverse scientific and engineering disciplines. As technology advances and research in fractional calculus progresses, its impact on these fields is expected to grow, opening new avenues for innovation and problem-solving.

Mathematical Foundations:

Fractional calculus involves two main operators, the Riemann-Liouville fractional derivative and the Caputo fractional derivative. These operators form the mathematical foundations of fractional calculus, providing tools to describe non-integer order derivatives and integrals.
Riemann-Liouville Fractional Derivative:
The Riemann-Liouville fractional derivative is a historical and widely used formulation in fractional calculus. For a function f(t), the Riemann-Liouville fractional derivative ($D^{\alpha}f(t)$) of order α with $0 < \alpha < 1$ is defined as:

$$D^{\alpha}f(t)=\Gamma 1/(1-\alpha)d/dt\int(t-\tau)^{-\alpha}f(\tau)d\tau$$

Here, $\Gamma(\cdot)$ is the gamma function, and $(t-\tau)^{-\alpha}$ is the kernel that characterizes the fractional order. This definition reflects the historical roots of fractional calculus and provides a way to extend the concept of derivatives to non-integer orders.
Properties of Fractional Derivatives:
Both the Riemann-Liouville and Caputo derivatives exhibit unique properties:
- Non-locality: Fractional derivatives involve integration over a range, capturing information from the entire history of the function.
- Memory Effects: Fractional derivatives exhibit memory effects, meaning the rate of change depends on past values of the function.
- Relationship to Integer-Order Derivatives: As the order α approaches 1, the fractional derivative converges to the traditional first derivative.
Understanding these mathematical foundations is crucial for effectively applying fractional calculus in various scientific and engineering problems. The choice between Riemann-Liouville and Caputo derivatives depends on the specific characteristics of the problem and the desired mathematical properties of the solution.

Numerical Methods and Computational Techniques:
Solving fractional differential equations (FDEs) numerically is a challenging yet essential task, given the prevalence of fractional calculus in modelling real-world phenomena. Various numerical methods have been developed to address this challenge, providing efficient tools for approximating solutions to FDEs. The choice of method depends on the specific characteristics of the problem at hand. Here, we provide an overview of some commonly used numerical methods for solving fractional differential equations. The Grünwald-Letnikov discretization is a straightforward approach for approximating fractional derivatives.

Caputo Difference Operator:
The Caputo difference operator is commonly used when dealing with initial value problems involving fractional derivatives. This method is particularly suitable when initial conditions are involved, as it naturally incorporates the initial values into the numerical approximation. The Caputo difference operator, denoted by Δ^{α} is a fractional difference operator used in fractional calculus. Fractional calculus extends the concepts of differentiation and integration to non-integer orders. The Caputo difference operator is particularly useful for defining fractional derivatives in a way that is compatible with initial conditions.
For a function f(t) defined on the interval [a, b] and for alpha > 0, the Caputo fractional derivative of order α is given by:

$$\Delta^{\alpha}f(t)=1/\Gamma(n-\alpha)\int(t-\tau)^{n-\alpha-1}f^{(n)}(\tau)d\tau$$

Here:
- n is the smallest integer greater than α.
- $\Gamma(\cdot)$ is the gamma function.
- $f^{(n)}(\tau)$ denotes the n-th derivative of f(t).

The Caputo derivative is particularly useful in fractional differential equations with initial conditions, as it naturally incorporates the effects of the initial state of the system. Unlike the Riemann-Liouville fractional derivative, which requires knowledge of the entire history of the function, the Caputo derivative is defined using standard derivatives, making it more applicable in physical problems with initial conditions.

Fractional calculus has found applications in various fields, including physics, engineering, and finance, where systems with memory effects or non-local behaviour are encountered. The Caputo difference operator is a key tool in analysing and modelling such systems.

The Caputo fractional differential operator, denoted by α is a generalization of the ordinary derivative to non-integer orders. Here are some properties and expressions associated with the Caputo fractional differential operator:

Properties:

1. Linearity:
   - The Caputo fractional derivative is a linear operator:

$$\partial^{\alpha}[af(t)+bg(t)] = a\partial^{\alpha}f(t)+b\partial^{\alpha}g(t)$$

2. Fractional Chain Rule:
   - The fractional chain rule for the Caputo derivative is given by:

$$\partial^{\alpha}[g(f(t))] = \partial^{\alpha}g(f(t))\cdot\partial^{\alpha}f(t)$$

3. Initial Conditions:
   - The Caputo fractional derivative naturally incorporates initial conditions.

$\partial^{\alpha}f(t)$, if f(t)=0, for t<0, then $\partial^{\alpha}f(t)=0$

4. Semigroup Property:
   - For α, β >0, the Caputo fractional derivative has the semigroup property:

$$\partial^{\alpha+\beta}f(t)=\partial^{\alpha}(\partial^{\beta}f(t))$$

5. Integral Representation:
   - The Caputo fractional derivative can be represented in an integral form:

$$\partial^{\alpha}f(t)=1/\Gamma(1-\alpha)\int(t-\tau)^{-\alpha}\partial f(\tau)d\tau$$

6. Caputo-Mittag-Leffler Function:
   - The solution to a linear fractional differential equation involving the Caputo derivative often includes the Caputo-Mittag-Leffler function.

These properties make the Caputo fractional differential operator a powerful tool in the analysis and modelling of systems with non-local behaviour and memory effects. The Caputo derivative is particularly suitable for fractional differential equations with initial conditions.

Finite Difference Methods:

Finite difference methods involve discretizing the spatial and temporal derivatives in fractional differential equations using finite differences. For example, the Grünwald-Letnikov discretization can be applied to both spatial and temporal derivatives, leading to a numerical scheme suitable for partial fractional differential equations. Finite Difference Methods (FDM) are numerical techniques used for solving differential equations, including ordinary differential equations (ODEs) and partial differential equations (PDEs). These methods discretize the spatial and/or temporal domains and replace derivatives with finite difference approximations. Finite Difference Methods are widely employed in various fields, such as physics, engineering, and finance. Here are some key aspects of Finite Difference Methods:

1. Grid Discretization:
 - The continuous domain is discretized into a grid. In one dimension, this involves dividing the domain into points, forming a grid.
2. Discretization of Derivatives:
 - Derivatives in the differential equations are approximated by finite differences. Common difference formulas include the forward difference, backward difference, and central difference.

Finite Difference Schemes:
1. Explicit Method:
 - In an explicit finite difference scheme, the future values of the solution are expressed explicitly in terms of the current values and known parameters. This method is straightforward but may have stability constraints.
2. Implicit Method:
 - In an implicit scheme, the future values are determined by an implicit equation involving both current and future values. Implicit methods are often unconditionally stable but require solving a system of equations at each time step.
3. Crank-Nicolson Method:
 - The Crank-Nicolson method is a semi-implicit scheme that combines aspects of explicit and implicit methods. It is widely used for solving parabolic PDEs.

Stability and Convergence:
1. Stability Analysis:
 - Stability is a crucial aspect of finite difference methods. Methods must be chosen such that the numerical solution remains bounded over time.
2. Convergence Analysis:
 - Convergence ensures that as the spatial and temporal discretization's become finer, the numerical solution approaches the true solution of the differential equation.

Examples:
1. Heat Equation:
 - Finite Difference Methods are commonly used to solve the heat equation, describing the distribution of temperature over time.
2. Wave Equation:
 - The wave equation, representing the propagation of waves, can be solved using finite difference methods.

Numerical Stability and CFL Condition:
1. Courant-Friedrichs-Lewy (CFL) Condition:
 - The CFL condition is a stability criterion for time-stepping methods, including finite difference methods. It ensures that the time step size is small enough to prevent numerical instability.

Finite Difference for Option Pricing:

1. Black-Scholes Equation:

 - Finite Difference Methods are applied to solve the Black-Scholes partial differential equation for pricing financial options.

Finite Difference Methods provide a versatile and widely applicable approach for solving differential equations numerically. The choice of method depends on the nature of the problem and considerations such as stability and computational efficiency.

The Finite Difference Method involves discretizing derivatives using finite differences on a grid. The specific mathematical expressions depend on the type of finite difference scheme (e.g., explicit, implicit, Crank-Nicolson) and the nature of the differential equation being solved (e.g., ordinary or partial differential equations). Here, I'll provide a general overview of the expressions for one-dimensional problems.

Explicit Finite Difference Method (Forward Difference):

Consider the one-dimensional heat equation:

$$\partial u/\partial t = \alpha \partial^2 u/\partial x^2$$

Using the forward difference for time and central difference for space, the explicit finite difference scheme is given by:

$$u_i^{n+1} = u_i^n + \alpha\Delta t/(\Delta x)^2 \left[(u_{i+1}^{\ n} - 2u_i^n + u_{i-1}^{\ n})\right]$$

where:

- u_i^n is the numerical solution at spatial point i and time step n,
- α is the diffusion coefficient,
- **Δ**x is the spatial step size,
- **Δ**t is the time step size.

Implicit Finite Difference Method (Backward Difference):

The implicit scheme involves discretizing the time derivative implicitly. Using backward difference for time and central difference for space, the implicit finite difference scheme is given by:

$$u_i^{n+1} = u_i^n + \alpha\Delta t/(\Delta x)^2 \left[(u_{i+1}^{\ n+1} - 2u_i^{n+1} + u_{i-1}^{\ n+1})\right]$$

This implicit scheme requires solving a system of linear equations at each time step, which often involves iterative methods.

Crank-Nicolson Finite Difference Method:

The Crank-Nicolson scheme is a combination of the explicit and implicit methods. It averages the explicit and implicit schemes. For the heat equation, it can be written as:

$$u_i^{n+1} = u_i^n + \alpha\Delta t/2(\Delta x)^2 \left[(u_{i+1}^{\ n+1} - 2u_i^{n+1} + u_{i-1}^{\ n+1} + u_{i+1}^{\ n} - 2u_i^n + u_{i-1}^{\ n}\right]$$

Example: 1D Heat Equation:

For the 1D heat equation

$$\partial u/\partial t = \alpha \partial^2 u/\partial x^2$$

the finite difference method expressions are derived by discretizing the derivatives. The specific form depends on the chosen scheme and boundary conditions.

These are simplified expressions, and the actual implementation might involve handling boundary conditions, stability analysis, and considerations specific to the problem being solved.

Fractional Adams Methods:

Fractional Adams methods extend the classical Adams methods to solve fractional differential equations. These methods provide a systematic way of integrating fractional derivatives over time steps, making them suitable for solving both ordinary and partial fractional differential equations. Adams methods are a family of numerical techniques for solving ordinary differential equations (ODEs) by approximating the solution at discrete points in time. Fractional Adams methods extend this approach to fractional differential equations (FDEs), which involve derivatives of non-integer order.

The Adams methods are based on the idea of constructing the solution incrementally by using previous values of the solution. The fractional Adams methods similarly aim to solve fractional differential equations by extending this concept to fractional orders.

Here is a general outline of the Adams-Bashforth method, an Adams method commonly used for integer-order ODEs:

Adams-Bashforth Method for Integer-Order ODEs:

Given an ODE du/dt = f(t,u) , the Adams-Bashforth method of order k is given by:

$$u_{n+1}=u_n+\Delta t[\beta_0 f(t_n,u_n)+\beta_1 f(t_{n-1},u_{n-1})+\ldots+\beta_k f(t_{n-k+1},u_{n-k+1})]$$

Here, Δt is the time step, t_n are the discrete time points, u_n are the approximations to the solution at time t_n.

Fractional Adams Methods:

Extending Adams methods to fractional orders involves adapting the discrete approximations to fractional derivatives. The general idea is to replace integer-order differences with fractional differences. The fractional Adams methods may take different forms depending on the specific fractional derivative used (e.g., Riemann-Liouville, Caputo) and the order of the method.

For example, a fractional Adams-Bashforth method might be expressed as:

$$u_{n+1}=u_n+\Delta t^{\alpha}[\beta_0 D^{\alpha} f(t_n,u_n)+\beta_1 D^{\alpha} f(t_{n-1},u_{n-1}) +\ldots+\beta_k D^{\alpha} f(t_{n-k+1},u_{n-k+1})]$$

Here, D^{α} represents the fractional derivative, and α is the order of the fractional derivative.

It's important to note that the implementation details and stability considerations for fractional Adams methods can be more intricate than their integer-order counterparts. The specific choice of the fractional derivative and numerical methods for approximating fractional derivatives play a crucial role in the accuracy and stability of the method. Additionally, the development and analysis of fractional Adams methods are active areas of research in the field of fractional calculus.

Fractional Fourier Series Methods:

Fractional Fourier series methods approximate the solution of fractional differential equations using a truncated series representation. This approach is particularly useful for problems with periodic solutions.

Fractional Finite Element Methods:

Fractional finite element methods involve discretizing the spatial domain using finite elements and approximating the fractional derivatives within each element. This approach is advantageous for solving problems defined on irregular domains. Understanding the characteristics and limitations of these numerical methods is crucial for selecting the most appropriate approach based on the specific requirements of the fractional differential equation under consideration. Additionally, the development of efficient and accurate numerical techniques remains an active area of research in the field of fractional calculus. The fractional Fourier series (FRS) is an extension of the classical Fourier series to functions with fractional periodicity. While traditional Fourier series deal with functions periodic over an integer number of cycles, fractional Fourier series considers functions with non-integer periods. The fractional Fourier series methods are particularly useful in handling signals or functions with non-integer periodicities, making them applicable in various fields such as signal processing and wave analysis.

The fractional Fourier series of a periodic function f(t) with a non-integer period T can be expressed as:

$$f(t)=\sum c_n e^{j2\pi nt/T\alpha}$$

where:

- C_n are the Fourier coefficients,
- T^α is the non-integer period, and
- α is the fractional order.

Fractional Fourier series methods can be used for various applications, such as signal representation, analysis, and synthesis, just like their integer-order counterparts. The extension to fractional order allows for more flexibility in handling signals with non-integer periodicities.

Here are some key aspects related to fractional Fourier series methods:

Fractional Fourier Coefficients:

The Fourier coefficients C_n in the fractional Fourier series are computed using integrals over the fractional period T^α:

$$C_n=1/T^\alpha\int f(t)e^{-j2\pi n/T\alpha t}dt$$

Fractional Fourier Transform:

The fractional Fourier series is closely related to the fractional Fourier transform (FRFT), which is a generalization of the classical Fourier transform. The FRFT allows the transformation of signals with fractional periodicities.

Applications:

1. Signal Processing:
   - Fractional Fourier series methods find applications in analysing and processing signals with non-integer periodicities, which is common in some communication systems and wave phenomena.
2. Wave Analysis:
   - The methods are useful in analysing waves with non-integer periodicities, providing an efficient representation for signals that do not fit the traditional Fourier series framework.
3. Fractional Periodic Functions:
   - FRS can be applied to functions with periodicities that are powers of a fundamental period, allowing for a more flexible representation.

It's important to note that fractional Fourier series methods are part of the broader field of fractional calculus, which deals with derivatives and integrals of non-integer order. The use of fractional derivatives and integrals allows for a more nuanced and versatile treatment of signals and functions with non-integer characteristics.

Challenges and Advancements in Computational Techniques for Fractional Calculus:

Computational techniques for fractional calculus play a pivotal role in applying fractional calculus to real-world problems. While advancements have been made, several challenges persist, necessitating ongoing research efforts. Here, we discuss both challenges and recent advancements in computational techniques for fractional calculus.

Challenges:

The non-local nature of fractional derivatives, involving integrals over infinite intervals, poses challenges in discretization and computational efficiency. Memory effects in fractional calculus imply that the solution at any point depends on the entire history of the function, increasing the complexity of numerical methods. Fractional derivatives often exhibit singular behaviour, leading to difficulties in numerical stability and convergence. Singularities can arise when approximating functions with sharp transitions or corners. The computational cost of numerical methods for fractional calculus can be high due to the intricate nature of fractional operators. Efficient algorithms and optimization strategies are needed to reduce computational expenses. Incorporating initial and boundary conditions in numerical methods for fractional differential equations can be challenging. Ensuring consistency with the underlying physical problem adds complexity to the solution process. Extending numerical techniques for fractional calculus to multi-dimensional problems, such as partial fractional differential equations, presents additional challenges. Discretizing and solving problems in multiple dimensions require sophisticated computational tools.

Advancements:

Advancements in fractional Fourier transform techniques provide efficient ways to handle non-locality and memory effects, particularly in signal processing and image analysis applications. Hybrid methods that combine traditional integer-order numerical techniques with fractional calculus provide a balance between accuracy and computational efficiency. These methods often exploit the strengths of both approaches to address specific challenges in solving fractional differential equations. Fractional finite element methods have evolved to handle irregular geometries and boundary conditions in a more robust manner, enabling the solution of complex problems involving fractional derivatives. The increasing availability of high-performance computing resources allows for the exploration of more sophisticated numerical methods and algorithms for fractional calculus. Parallel computing strategies can significantly reduce the computational time for solving large-scale fractional differential equations. Machine learning techniques, including neural networks and data-driven models, show promise in predicting and approximating solutions to fractional differential equations. These methods can complement traditional numerical approaches, especially in scenarios with complex and unknown dynamics. Enhanced discretization schemes, such as refined finite difference methods and adaptive mesh refinement, contribute to the accurate and efficient numerical solution of fractional calculus problems. Navigating the challenges in computational techniques for fractional calculus requires interdisciplinary collaboration between mathematicians, computer scientists, and domain experts. As advancements continue, the field is poised to address increasingly complex problems and further expand the applications of fractional calculus in various scientific and engineering domains.

II. CONCLUSION

In the realm of control systems, fractional calculus finds practical utility, with a dedicated section discussing its application in designing fractional-order controllers. Recent developments and advances are surveyed, presenting cutting-edge research papers and trends that shape the current landscape of fractional calculus. The paper concludes by identifying challenges and open problems, suggesting avenues for future research to further unravel the potential of fractional calculus. Through this comprehensive exploration, our paper aims to provide researchers, academics, and practitioners with a nuanced understanding of fractional calculus, showcasing its theoretical underpinnings, diverse applications, and recent progress. As a dynamic field with continually evolving facets, fractional calculus promises to play an increasingly pivotal role in advancing scientific and engineering research in the years to come.

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Application of Laplace Transformation in Differential Equation

[1]Santosh C. Rudrawar, [2] Dr. Ganesh L. Rokade
[1]Department of mathematics, Siddharth College Jafrabad Dist-Jalna, Maharashtra
[2]Department of mathematics, JES College Jalna, Maharashtra

ABSTRACT

In this paper we propose to discuss application of L.T. in various fields. Many researchers, Scientists and Mathematicians use L.T. to find solution for their problems. In this paper we will try to solve Mixture problems, Air resistance problems, Population growth problems, Vibrating Mechanical Systems. This helps for the study in the fields of Science and Engineering.

Keywords : Laplace transforms, Differential equation, Mixture, Air resistance, Population growth, Vibrating Mechanical.

I. INTRODUCTION

Laplace transform methods have a key role to play in the modern approach to the analysis and design of engineering system. The concepts of Laplace transforms are applied in the area of science and technology. Laplace transform is one of the most important tools used for solving linear ordinary differential equation. Laplace transform takes a time domain function f(t) and converts it into a Laplace domain functionF(p). In this paper we will try to solve Mixture problems [2], Air resistance problems [8][9], Population growth problems[2][8].

II. APPLICATION

A. Air Resistance problem

We studied an application of a first-order differential equation that involved solving for the velocity of an object. In particular, if a ball is thrown upward with an initial velocity of v_0 m/s, allowing for the possibility of air resistance acting on the ball.

Air resistance always acts in the direction opposite to motion. Therefore if an object is rising, air resistance acts in a downward direction. If the object is falling, air resistance acts in an upward direction. There is no exact relationship between the velocity of an object and the air resistance acting on it. For very small objects, air resistance is proportional to velocity; that is, the force due to air resistance is numerically equal to some constant k times v.For larger (e.g., baseball-sized) objects, depending on the shape, air resistance can be approximately proportional to the square of the velocity. In fact, air resistance may be proportional to $v^{1.5}$or $v^{0.9}$or some other power of v.

In this paper we would use the linear approximation for air resistance. If we assume $k > 0$, then the expression for the force F_A due to air resistance is given by $F_A = -kv$.Therefore the sum of the forces acting on the object is equal to the sum of the gravitational force and the force due to air resistance. This, in turn, is equal to the mass of the object multiplied by its acceleration at time t (Newton's second law). This gives us the differential equation

$$m\frac{dv}{dt} = -kv - mg \tag{1}$$

Finally, we impose an initial condition $v(0)=v_0$, where v_0 is the initial velocity measured in meters per second. This makes $g = 9.8\ m/s^2$.

By using Laplace transform we find solution of (1).

$$\mathcal{L}[mv'] = -k\mathcal{L}[v] - \mathcal{L}[mg]$$

$$m\big(p\mathcal{L}[v] - v(0)\big) = -k\mathcal{L}[v] - \frac{mg}{p}$$

$(mp + k)\mathcal{L}[v] = -\frac{mg}{p} + mv_0$

By using $v(0)=v_0$

$$\mathcal{L}[v] = -\frac{mg}{p(mp+k)} + \frac{mv_0}{(mp+k)}$$

Taking inverse Laplace both side

$$v(t) = -g\mathcal{L}^{-1}\left[\frac{1}{p\left(p+\frac{k}{m}\right)}\right] + \mathcal{L}^{-1}\left[\frac{v_0}{\left(p+\frac{k}{m}\right)}\right]$$

$$= -\frac{gm}{k}\mathcal{L}^{-1}\left[\frac{1}{p} - \frac{1}{\left(p+\frac{k}{m}\right)}\right] + \mathcal{L}^{-1}\left[\frac{v_0}{\left(p+\frac{k}{m}\right)}\right]$$

$$v(t) = -\frac{gm}{k}\left[1 - e^{-\frac{k}{m}t}\right] + v_0 e^{-\frac{k}{m}t}$$

$$= -\frac{gm}{k} + [v_0 + \frac{gm}{k}]\, e^{-\frac{k}{m}t}$$

This is solution of velocity after time t allowing air resistance.

Example. A racquetball is hit straight upward with an initial velocity of 2 m/s. The mass of racquetball is approximately 0.0427 kg. Air resistance acts on the ball with a force numerically equal to 0.5 v, where v represents the velocity of the ball at time t. Find the velocity of the ball as function of time.

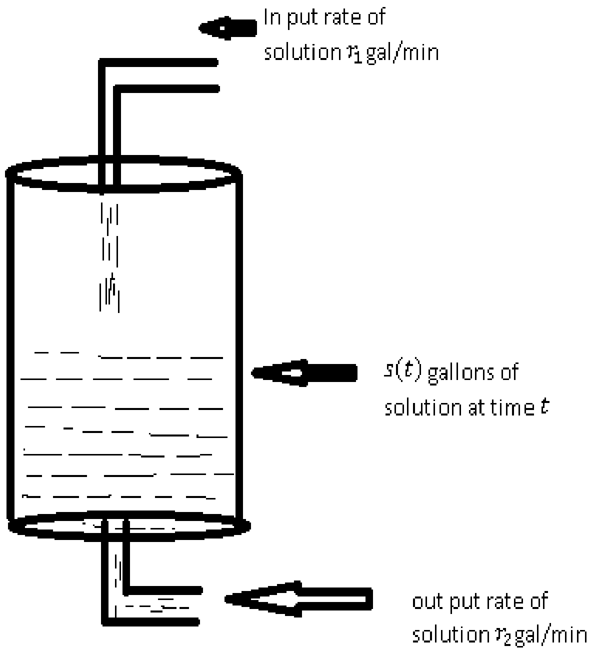
$m = 0.0427kg,\ k = 0.5,\ g = 9.8\ m/s^2,\ v_0 = 2\ m/s$

$$v(t) = -\frac{gm}{k} + \left[v_0 + \frac{gm}{k}\right] e^{-\frac{k}{m}t}$$

$$v(t) = -\frac{(9.8)(0.0427)}{0.5} + \left[2 + \frac{(9.8)(0.0427)}{0.5}\right] e^{-\frac{0.5}{0.0427}t} = -0.8369 + 2.8369e^{-11.7096t}$$

B. Mixture Problem

Let us suppose that a large mixing tank initially holds s_0 gallons of a solution in which x_0 pounds of a substance S is dissolved. Let another solution containing x_1 lb/gal of S, flows into the tank at a given rate $r_1$gal/min. When the solution in the tank is well stirred it in pumped out at a given rate $r_2$gal/min.



Let x(t) denote the amount of substance S (measured in pounds) in the tank at time t. Then the rate at which x change with time t is given by

$$\frac{dx}{dt} = (\text{input rate of S}) - (\text{Out put rate of S}) = R_1 - R_2 \quad (2)$$

Now the input rate R_1 at which S enters the tank is the product of the inflow concentration x_1 lb/gal of S and the inflow rate $r_1$gal/min of the fluid. Note that R_1 is measured in lb/min thus we have

$R_1 = (x_1 \text{ lb/gal})(r_1\text{gal/min}) = x_1 r_1 \text{lb/min}$ (3)

Let $s(t)$ denote the number of gallons of solution in the tank at timet. Then the concentration of S in the tank as well as in the out flow in $(^X/_S)$ lb/gal at any time t. Hence the out put rate R_2 of S is given by

$$R_2 = ((^X/_S)\text{lb/gal})(r_2\text{gal/min}) = (\frac{xr_2}{s})\text{lb/min} \quad (4)$$

From(2),(3) & (4)

$$\frac{dx}{dt} = x_1 r_1 - \left(\frac{xr_2}{s}\right) \quad (5)$$

Which is a first order differential equation on solving (5) we obtain the amount of substance S in the tank at any time t.

By use L.T. we try to find solution under the condition

i) if $r_1 = r_2$then clearly $s(t) = s_0 = constant\ value$

ii) x_0 may be zero and x_1, r_1, r_2 are non zero

$$\frac{dx}{dt} = x_1 r_1 - \left(\frac{xr_2}{s}\right)$$

Taking L.T. both side $\mathcal{L}\left[\frac{dx}{dt}\right] = \mathcal{L}[x_1 r_1] - \mathcal{L}[\frac{xr_2}{s}]$

Using L.T. derivative property

$$p\mathcal{L}[x] - x(0) = \frac{x_1 r_1}{p} - \frac{r_2}{s}\mathcal{L}[x]$$

$$\text{Use } x_1 r_1 = k \text{ and } \frac{r_2}{s} = m \quad (6)$$

$$p\mathcal{L}[x] - x(0) = \frac{k}{p} - m\mathcal{L}[x]$$
$$[p+m]\mathcal{L}[x] = \frac{k}{p} + x(0)$$
$$\mathcal{L}[x] = \frac{k}{p(p+m)} + \frac{x(0)}{(p+m)}$$

Taking I.L.T. both side

$$x = k\mathcal{L}^{-1}\left[\frac{1}{p(p+m)}\right] + x(0)\mathcal{L}^{-1}\left[\frac{1}{(p+m)}\right]$$
$$x = \frac{k}{m}[1 - e^{-mt}] + x(0)e^{-mt}$$

By (6)

$$x = \frac{x_1 r_1 s}{r_2}\left[1 - e^{-\frac{r_2}{s}t}\right] + x(0)e^{-\frac{r_2}{s}t}$$

This solution give amount of substance S in the tank at any time t.

Example. Initially 50 pounds of salt is dissolved in a large tank having 300 gallons of water. A brine solution in pumper into the tank at a rate of 3 gal/min and well stirred solution is then pumped out at the same rate. If the concentration of the solution entering is $2lb/gal$ find the amount of salt in the tank at any time. How much salt is present after 50 min?

$$x = \frac{x_1 r_1 s}{r_2}\left[1 - e^{-\frac{r_2}{s}t}\right] + x(0)e^{-\frac{r_2}{s}t}$$

This solution give amount of substance S in the tank at any time t.

$t = 0, x(0) = 50\, pounds, r_1 = r_2 = 3\, gal/min,$

$s = 300\, gallons, x_1 = 2\, lb/gal$

$$\therefore x = \frac{2 \times 3 \times 300}{3}\left[1 - e^{-\frac{3}{300}t}\right] + 50e^{-\frac{3}{300}t}$$

$x = 600 - 550e^{-\frac{1}{100}t}$

Is amount of salt in the tank at any time t

Now find $t = 50\, min, x = ?$

$$x = 600 - 550e^{-\frac{50}{100}} = 600 - 550e^{-0.5} = 266.41lb$$

C. Population growth problem

The growth of population (growth of species, an organ, or plant) it can be written as first order linear ordinary differential equation.

$$\frac{dN}{dt} = \propto N \tag{7}$$

With the initial condition as $N(t_0) = N_0$

Where $\propto$ is positive real number, N is amount of population at time t and N_0 is the initial population at time t_0. Equation (7) is also known as Malthusian law of population growth.

Taking Laplace transform both side of (7)

$$\mathcal{L}\left[\frac{dN}{dt}\right] = \propto \mathcal{L}[N]$$
$$p\mathcal{L}[N] - N_0 = \mathcal{L}[N]$$
$$(p - \propto)\mathcal{L}[N] = N_0$$
$$\mathcal{L}[N] = \frac{N_0}{(p-\propto)}$$
$$N = N_0 e^{\propto t}$$

Is required amount of population of time t.

Example. If the population of a country doubles in 50 years in how many years will it treble under the assumption that the rate of increases is proportional to the number of inhabitants.

$$N = N_0 e^{\propto t}$$
$$t = 50, N = 2N_0$$
$$\therefore 2N_0 = N_0 e^{50\propto}$$
$$\propto = \frac{log2}{50} \tag{8}$$

Let the population treble in t_1 year

$$\therefore 3N_0 = N_0 e^{\propto t_1}$$
$$\Rightarrow t_1 = \frac{\log 3}{\propto}$$

Using (8)

$$t_1 = \left[\frac{\log 3}{\log 2}\right] \times 50 = 79.25 \text{ years}$$

D. Vibrating Mechanical Systems [2]

If we discuss the suspension system of the car the mass is an important, damper and springs may be used to join the body of the car. Mechanical systems may be used to model many situations and involve there basic elements masses (mass m measured in kg), dampers (damping coefficient a measured inNsm^{-1}.The associated variables are force $F(t)$ (measured in N) displacement $x(t)$ (measured in m).Newton's Hooke's law

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} + kx = F(t)$$

III. CONCLUSION

Through this paper we showed the application of L.T. in various fields. We try to solve linear differential equation in air resistance, mixture problems, population growth problem. See in other fields we use L.T. to find easy solution .L.T. is very effective tool to solve very complex problems engineering and science fields. Now a days there is tremendous use of L.T. to find the solution of different problems

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Module Theories on Finite Goldie Dimension

Dr. Dale T. B.
Head of Department of Mathematic, Rajashri Shahu Arts, Comm., Science College, Pathari TQ-Phulambri, Aurangabad, India

ABSTRACT

In the given seminal paper we are explain modules, Definition , finite Goldie dimension theory(FGD), properties of Goldie, some im complementportant theorem with proofs.
Keywords : Modules, finite Goldie dimension theory.

I. INTRODUCTION

It is well known that the dimension of a vector space is defined as the number of elements in the basis.one can define a basis of a vector space as a maximal set of linearly independent vector s or a minimal set of vectors,which span the space.The former,when generalized to modules over rings, becomes the concept of goldie dimension.the concept of goldie dimension in modules theories was studies by many authors camillo,goldie,reddy and satyanarayan.let R be a fixed ring.throughout this paper we are concerned with left R modules M. like in goldie.Anon zero submodule K of M is called essential in M.if $K \cap A = (0)$for any other submodule A of M,implies A= (0).M has finite goldie dimension dimension.if M does not contain a direct sum of infinite number of non zero submodules.Equivalent ,M has a finite goldie dimension if for any strictly increasing sequence H_0 subset ofH_1 subet of H_2 of submodules of M,there is an integer I such that $H_K$is essential sub module in H_{k+1} for every k≥ i.the number n is called the goldie dimension of M and is denoted by dim M.it canbe esalily proved that if M has FGD then every submodule K of M has also FGDand dim $K \leq \dim M$.furthermore if K ,A are submodule of M,and K is maximal submodule of M such that K ∩ A=(0),then we say that K is complement of A .it is easy to prove that if K is submodule of M then K is a complement in M there exits a submodules A in M such that A ∩ K =(0) and $K' \cap A \neq 0$ for any submodule k1 of M such that k1 properly contains K.in this case we have K+A is essential in M.its proved that if M has FGD then a submodule K is complement $\frac{M}{K}$ has FGD and dim ( ) =dim M-dim k

II. Modules

Some Definition Modules:

1) an R module P is projective if given an exact sequence $\beta: M \to W \to 0$ (β is onto)and an R homomorphism g:p→W there exits an R homomorphism $f: P \to M$ such that β of $=$ g.

2) an R module E is injective if for every mono morphism α:L→M and every $\varphi: L \to E$ there exits $\pi: M \to E$ such that $\pi 0 \alpha = \emptyset$.

Finite Goldie Dimension in module theory

In this section, we collect the fundamental definitions in Module Theory, and some existing results on Modules with Finite Goldie Dimension.

Definition: Let R be an associative ring. An Abelian group (M, +) is said to be a module over R if there exists a mapping (called scalar multiplication) f : R → M : M (the image of (r, m) is denoted by rm satisfying the following three conditions:

(i) r(a + b) = ra + rb

ii)(r + s)a = ra + sa; and

iii)r(sa) = (rs)a for all a, b ϵ M and r, s ϵ R.

Moreover, if R is ring with identity 1 and 1m = m for all m M, then M is called a unital R–Module. Every vector space is a module. Every Abelian group is a module over the ring of integers . Every ring R is a module over itself. So the study of module theory includes the study of vector space theory, Abelian group theory and ring theory. Consequently module theory became an important part of Algebra.

Definition: An R-module M is said to have Finite Goldie Dimension (written as **FGD) if** M does not contain a direct sum of infinite number of non-zero sub modules.

Lemma: Let M beanon-zeromodule with FGD. Then every non-zero sub module o{Existence) There exist uniform sub modules Ut, U2 • • .p UnWhose sum is direct and essential in M.

ii) **Uniqueness:** If there exist uniform sub modules Ve. +2, Vk whosesum is direct and essential in M, then k = n.

Definition: a) Let M beamodulewithFGD. ThenbyTheoremthere exist uniform sub modules Ui, 1 iSn whose sum is direct and essential in M. The number 'n' is independent of the choice of the uniform sub modules. This number n is called the Goldie dimension of M, and it is denoted by dim M.

Definition: b) Let M be an R-Module. A non-empty subset N of M is called an R-sub module (or a sub module) of M if

Properties: (Goldie)

i) If M has finite Goldie dimension and K is a complement sub module, then dim (M/K) = dim M – dim K.

ii) M be an R-module with Finite Goldie Dimension and let K be a sub module of M such that dim M = dim K + dim(M/K). Then K is a complement sub module of M.

iii) M be an R-module with Finite Goldie Dimension and K be a sub module of M. Then the following two conditions are equivalent:

(i). K is a complement sub module of M; and

(ii). dim M = dim K + dim(M/K).

Theorem: If M has FGD and K1 and K2are two sub modules of M such that

K = $K_1 \cap K_2$ *is a complement. Then* $dim K_1 + dim K_2 = dim(K_1 + K_2) + dim(K_1 \cap K_2)$

Proof: Let A be a complement of K in K , and B be a complement of K in K2

we get that A ⊕K≤K1 and B ⊕K≤ K_2

$$\Rightarrow (A + K)/K \leq eK1/K \quad \text{and} \quad (B + K)/K \leq K2/K$$

Now $\frac{K_1}{K} \cap \frac{K_2}{K} = \frac{K_1 \cap K_2}{K} = 0$

We have that (A+B+K)/K= (A+K)/K $\oplus \frac{B+K}{K} \leq \frac{K_1}{K} \oplus (\frac{k2}{k}) = \frac{k1+k2}{k}$

$=$ (A+B+K)/K$\leq (K1 + K2)/K$

Therefore (A+B+K)$\leq (K1 + k2)$

= dim(A+B+K) =dim(k1+k2)

Now we show that A+ $B + K$ *is direct. let* $a + b + k = 0$ *for some* $a \epsilon A, b \in B, k \in K$. *it follows*

That $b = -a - k \in K_1 \cap K_2 = K$. *then* $b \epsilon B \cap K = 0$, *hence* $b = 0$. *now* $a \epsilon A \cap K = 0$ *hence*

$a = 0$ *and then* $k = 0$ *thus the sum* $A + B + K$ *is direct. since* $A \oplus B \oplus K \leq\leq (K1 + K2)$

We have that,

Thus sum is direct.

dim(K1 + K2) = dim(A$\oplus$ B $\oplus$k)

$$= dim\, A + dim\, B + dim\, K$$

$$= (dim\, K1 - dim\, K) + (dim\, K2 - dim\, K)$$

$$= dim\, K1 + dim\, K2 - dim\, K$$

$$= dim + k1 + dim\, k2 - dim\, (k1 \cap k2)$$

This proof is completed.
corollary : Suppose
Suppose V is a finite dimensional vector space. then every subspaces W of V is a complement *submodule of V when we consider V as amodule over the same field.*we can conclude that for any two subspaces $K1$ *and* $K2$ *of V , we* have that $dimK_1 + dimK_2 = dim(K_1 + K_2) + dim(K_1 \cap K_2)$

Some Result : **(i)** If A_1 and A_2 are sub modules of M with A_1 A_2 = (0), then

dim(A_1 + A_2) = dim A_1 + dim A_2.

i)If A_1, A_2, …, A_n are sub modules of M whose sum is direct,

ii)Then dim($A_1 \oplus A_2 \oplus \ldots A_n$) = dim A_1 + dim A_2 + … + dim A_n.

iii)If M has FGD with dim M = n and H is a sub module of M, then H is essential in M if and only if dim H = dim M if and only if H contains a direct sum of n uniform ideals.

Conclusion: we have conclude that that for any two subspaces $K1$ *and* $K2$ *of V , we* have that $dimK_1 + dimK_2 = dim(K_1 + K_2) + dim(K_1 \cap K_2)$

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Some Fixed-Point Theorems in Rectangular S-Metric Spaces

P. G. Golhare[1], C. T. Aage[2]
[1]Department of Mathematics, Sant Dnyaneshwar Mahavidyalaya, Soegaon, Aurangabad, India
[2]Department of Mathematics Kbcnmu, Jalgaon, India

ABSTRACT
In this paper we prove some fixed-point theorems in complete rectangular S–metric spaces for mappings satisfying generalized contractive conditions. Also, we have established a fixed-point theorem for mapping satisfying weak contractive conditions in such spaces.
Keywords : rectangular S–metric spaces, weak contraction, generalized contraction.

1. Introduction and Preliminaries

Definition 1.1. For a non-empty set X and a function $d : X^2 \to [0, \infty)$ satisfying the following properties:

(i) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;
(ii) $d(x, y) = d(y, x)$, $\forall\, x, y \in X$;
(iii) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$ $\forall\, x, y \in X$ and all distinct points $u, v \in X - \{x, y\}$.

d is called a rectangular metric on X and (X, d) is called a rectangular metric space.

Sedghi et al. [28] introduced the notion of an S-metric space as stated below.

Definition 1.2. [28] Let X be a non-empty set and $S : X^3 \to R^+$, a function satisfying the following properties:

(i) $S(x, y, z) = 0$ if and only if $x = y = z$
(ii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$,$\forall a, x, y, z \in X$ (rectangle inequality).

Then (X, S) is called a S-metric-metric space.

Definition 1.3. [1] Let X be a non-empty set and $S : X^3 \to R^+$, a function satisfying the following properties:

(i) $S(x, y, z) = 0$ if and only if $x = y = z$
(ii) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ $\forall x, y, z \in X$ and all distinct points $a \in X - \{x, y, z\}$.

Then (X, S) is called a rectangular S-metric-metric space.

Example 1.4. [1] Let $X = N \cup \{0\}$ and define $S : X \times X \times X \to R^+ \cup \{0\}$ by

$$S(x, y, z) = \begin{cases} 0, & \text{if } x = y = z; \\ xyz, & \text{otherwise} \end{cases}$$

Then (X, S) is a rectangular S-metric space but neither a G-metric space nor rectangular metric space because $G(6, 4, 2) < G(6, 6, 2)$.

Example 1.5. [1] Let $X = N \cup \{0\}$ and define $S : X \times X \times X \to R^+ \cup \{0\}$ by

$$S(x, y, z) = \begin{cases} 0, & \text{if } x = y = z; \\ x + y + z, & \text{otherwise} \end{cases}$$

Then (X, S) is a rectangular S-metric space but neither a G-metric space nor rectangular metric space because $G(6, 4, 2) < G(6, 6, 2)$.

Example 1.6. [1] Let $X = N \cup \{0\}$ and define $S : X \times X \times X \to R^+ \cup \{0\}$ by

$$S(x, y, z) = \begin{cases} 0, & \text{if } x = y = z; \\ a, \forall a \in N, & \text{otherwise} \end{cases}$$

Then (X, S) is a rectangular S-metric space but neither a G-metric space nor rectangular metric space because $G(x, y, z) = G(x, x, y)$.

Definition 1.7. [1] Let (X, G) be a rectangular S-metric space. For $y \in X, r > 0$, the G-sphere with centre y and radius r is $S_G(y, r) = \{z \in X : G(y, z, z) < r\}$

Definition 1.8. [1] Let (X, G) be a rectangular S-metric space. A sequence $\{x_n\} \subset X$ is G-convergent to z if it converges to z in the rectangular S-metric topology.

Definition 1.9. [1] Let (X, G) and $(\overline{X}, \overline{G})$ be two rectangular S-metric spaces, a function $T : X \to \overline{X}$ is G-continuous at a point $x \in X$ if $T^{-1}(S_{\overline{G}}(f(x), r)) \in \tau(G), \forall r > 0$. T is G-continuous if it is G-continuous at all points of X.

Lemma 1.10. [1] *Let (X, S) be a rectangular S-metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ converges to x if and only if $S(x_n, x, x) \to 0$ as $n \to \infty$.*

Lemma 1.11. [1] *Let (X, S) be a rectangular S-metric space and $\{x_n\}$ a sequence in X. Then $\{x_n\}$ is said to be a Cauchy sequence if and only if $S(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.*

2. Main results

In [1] Adewale and Iluno have proved the following fixed point theorem analogous to Banach's fixed point theorem.

Theorem 2.1. *Let X be a complete rectangular S-metric space and $T : X \to X$ a map for which there exists the real number, k satisfying $0 \le k < 0.5$ such that for each pair $x, y, z \in X$.*

$$S(Tx, Ty, Tz) \le kS(x, y, z).$$

Then T has a unique fixed point.

Also, they have proved the theorem given below.

Theorem 2.2. [1] *Let X be a complete rectangular S-metric space and $T : X \to X$ a map for which there exists the real number, b satisfying $0 \le b < 0.2$ such that for each pair $x, y, z \in X$.*

$$\begin{aligned} S(Tx, Ty, Tz) \le b[&S(x, Tx, Tx) + S(y, Ty, Ty) \\ &+ S(z, Tz, Tz)] \end{aligned}$$

Then T has a unique fixed point.

Now we give our first result.

Theorem 2.3. *Let X be a complete rectangular S-metric space and $T : X \to X$ a map for which there exists the real numbers, $\alpha, \beta, \gamma, \delta$ satisfying $0 \le \alpha+\beta+\gamma+\frac{1}{2}\delta < \frac{1}{2}$ such that for each $x, y, z \in X$*

$$\begin{aligned} S(Tx, Ty, Tz) \le\ & \alpha S(x, y, z) + \beta S(x, x, Tx) \\ &+ \gamma S(y, y, Ty) + \delta S(z, z, Tz)] \end{aligned} \tag{2.1}$$

Then T has a unique fixed point.

Proof. Let x_0 be an arbitrary point in X. Define a sequence by $x_1 = Tx_0, x_2 = Tx_1$, continuing in this manner, $x_{n+1} = Tx_n$, for $n = 0, 1, 2 \cdots$. In (2.1), taking $x = x_{n-1}, y = x_{n-1}, z = x_n$, we can write

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &= S(Tx_{n-1}, Tx_{n-1}, Tx_n) \\ &\le \alpha S(x_{n-1}, x_{n-1}, x_n) \\ &\quad + \beta S(x_{n-1}, x_{n-1}, Tx_{n-1}) \\ &\quad + \gamma S(x_{n-1}, x_{n-1}, Tx_{n-1}) \\ &\quad + \delta S(x_n, x_n, Tx_n) \\ &= \alpha S(x_{n-1}, x_{n-1}, x_n) \\ &\quad + \beta S(x_{n-1}, x_{n-1}, x_n) \\ &\quad + \gamma S(x_{n-1}, x_{n-1}, x_n) \\ &\quad + \delta S(x_n, x_n, x_{n+1}) \end{aligned}$$

This gives that

$$S(x_n, x_n, x_{n+1}) \le \frac{\alpha+\beta+\gamma}{1-\delta} S(x_{n-1}, x_{n-1}, x_n).$$

Let $\xi = \frac{\alpha+\beta+\gamma}{1-\delta}$. Certainly $\xi < \frac{1}{2}$. Setting $d_n = S(x_n, x_n, x_{n+1})$, we have

$$\begin{aligned} d_n = S(x_n, x_n, x_{n+1}) &\le \xi S(x_{n-1}, x_{n-1}, x_n) \\ &= \xi d_{n-1}, \forall n = 1, 2, \cdots. \end{aligned}$$

Thus $d_n \le \xi d_{n-1} \le \xi^2 d_{n-2} \cdots \le \xi^n d_0$. We prove that all the elements in the sequence $\{x_n\}$ are distinct. On the contrary assume that $x_n = x_m$ for some $n, m \in N$. Without loss of generality, we assume that $m > n$. With this $d_n \le \xi^{n-m} d_m \le \xi^{n-m} d_n$. This again is a contradiction as $\xi < 1$.

$$\begin{aligned} S(x_n, x_m, x_m) &\le S(x_n, x_n, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &= S(x_n, x_n, x_{n+1}) \\ &\quad + 2S(x_m, x_m, x_{n+1}) \\ &= d_n + 2S(x_m, x_m, x_{n+1}) \\ &\le d_n + 2d_{n+1} \\ &\quad + 2^2 S(x_m, x_m, x_{n+2}) \\ &\le d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 S(x_m, x_m, x_{n+3}) \\ &\le d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 d_{n+3} + \cdots + 2^{m-1} s_m \end{aligned}$$

$$
\begin{aligned}
S(x_n, x_m, x_m) &\leq \xi^n d_0 + 2\xi^{n+1} d_0 + 2^2 \xi^{n+2} d_0 \\
&\quad + \cdots + 2^{m-1} \xi^{m-1} d_0 \\
&\leq [\xi^n d_0 + 2\xi^{n+1} + 2^2 \xi^{n+2} \\
&\quad + \cdots + 2^{m-1} \xi^{m-1}] d_0 \\
&\leq \xi^n [1 + 2\xi + (2\xi)^2 + (2\xi)^3 + \\
&\quad + \cdots + (2\xi)^{m-n-1}] d_0 \\
&\leq \xi^n [1 + 2\xi + (2\xi)^2 + (2\xi)^3 + (2\xi)^4 \\
&\quad + \cdots] d_0 \\
&\leq \xi^n \frac{1}{1-2\xi} d_0.
\end{aligned}
$$

Now taking limit as $n, m \to \infty$, we obtain $\lim_{n,m\to\infty} S(x_n, x_m, x_m) = \lim_{n,m\to\infty} \xi^n \frac{1}{1-2\xi} d_0 = 0$. Thus for $n, m, l \in N$ with $n > m > l$,

$$
\begin{aligned}
S(x_n, x_m, x_l) &\leq S(x_n, x_n, x_{n-1}) \\
&\quad + S(x_m, x_m, x_{n-1}) \\
&\quad + S(x_l, x_l, x_{n-1}).
\end{aligned}
$$

If we take limit as $n, m, l \to \infty$, we get $lim_{n,m,l\to\infty} S(x_n, x_m, x_l) = 0$. So that sequence $\{x_n\}$ is a Cauchy sequence in X and therefore converges to some $\omega \in X$. We claim that ω is fixed point of T that is $T\omega = \omega$. On the contrary, we assume that $T\omega \neq \omega$. Then using inequality in (2.1) with $x = x_{n-1}, y = \omega, z = \omega$, we get

$$
\begin{aligned}
S(x_n, T\omega, T\omega) &\leq \alpha S(x_{n-1}, \omega, \omega) \\
&\quad + \beta S(x_{n-1}, x_n, x_n) \\
&\quad + \gamma S(\omega, T\omega, T\omega) \\
&\quad + \delta S(\omega, T\omega, T\omega)
\end{aligned}
$$

Now if we take limit as $n \to \infty$, then using continuity of S in its variables and using (i) of Definition 1.3, we get

$$
\begin{aligned}
S(\omega, T\omega, T\omega) &\leq \alpha S(\omega, \omega, \omega) + \beta S(\omega, \omega, \omega) \\
&\quad + \gamma S(\omega, T\omega, T\omega) \\
&\quad + \delta S(\omega, T\omega, T\omega) \\
S(\omega, T\omega, T\omega) &\leq (\gamma + \delta) S(\omega, T\omega, T\omega).
\end{aligned}
$$

This gives contradiction to $(\gamma + \delta) < \frac{1}{2}$. Therefore we must have $T\omega = \omega$. Now to prove that ω is a unique fixed point of T, let us assume that u is another fixed point of T in X distinct from ω. Then

$$
\begin{aligned}
S(T\omega, Tu, Tu) &\leq \alpha S(\omega, u, u) + \beta S(\omega, T\omega, T\omega) \\
&\quad + \gamma S(u, Tu, Tu) \\
&\quad + \delta S(u, Tu, Tu).
\end{aligned}
$$

Using (i) of Definition 1.3 and $T\omega = \omega, Tu = u$, we get

$$
S(\omega, u, u) \leq \alpha S(\omega, u, u).
$$

This gives contradiction to the fact that $\alpha < 1$. Thus ω is a unique fixed point of T in X. □

Theorem 2.4. *Let X be a complete rectangular S-metric space and $T : X \to X$ a map for which there exists the real numbers, $\alpha, \beta, \gamma, \delta$ satisfying $0 \leq \alpha < \frac{1}{2}$ such that for each $x, y, z \in X$*

$$
\begin{aligned}
S(Tx, Ty, Tz) \leq \alpha \max\{&S(x, x, Tx), \\
&S(y, y, Ty), \\
&S(z, z, Tz)\}
\end{aligned}
\tag{2.2}
$$

Then T has a unique fixed point.

Proof. Let x_0 be an arbitrary point in X. Define a sequence by $x_1 = Tx_0, x_2 = Tx_1$, continuing in this manner, $x_{n+1} = Tx_n$, for $n = 0, 1, 2 \cdots$. In (2.2), taking $x = x_{n-1}, y = x_n, z = x_n$, we can write

$$
\begin{aligned}
S(x_n, x_n, x_{n+1}) &= S(Tx_{n-1}, Tx_{n-1}, Tx_n) \\
&\leq \alpha \max\{S(x_{n-1}, x_{n-1}, Tx_{n-1}), \\
&\quad S(x_{n-1}, x_{n-1}, Tx_{n-1}), \\
&\quad S(x_n, x_n, Tx_n)\} \\
&= \alpha \max\{S(x_{n-1}, x_{n-1}, x_n), \\
&\quad S(x_{n-1}, x_{n-1}, x_n), \\
&\quad S(x_n, x_n, x_{n+1})\} \\
&= \alpha \max\{S(x_{n-1}, x_{n-1}, x_n), \\
&\quad S(x_n, x_n, x_{n+1})\}
\end{aligned}
$$

If $\max\{S(x_{n-1}, x_{n-1}, x_n), S(x_n, x_n, x_{n+1})\} = S(x_n, x_n, x_{n+1})$, we get contradiction to the fact that $\alpha < \frac{1}{2}$. Therefore

$$
S(x_n, x_n, x_{n+1}) \leq \alpha S(x_{n-1}, x_{n-1}, x_n).
$$

Repeating, we get $S(x_n, x_n, x_{n+1}) \leq \alpha^n S(x_0, x_0, x_1)$, $n \in N$. Setting $d_n = S(x_n, x_n, x_{n+1})$, we have

$$
\begin{aligned}
d_n = S(x_n, x_n, x_{n+1}) &\leq \alpha S(x_{n-1}, x_{n-1}, x_n) \\
&= \alpha d_{n-1}, \forall n = 1, 2, \cdots.
\end{aligned}
$$

Thus

$$
d_n \leq \alpha d_{n-1} \leq \alpha^2 d_{n-2} \cdots \leq \alpha^n d_0.
$$

We prove that all the elements in the sequence $\{x_n\}$ are distinct. On the contrary assume that $x_n = x_m$ for some $n, m \in N$. Without loss of generality, we assume that

$m > n$. With this $d_n \leq \alpha^{n-m} d_m \leq \alpha^{n-m} d_n$.
This again is a contradiction as $\alpha < 1$.

$$\begin{aligned} S(x_n, x_m, x_m) &\leq S(x_n, x_n, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &= S(x_n, x_n, x_{n+1}) \\ &\quad + 2S(x_m, x_m, x_{n+1}) \\ &= d_n + 2S(x_m, x_m, x_{n+1}) \\ &\leq d_n + 2d_{n+1} \\ &\quad + 2^2 S(x_m, x_m, x_{n+2}) \\ &\leq d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 S(x_m, x_m, x_{n+3}) \\ &\leq d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 d_{n+3} + \cdots + 2^{m-1} s_m \end{aligned}$$

$$\begin{aligned} S(x_n, x_m, x_m) &\leq \alpha^n d_0 + 2\alpha^{n+1} d_0 + 2^2 \alpha^{n+2} d_0 \\ &\quad + \cdots + 2^{m-1} \alpha^{m-1} d_0 \\ &\leq [\alpha^n d_0 + 2\alpha^{n+1} + 2^2 \alpha^{n+2} \\ &\quad + \cdots + 2^{m-1} \alpha^{m-1}] d_0 \\ &\leq \alpha^n [1 + 2\alpha + (2\alpha)^2 + (2\alpha)^3 \\ &\quad + \cdots + (2\alpha)^{m-n-1}] d_0 \\ &\leq \alpha^n [1 + 2\alpha + (2\alpha)^2 + (2\alpha)^3 \\ &\quad + (2\alpha)^4 + \cdots] d_0 \\ &\leq \alpha^n \frac{1}{1 - 2\alpha} d_0. \end{aligned}$$

Now taking limit as $n, m \to \infty$, we obtain $\lim_{n,m\to\infty} S(x_n, x_m, x_m) = \lim_{n,m\to\infty} \xi^n \frac{1}{1-2\alpha} d_0 = 0$. Thus for $n, m, l \in N$ with $n > m > l$,

$$\begin{aligned} S(x_n, x_m, x_l) &\leq S(x_n, x_n, x_{n-1}) \\ &\quad + S(x_m, x_m, x_{n-1}) \\ &\quad + S(x_l, x_l, x_{n-1}). \end{aligned}$$

If we take limit as $n, m, l \to \infty$, we get $\lim_{n,m,l\to\infty} S(x_n, x_m, x_l) = 0$. So that sequence $\{x_n\}$ is a Cauchy sequence in X and therefore converges to some $\omega \in X$. We claim that ω is a fixed point of T that is $T\omega = \omega$. On the contrary, we assume that $T\omega \neq \omega$. Then using inequality in (2.2) with $x = x_{n-1}, y = \omega, z = \omega$, we get

$$\begin{aligned} S(x_n, T\omega, T\omega) &\leq \alpha \max\{S(x_{n-1}, x_{n-1}, x_n), \\ &\qquad S(\omega, \omega, T\omega), S(\omega, \omega, T\omega)\} \end{aligned}$$

Now if we take limit as $n \to \infty$, then using continuity of S in its variables and using (i) of Definition 1.3, we get

$$\begin{aligned} S(\omega, T\omega, T\omega) &\leq \alpha \max\{S(\omega, \omega, \omega), \\ &\qquad S(\omega, T\omega, T\omega), S(\omega, T\omega, T\omega) \\ S(\omega, T\omega, T\omega) &\leq \alpha S(\omega, T\omega, T\omega). \end{aligned}$$

This gives contradiction to $\alpha < \frac{1}{2}$. Therefore we must have $T\omega = \omega$. Now to prove that ω is a unique fixed point of T, let us assume that u is another fixed point of T in X distinct from ω. Then

$$\begin{aligned} S(T\omega, Tu, Tu) &\leq \alpha \max\{S(\omega, \omega, T\omega), \\ &\qquad S(u, u, Tu), S(u, u, Tu)\}. \end{aligned}$$

Using (i) of Definition 1.3 and $T\omega = \omega, Tu = u$, we get

$$S(\omega, u, u) = 0.$$

This gives $\omega = u$. Thus ω is a unique fixed point of T in X. □

Definition 2.5. A mapping $T : X \to X$, where (X, d) is a metric space, is said to be a weak contraction if $d(Tx, Ty) \leq d(x, y) - \phi(d(x, y))$ where $x, y \in X$ and $\phi : [0, \infty) \to [0, \infty)$ is continuous and nondecreasing function such that $\phi(t) = 0$ if and only if $t = 0$.

Theorem 2.6. *Let (X, S) be a complete rectangular S-metric space and $T : X \to X$ a mapping satisfying*

$$S(Tx, Ty, Tz) \leq S(x, y, z) - \phi(S(x, y, z)) \tag{2.3}$$

for all $x, y, z \in X$. If $\phi : [0, \infty) \to [0, \infty)$ is a continuous and nondecreasing function with $\phi^{-1}(0) = 0; \phi(t) > 0$ for all $t \in (0, \infty)$, then T has a unique fixed point in X.

Proof. Let x_0 be an arbitrary point in X. Define a sequence by $x_1 = Tx_0, x_2 = Tx_1$, continuing in this manner, $x_{n+1} = Tx_n$, for $n = 0, 1, 2 \cdots$. In (2.3), taking $x = x_{n-1}, y = x_{n-1}, z = x_n$, we can write

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &= S(Tx_{n-1}, Tx_{n-1}, Tx_n) \\ &\leq S(x_{n-1}, x_{n-1}, x_n) \\ &\quad - \phi(S(x_{n-1}, x_{n-1}, x_n)). \end{aligned} \tag{2.4}$$

Using property of ϕ, we get

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &= S(Tx_{n-1}, Tx_{n-1}, Tx_n) \\ &\leq S(x_{n-1}, x_{n-1}, x_n). \end{aligned}$$

Similarly, we can show that

$$S(x_{n-1}, x_{n-1}, x_n) \leq S(x_{n-2}, x_{n-2}, x_{n-1}).$$

This shows that $S(x_n, x_n, x_{n+1})$ is monotone decreasing and consequently there exists $r \geq 0$ such that $\lim_{n\to\infty} S(x_n, x_n, x_{n+1}) \to r$ as $n \to \infty$. By taking

$n \to \infty$ in (2.4), we obtain $r \leq r - \phi(r)$ which is a contradiction unless $r = 0$. Hence

$$\lim_{n\to\infty} S(x_n, x_n, x_{n+1}) \to 0 \text{ as } n \to \infty. \tag{2.5}$$

Now we prove that $\{x_n\}$ is a Cauchy sequence. We set $d_n = S(x_n, x_n, x_{n+1})$ then

$$\lim_{n\to\infty} d_n = \lim_{n\to\infty} S(x_n, x_n, x_{n+1}) = 0. \tag{2.6}$$

Consider for $m > n$,

$$\begin{aligned} S(x_n, x_m, x_m) &\leq S(x_n, x_n, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &\quad + S(x_m, x_m, x_{n+1}) \\ &= S(x_n, x_n, x_{n+1}) \\ &\quad + 2S(x_m, x_m, x_{n+1}) \\ &= d_n + 2S(x_m, x_m, x_{n+1}) \\ &\leq d_n + 2d_{n+1} \\ &\quad + 2^2 S(x_m, x_m, x_{n+2}) \\ &\leq d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 S(x_m, x_m, x_{n+3}) \\ &\leq d_n + 2d_{n+1} + 2^2 d_{n+2} \\ &\quad + 2^3 d_{n+3} + \cdots + 2^{m-1} s_m. \end{aligned}$$

Now taking limit as $n, m \to \infty$, we obtain $\lim_{n,m\to\infty} S(x_n, x_m, x_m) = 0$. Thus for $n, m, l \in N$ with $n > m > l$,

$$\begin{aligned} S(x_n, x_m, x_l) &\leq S(x_n, x_n, x_{n-1}) \\ &\quad + S(x_m, x_m, x_{n-1}) \\ &\quad + S(x_l, x_l, x_{n-1}). \end{aligned}$$

If we take limit as $n, m, l \to \infty$, we get $\lim_{n,m,l\to\infty} S(x_n, x_m, x_l) = 0$. So that sequence $\{x_n\}$ is a Cauchy sequence in X and therefore converges to some $\omega \in X$. Now we claim that $T\omega = \omega$. For this we consider

$$\begin{aligned} S(x_n, T\omega, T\omega) &= S(Tx_{n-1}, T\omega, T\omega) \\ &\leq S(x_{n-1}, \omega, \omega) - \phi(S(x_{n-1}, \omega, \omega)). \end{aligned}$$

By taking $n \to \infty$ we obtain $S(\omega, T\omega, T\omega) \leq 0$. But we know that $S(\omega, T\omega, T\omega) \geq 0$. So we have $T\omega = \omega$. Suppose T has another fixed point p, then

$$\begin{aligned} S(\omega, p, p) &= S(T\omega, Tp, Tp) \\ &\leq S(\omega, p, p) - \phi(S(\omega, p, p)), \end{aligned}$$

by the property of ϕ this is contradiction if $S(\omega, p, p) > 0$. Hence we must have $S(\omega, p, p) = 0$. This in turn implies that $\omega = p$. This proves uniqueness of fixed point of T in X. □

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Cosmological model with Hybrid Expansion Law (HEL) of Higher Dimensional Universe in f(R, T) Gravity

Salam Kiranmala Chanu*[1], Laishram Nevy Singh[2], S. Surendra Singh[3]

*[1]Department of Mathematics, D.M. College of Arts, Imphal, India

[2]Department of Mathematics, Kamakhya Pemton College, Hiyangthang, India

[3]Department of Mathematics, NIT Manipur, India

ABSTRACT

We investigate the development of n-dimensional Friedmann Robertson Walker (FRW) cosmological model in the conceptual structure of f (R; T) theory of gravitation. The particular form of f(R; T) = R + eT is taken for this particular dust model to obtain Universe with hybrid expansion. There is initial singularity which establishes the hypothesis of Big Bang singularity. We also observe the transitional phase of our Universe from decelerated epoch to accelerated epoch in this derived model. By investigating state finder diagnostic, we also found the interesting result that our model overlaps with ΛCDM model. These cosmological solutions have been investigated geometrically and kinematic behaviors are also investigated.

Keywords : Higher Dimension; Modified Gravity; State Finder Diagnostic; Initial Singularity; Dark Energy.

1. Introduction

Today, there have been a current interest in the study of models that involve higher dimension. At present, the most dated effort about the dimensions of space-time is taken to be more than four. In many aspects of the early evolution of the Universe, higher dimensional space time plays an important role. Our Universe is assumed to be smaller at the beginning of its evolution as compared to today's Universe. This finding is given by the research of Kaluza and Klein model which is a notable research by in order to unify gravity with other fundamental forces in nature by taking the concept of higher dimensional space time. In recent years, multidimensional cosmological models in general relativity (GR) are researched by many researchers. Inhomogeneous Kaluza-klein Universe are studied with or without cosmological constants by Banerjee et al. (1995) and Chaterzee and Banerjee (1993) respectively. In generalizes scalar tensor theory and Lyra's geometry, Kaluza-klein cosmological models are investigated by Chakraborty and Ghosh (2000) and Rahaman and Bera (2001), respectively. Multi dimensional Cosmological models in GR have studied by Lorentz and Petzold (1985), Ibanez et al. (1986), Khadekar and Gaikward (2001). The observational prediction by the measuring of the brightness of distant supernovae Ia has shown that that the present Universe is passing through a phase of accelerated expansion which was mentioned by A. G. Riess and S. Perlmutter (1998, 1999). This epoch of Universe is the most interesting findings of investigation. This surprising finding of the accelerating expansion has given the way of the most mysteriousas well as significant problem in cosmology today. This finding leads to the new direction of research for either changing the right hand side of Einstein field equations (EFE) or adding the energy momentum tensor by an exotic matter with negative pressure called dark energy. A lot of authors introduced other theories which is a modification to the geometric part or the matter part of general theory relativity (GR) as an instrument for the explanation of the late epoch scenario of the Universe. The many gravity theories such as Gauss-Bonnet theory, Brans-Dick theory, string theory, Scalar tensor gravity, $f(R)$ theory and $f(R, T)$ theory of gravity have ex- tended GR. Recently, a new modified theory has been found by Harko et al. (2011), called the $f(R, T)$ gravity which is formed by replacing the relation $f(R)$ with the scalar curvature R andthe trace of energy-momentum tensor T in the Einstein-Hilbert Lagrangian of GR. The $f(R, T)$ theory of gravity has been the most attractive modified theory in the modified theories of gravity. Different models in $f(R, T)$ gravity have been studied by Adhav (2012), Reddy et al. (2012) and Shamir et al. (2012). Perfect fluid cosmological models in $f(R, T)$ gravity have been investigated by Samanta (2013)in the context of higher dimensions. For $f(R, T)$ gravity, Sadia Sattar et al. (2019) have evaluated the N-dimensional plane-symmetric space-time solutions. Moraes et al. (2020) have discussed $f(R, T)$ gravity that derives by taking an exponential of the trace of the energy-momentum tensor which depends on the action of gravitation, leads to the Ricci scalar.

Pettorino et al. (2005) have verified the role of scalar field as dark energy and is associated with exponential form of the Ricci scalar. Harko (2010) started the change of R by using $\Lambda e^{\frac{R+L}{\Lambda}}$ in Einstein Hilbert action, with L being the matter Lagrangian. Surendra et al.(2021) investigated the Kaluza-Klein Universe with interacting wet dark fluids in $f(R, T)$ theory and found the accelerated expansion of Universe which is approaching to ΛCDM . Surendra and Kiranmala (2021) investigated the thermodynamic background of particle production in higher dimensional Universe. Surendra and Anjana (2022) also investigated anisotropic Bianchi type-I in $f(R, T)$ gravity and investigated many interesting solutions which are in agreement with observations. Saratchandra and Surendra (2019) investigated the geometrical and kinematic behaviors of the Locally Rotationally Symmetric (LRS) Bianchi type-I metric with dark energy in $f(R, T)$ gravity. Recently, Khurshid et al.(2022) also shown the dark energy in anisotropic cosmological model with $f(R, T)$ gravity with a new hybrid expansion law and also determined the age of Universe from the geometrical plot.

The main reason of our present paper is to study higher dimensional cosmological model in $f(R, T)$ theory where $f(R, T)$ is replaced by $R + f(T)$ in gravitational action. In our work we are using T as $f(T) \sim e^{\aleph}T$ where $\aleph$ is a free parameter. With the intentions of works done by other authors, the higher dimensional Universe is formulated in the $R + f(T)$ cosmology. $R + e^{T}$ cosmology is presented in sec 2, model and the field equations in sec 3, state finder diagnostic in sec 4 and lastly conclusions are given in sec 5.

2. $R + e^{T}$ Cosmology

In 2011, Harko et al. introduced the modified gravity called $f(R, T)$ theory of gravitation. On taking $f(R, T) = R+e^{T}$, we obtain the field equations of $R+e^{T}$ gravity from the Hilbert-Einstein theory variational principle. Respective action based on this approach is given by

$$s = \int \sqrt{-g}\left(\frac{R+\gamma e^{\aleph T}}{16\pi}\right) d^{n}x + \int \sqrt{-g}L d^{n}x \tag{1}$$

where g is the metric determinant, γ a constant, R is the Ricci scalar, T is the trace of the energy momentum tensor $T_{\mu\nu}$ and L represents the matter Lagrangian. Here the only independent variable is taken to be the metric tensor $g_{\mu\nu}$. We are taking the variation of action (1) with respect to $g_{\mu\nu}$ which gives related field equations as

$$G_{\mu\nu} = 8\pi T^{eff}_{\mu\nu}, \tag{2}$$

$$T^{eff}_{\mu\nu} = T_{\mu\nu} + \frac{\gamma e^{\aleph T}}{8\pi}\left[\frac{g_{\mu\nu}}{2} + \aleph(T_{\mu\nu} + pg_{\mu\nu})\right] \tag{3}$$

where $T_{\mu\nu} = -pg_{\mu\nu} + (p+\rho)u_\mu u_\nu$ denotes the usual energy-momentum tensor of matter, p the pressure and ρ the energy density. The covariant derivative of the equation (3) yields

$$\nabla^\mu T_{\mu\nu} = -\frac{\gamma\aleph e^{\aleph T}}{8\pi + \gamma\aleph e^{\aleph T}}\left[\aleph(T_{\mu\nu} + pg_{\mu\nu})\nabla^\mu T + \nabla^\mu\left(\frac{T}{2} + p\right)\right] \tag{4}$$

We have seen the non conservancy of energy-momentum tensor that observes in other theories studied by Harko et. al. (2010, 2011). J.A.S. Lima et. al. (2010, 2012) noticed that such features can be related to a process of particle creation.

3. Models and Field equations

In this present work, investigation is for n dimensional Friedmann Robertson Walker (FRW) Universe that is homogeneous, flat and isotropic. The line element of this concerned model is given as

$$ds^2 = -dt^2 + a^2(t)\delta_{\mu\nu}dx^\mu dx^\nu,\ \mu,\nu = 1,\dots\dots,n-1 \tag{5}$$

For this line element, $a(t)$ stands for the scale factor, x^μ for comoving co-ordinate set and$\delta_{\mu\nu}$ for the Kronecker delta symbol. The n dimensional velocity of the field which fills the Universe is given by u^i = (1, 0, 0, 0, ..., 0). In order to find the possibility of the accelerated expansion of the Universe as a result of the correction terms of the present model rather than as because of the existence of some mysterious fluid called dark energy passing through the Universe. We assume the dust model $p = 0$ from the equation of state $p = \omega\rho$ in the dynamical equations. For the metric (5) and the equation (3), the field equation (2) yields two independent equations

$$3(n-2)H^2 = 8\pi\rho + \gamma e^{\aleph\rho}\left(\frac{1}{2} + \aleph\rho\right) \tag{6}$$

$$-(n-1)H - (n-3)(n-4)H^2 = \left(8\pi + \gamma\aleph e^{\aleph\rho}\right)\rho. \tag{7}$$

$$\dot{\rho}\left[\frac{\gamma\aleph e^{\aleph\rho}}{8\pi + \gamma\aleph e^{\aleph\rho}}\left(\frac{1}{2} + \aleph\rho\right) + 1\right] + nH\rho = 0 \tag{8}$$

Here $H = \frac{\dot{a}}{a}$ is the Hubble parameter, a is the scale factor and overdot denotes differentiation with respect to cosmic time t. This dust model can be retrieved by taking $\gamma = 0$. If we consider dust model $p = 0$, then from equations (6) and (7) we get

$$\rho = log\left(\frac{2\chi}{\gamma}\right)^{\frac{1}{\aleph}} \tag{9}$$

$$\dot{\rho} = \frac{(n-1)[\ddot{H} + 2(n^2 - 4n + 6)H\dot{H}}{\aleph\chi} \tag{10}$$

In which it was defined $\chi = (n-1)\dot{H} + (n^2 - 4n + 6)H^2$. By use of equations (9) and (10) in equation (8) we get

$$\gamma[(n-1)[\ddot{H}+2(n^2-4n+6)H\dot{H}]\Big[1+\frac{2\aleph\chi^2\left[\frac{1}{2}+log\left(\frac{2\chi}{\gamma}\right)\right]}{8\pi+2\aleph\chi}\Big]+nlog2\chi^2H=0 \quad (11)$$

By inspecting the nature of (10), we assume the condition

$$\chi > 0, \quad (12)$$

which gives the hybrid expansion form of the scale factor $a(t)$ as

$$a(t) = a_0 t^{\beta} e^{\alpha t} \quad (13)$$

where $\beta = \frac{n-1}{n^2-4n+6}$. (14)

Here a_0 and α are integrating constants. The deceleration parameter $(q) = \frac{\ddot{a}a}{\dot{a}^2}$ is calculated as

$$q(t) = \frac{\beta}{(\alpha t+\beta)^2} - 1 \quad (15)$$

Here q is a mapping of variable 'time'. As $t \to \infty$, $q = -1$ which is interpreted as the accelerated expansion of the Universe at late epoch.

As $t \to 0$ $q = \frac{1}{\beta} - 1$. (16)

Using (14) in (16), we obtain

$$q(t) = n - 4 + \frac{3}{n-1} \quad (17)$$

From equation (17) we observe that $q > 1$ when $n \geq 2$ at the early evolution of Universe. This shows that at early Universe, the Universe has decelerated expansion for valid range of $n \geq 2$. Thus, for this derived model, the transitional phase of decelerated expansion to accelerated expansion is observed for valid range of $n \geq 2$. The referred Hubble parameter (H) is

$$H = \alpha + \frac{\beta}{t} \quad (18)$$

$$\rho = log\left(\left[\frac{2\alpha(n^2-4n+6)\alpha t+2(n-1)}{\gamma t}\right]^{\frac{1}{\aleph}}\right) \quad (19)$$

For our convenience, cosmological parameters are rewriting in terms of the redshift z rather than in terms of time. For this, we use $a(t) = a_o/(1+z)$, with the present value of the scale factor $a_o = 1$. The relation of time vs redshift in this derived model is written as

$$t = \frac{\beta}{\alpha} W\left[\frac{\alpha}{\beta}\frac{1}{[a_0(1+z)]^{\frac{1}{\beta}}}\right] \quad (20)$$

For this relation, W express the Lambert function, also called as product logarithm. Now we will take

$$\Gamma = \frac{\alpha}{\beta}\frac{1}{[a_0(1+z)]^{\frac{1}{\beta}}}.$$

Equations (15) and (18) can be written as

$$q(z) = \frac{1}{\beta[W(\Gamma)+1]^2} - 1, \tag{21}$$

$$H(z) = \alpha\left(1+\frac{1}{W(\Gamma)}\right) \tag{22}$$

The cosmological parameters with their respective asymptotic behaviors are also investigated and presented in Table 1.

Parameters	$t \to 0 (z \to \infty)$	$t \to \infty (z \to -1)$
$a(t)$	0	∞
ρ	∞	$log\left[\frac{2\alpha(n^2-4n+6)\alpha}{\gamma}\right]^{\frac{1}{\aleph}}$
H	∞	α
q	1/2	-1

Table 1: Asymptotic behavior of the cosmological parameters are givenas follow

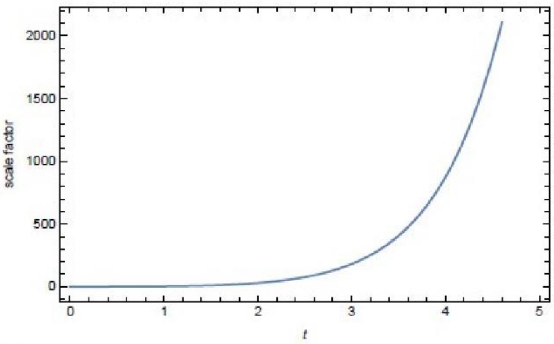

Fig.1

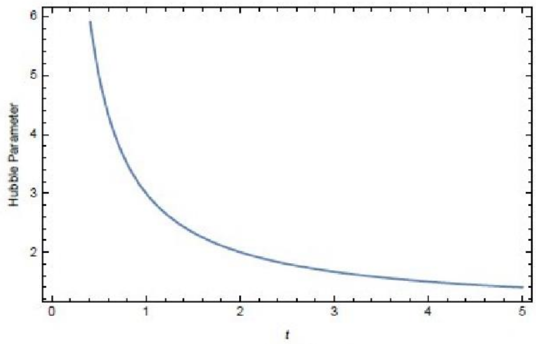

Fig.2

Fig.1: Plot of Scale factor vs time (t). Fig.2: Plot of Hubble parameter vs. time(t).

4.Statefinder diagnostic

A cosmological diagnostic parameter set (r, s) called state-finder pair, is firstly introduced by Sahni et.al. (2003) in view of the dark energy models. The state finder parameters are formulated with the scale factor and its third order derivatives and its mathematical formulas are given as

$$r = \frac{\dddot{a}}{aH^3}. \tag{23}$$

$$s = \frac{1}{3}\left(\frac{-1+r}{-\frac{1}{2}+q}\right) \tag{24}$$

where $q \neq 1/2$.

The trajectories of plot of $q - r$ and $r - s$ pairs are an useful way of defining different types of dark energy models that posses equivalent behaviors. This derived pair (r, s) is calculated as

$$r = \frac{\alpha^3 t^3 + 3\alpha^2 \beta t^2 + 3\alpha\beta(\beta-1)t + \beta(\beta-1)(\beta-2)}{(\alpha t+\beta)^3}. \tag{25}$$

$$s = \frac{2}{3}\frac{(2\beta - 3\beta^2 - 3\alpha\beta t)}{(\alpha t+\beta)(2\beta - 3(\alpha t+\beta)^2)} \tag{26}$$

We can observe from this mathematical expression of (r, s) that for $\alpha = 1$, the model becomes ΛCDM at late times as (r, s) tends to $(1, 0)$. This implies that the model coincides with ΛCDM at late era. Thus this model is compatible with the present observation. We plot the Figures for $\alpha = 1$, $\beta = 1$ and $\gamma = 1$. We plot scale factor vs time in Fig. 1 and it is found that expansion of Universe is increasing. In Figure 2, plot of Hubble parameter vs time is always decreasing butit in positive region which is in agreement with observations. In Fig. 3, plot of energy density with the age of Universe is always in positive region. In Fig.4, the plot of statefinder parameters approaches to $(1, 0)$ at late epoch which denotes the ΛCDM model.

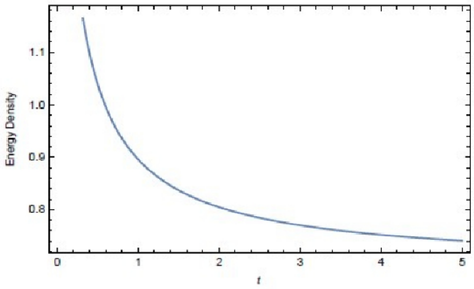

Fig.3

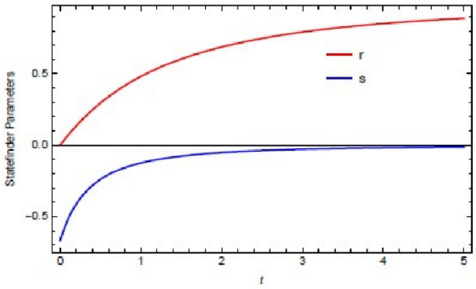

Fig.4

Fig.3: Plot of energy density vs time (t). Fig.4: Plot of statefinder parameter vs time (t)

In Fig. 5, we see the graphof deceleration parameter with time and q approaches to -1 which indicates the expansion of the Universe is Accelerating with the evolution of time.

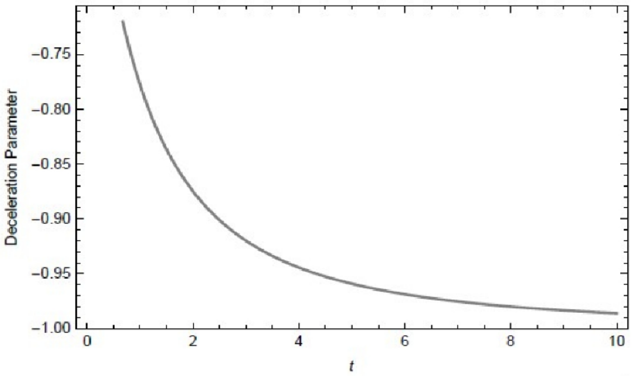

Fig.5: Plot of deceleration parameter vs time (t).

5. **Conclusion**

We developed a flat n-dimensional FRW dust cosmological models based on $R+e^{T}$ cosmology. In this dust model, non-negativity of energy density (ρ) is observed and is a decreasing with respect to time which supports the predictions of the observations. We also see that the model starts from an initial singularity which is in agreement with the hypothesis that Universe begins with a big bang. We obtain a cosmological model with hybrid expansion which exhibits the transition phase of Universe from deceleration to acceleration epoch of the Universe. At the beginning of the evolution of the Universe as $t \rightarrow 0$, deceleration parameter approaches to constant then it tends to -1 as $t \rightarrow \infty$. This implies accelerating expansion of the model with the age of evolution Universe. We found an increasing scale factor with the evolution of time that implies the expansion of Universe which coincides with recent observations. Deceleration parameter is in negative region that explains the accelerating expansion which is helped by the WMAP satellite's data. This result is in accordance with the results obtained by Khurshid et al. (2022). Since statefinder diagnostic (r, s) approaches to $(1, 0)$, our model becomes ΛCDM model. Although, this developed model is simple but it can highlight the basic evolutionary behaviors of the Universe and it may be useful in future research of the cosmology.

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The Combined Double Sumudu Transform and Adomian Decomposition Method for Solving Nonlinear Klein Gordon Equations

Anil D. Chindhe

Department of Mathematics, Balbhim College Beed, Maharashtra, India

ABSTRACT

In this paper, double Sumudu transform is combined with Adomian decomposition method to solve the nonlinear Klein Gordon equations. The nonlinear terms in the Klein Gordon equations are treated by Adomian polynomials which gives faster solution to given equations. The effectiveness and stability of the method is illustrated by solving some nonlinear Klein Gordon equation.
Keywords: Double Sumudu Transform, Non-linear Equations, Adomian Polynomial, Klein
AMS Sub ject Classification: 44A05, 81Q05

1 Introduction

The OsKar Klein and Walter Gordon in 1926 presented an equation which is known as Klein Gordon equation which plays a significant role in the field like Mathematical Physics, Relativistic physics, plasma physics, dispersive wave phenomena, quantum field theory, and nonlinear optics.According to the literature review numerous writers have made contributions to the approximate and exact solution of linear and nonlinear Klein Gordon equations.The mathematical method like iterative method, Homotopy Perturbation method,Adomian decomposition method, variational iteration method etc. are more efficient to possess the solution of integro-differential equations, partial differential equations, algebraic equations which is having non-linear and linear factors. A.M. Wazwaz[**1**] established the necessary condition to make certain that the noise term appears in the non homogeneous equation. Deeba and khuri[**2**] used Adomian decomposition method to have solution of Klein Gordon equations.Further in[**3**, **4**, **5**] used variational iteration method based on He polynomials for solving Klein Gordon equations.In order to solve numerous Klein Gordon equations, some authors have combined the integral transforms with other techniques like the variational iteration method, homotophy perturbation method, Adomian decomposition method,and differential transform method [**6**, **7**, **8**, **9**, **10**, **11**, **12**, **13**].

In the literature,various integral transforms have been used to solve many problems in mathematics,mathematical physics and engineering field.Laplace transform and Sumudu transform are the most widely used integral transforms among all other integral transforms.The Sumudu transform has an advantage over the Laplace transform is the unit preserving property.Moreover, Sumudu transform is coupled with different mathematical /computational methods to possess the solution of partial differential equation ,integro differential equations and many more.[**14**, **15**, **16**, **17**, **18**, **19**]There are some important applications and work has been done on double Sumudu transform [**20**, **21**, **22**]

1.1 Sumudu Transform

Watugala G.K.[**23**] defines the Sumudu transform for the function $f(t)$ by

$$S[f(t)] = G(u) = \int_0^\infty e^{-t} f(ut)dt \tag{1}$$

$u \in (-\tau_1, \tau_2)$ given that the set over which the function $f(t)$ defined is $A = [f(t)/\exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, if t \in (-1)^j \times [0, \infty)]$ With the right adjustments to the variable, the aforementioned transform can be reduced to the following form

$$S[f(t)] = G(u) = \frac{1}{u} \int_0^\infty e^{\frac{-t}{u}} f(t)dt \tag{2}$$

For the function $G(u)$ The inverse Sumudu transform is defined by

$$S^{-1}[G(u)] = f(t) = \lim_{T\to\infty} \frac{1}{2\Pi i} \int_{\gamma-iT}^{\gamma+iT} e^{st} G(u) du \tag{3}$$

1.2 Double Sumudu Transform

Double Sumudu transform of the function $f(t,x)$ is defined by [**24**]

$$S_2\{f(t,x):(u,v)\} = F(u,v) = \frac{1}{uv} \int_0^\infty \int_0^\infty e^{-(\frac{t}{u}+\frac{x}{v})} f(t,x) dt dx \qquad t,x \in R_+ \tag{4}$$

whenever the improper integral converges.

1.3 Double Sumudu Transform of Partial Derivative

For the function $g(x,t)$ with $S_2\{f(t,x)\} = G(u,v)$ then we have from[**21**]

1. $S_2\{\frac{\partial g(t,x)}{\partial x}\} = \frac{1}{u}G(u,v) - \frac{1}{u}G(0,v)$
2. $S_2\{\frac{\partial^2 g(t,x)}{\partial x^2}\} = \frac{1}{u^2}G(u,v) - \frac{1}{u^2}G(0,v) - \frac{1}{u}\frac{\partial G(0,v)}{\partial x}$
3. $S_2\{\frac{\partial g(t,x)}{\partial t}\} = \frac{1}{v}G(u,v) - \frac{1}{v}G(u,0)$
4. $S_2\{\frac{\partial^2 g(t,x)}{\partial t^2}\} = \frac{1}{v^2}G(u,v) - \frac{1}{v^2}G(u,0) - \frac{1}{v}\frac{\partial G(u,0)}{\partial t}$

2 Combine Double Sumudu Transform and Adomian Decomposition Method

This section outlines the methodology in which double Sumudu transform is combine with Adomian decomposition method for handling the nonlinear terms appearing in the K-G equations.We demonstrate the methodology by solving some nonlinear Klein Gordon equations. We examine the second-order nonlinear Klein-Gordon equation of the form

$$\Phi_{tt} - \Phi_{xx} + q\Phi(x,t) + N\Phi(x,t) = g(x,t) \tag{5}$$

subject to the initial conditions $\Phi(x,0) = f(x), \Phi_t(x,0) = f_1(x)$ and 'q' is constant, $N\Phi(x,t)$ is nonlinear term and $g(x,t)$ is the source term.

Double Sumudu transform applied to both sides of equation (5) yields

$$S_2\{\Phi_{tt}\}-S_2\{\Phi_{xx}\}+S_2\{q\Phi(x,t)\}+S_2\{N\Phi(x,t)\}= S_2\{g(x,t)\} \tag{6}$$

By using the double Sumudu transform's differential property, we have

$$[\frac{1}{v^2}\Phi(u,v)-\frac{1}{v^2}\Phi(u,0)-\frac{1}{v}\frac{\partial\Phi(u,0)}{\partial t}]-S_2\{\Phi_{xx}\}+S_2\{q\Phi(u,v)\}+S_2\{N\Phi(x,t)\}= S_2\{g(x,t)\} \tag{7}$$

Using a single Sumudu transform to the initial condition and using the result in above equation, we get

$$[\frac{1}{v^2}\Phi(u,v)-\frac{1}{v^2}F(u,0)-\frac{1}{v}F_1(u,0)]-S_2\{\Phi_{xx}\}+S_2\{q\Phi(u,v)\}+S_2\{N\Phi(x,t)\}= S_2\{g(x,t)\}$$

$$\Phi(u,v)=F(u,0)+vF_1(u,0)+v^2S_2\big[g(x,t)-q\Phi(u,v)\big]+v^2S_2\big[\Phi_{xx}-N\Phi(x,t)\big] \tag{8}$$

Using the inverse double Sumudu transform on the equation above, we obtain

$$\Phi(x,t)=S_2^{-1}\big[F(u,0)+vF_1(u,0)\big]+S_2^{-1}\big[v^2S_2\big[g(x,t)-q\Phi(u,v)\big]\big]+S_2^{-1}\big[v^2S_2\big[\Phi_{xx}-N\Phi(x,t)\big]\big] \tag{9}$$

Now, here we introduce the Adomian decomposition method[**25**], which gives the infinite series solution $\Phi(x,t)$ of the form $\Phi(x,t)=\sum_{\infty}^{n=0}\Phi_n(x,t)$ and the nonlinear term can be decomposed as $N\Phi(x,t)=\sum_{n=0}^{\infty}A_n$,where A_n are Adomian polynomials which can be calculated by the formula $A_n=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\big[N\big[\sum_{i=0}^{\infty}\lambda^i\Phi_i\big]\big]$

Then equation (9) becomes

$$\sum_{\infty}^{n=0}\Phi_n(x,t)=S_2^{-1}\big[F(u,0)+vF_1(u,0)\big]+S_2^{-1}\big[v^2S_2\big[g(x,t)-q\Phi(u,v)\big]\big]+$$
$$S_2^{-1}\big[v^2S_2\big[\big(\sum_{\infty}^{n=0}\Phi_n(x,t)\big)_{xx}-\sum_{n=0}^{\infty}A_n\big]\big]$$

From the equation (10), we have the recursive relations as

$$\Phi_0(x,t)=S_2^{-1}\big[F(u,0)+vF_1(u,0)\big]+S_2^{-1}\big[\big(v^2\big)S_2\big[g(x,t)-q\Phi(u,v)\big]\big]$$

$$\Phi_{k+1}(x,t) = S_2^{-1}\big[(v^2)S_2\big[\big(\sum_{\infty}^{n=0}\Phi_n(x,t)\big)_{xx} - \sum_{n=0}^{\infty}A_n\big]\big]$$

Thus the series solution of the nonlinear Klein Gordon equation (5) is of the form

$\Phi(x,t) = \Phi_0(x,t) + \Phi_1(x,t) + \Phi_2(x,t) + ...$

3 Illustarative Examples

In this section, we solve some nonlinear Klein Gordon equations using the combined double Sumudu transform and the Adomian decomposition method.

Example(1) Examine the nonlinear Klein-Gordon equation that follows

$$\Phi_{tt} - \Phi_{xx} + \Phi^2 = 2x^2 - 2t^2 + x^4t^4 \tag{10}$$

subject to the initial condition

$\Phi(x,0) = 0, \Phi_t(x,0) = 0$

Solution:- Double Sumudu transform applied to both sides of the equation (11)

$$S_2\{\Phi_{tt}\} - S_2\{\Phi_{xx}\} + S_2\{\Phi^2\} = 2S_2\{x^2\} - 2S_2\{t^2\} + S_2\{x^4t^4\} \tag{11}$$

By using the double Sumudu transform's differential property, we have

$$\Big[\frac{1}{v^2}\Phi(u,v) - \frac{1}{v^2}\Phi(u,0) - \frac{1}{v}\frac{\partial\Phi(u,0)}{\partial t}\Big] - S_2\{\Phi_{xx}\} + S_2\{\Phi^2\} = 4u^2 - 4v^2 + 576u^4v^4 \tag{12}$$

Using a single Sumudu transform to the initial condition and using the result in above equation, we get

$$\frac{1}{v^2}\Phi(u,v) = 4u^2 - 4v^2 + 576u^4v^4 + S_2\{\Phi_{xx} - \Phi^2\}$$

$$\Phi(u,v) = 4u^2v^2 - 4v^4 + 576u^4v^6 + v^2S_2\{\Phi_{xx} - \Phi^2\}$$

Using the inverse double Sumudu transform on the equation above, we obtain

$$\Phi(x,t) = x^2t^2 - \frac{t^4}{6} + \frac{1x^4t^6}{30} + S_2^{-1}\big[v^2S_2\{\Phi_{xx} - \Phi^2\}\big] \tag{13}$$

Now, here we introduce the Adomian decomposition method, which gives the infinite series solution $\Phi(x,t)$ is of the form $\Phi(x,t)=\sum_{\infty}^{n=0}\Phi_n(x,t)$, such that

$$\sum_{\infty}^{n=0}\Phi_n(x,t)=x^2t^2-\frac{t^4}{6}+\frac{1x^4t^6}{30}+S_2^{-1}\left[v^2S_2\left\{(\Phi_n)_{xx}-\sum_{n=0}^{\infty}A_n(\Phi)\right\}\right] \tag{14}$$

The Adomian polynomials for the nonlinear term $N\Phi(x,t)=\Phi^2$ are of the form

$$A_0(\Phi)=\Phi_0^2$$
$$A_1(\Phi)=2\Phi_0\Phi_1$$
$$A_2(\Phi)=2\Phi_0\Phi_2+\Phi_1^2$$
$$\vdots$$

From the equation (15) we have the recursive relations as

$\Phi_0(x,t)=x^2t^2-\frac{t^4}{6}+\frac{1x^4t^6}{30}$

$\Phi_{k+1}(x,t)=-S_2^{-1}\left[v^2S_2\left\{(\Phi_k)_{xx}-\sum_{k=0}^{\infty}A_k(\Phi)\right\}\right]$

From these recursive relation we have the series solution as

$\Phi_1(x,t)=-S_2^{-1}\left[v^2S_2\left\{(\Phi_0)_{xx}-\sum_{k=0}^{\infty}A_0(\Phi)\right\}\right]$

$\Phi_1(x,t)=-\frac{x^8t^4}{163800}+\frac{x^4t^{12}}{11880}-\frac{x^6t^{10}}{1350}+\frac{x^6t^{16}}{18662400}-\frac{t^{10}}{3240}+\frac{11x^2t^8}{840}-\frac{x^4t^6}{30}+\frac{t^6}{6}$

and so on ...

Hence the series solution of given equation (11) is given by

$\Phi(x,t)=\sum_{\infty}^{n=0}\Phi_n(x,t)=x^2t^2$

Example(2) Examine the nonlinear Klein-Gordon equation that follows

$$\Phi_{tt}-\Phi_{xx}+\Phi^2=6xt(x^2-t^2)+x^6t^6 \tag{15}$$

subject to the initial condition

$\Phi(x,0)=0,\Phi_t(x,0)=0$

Solution:- Double Sumudu transform applied to both sides of the equation (16)

$$S_2\{\Phi_{tt}\}-S_2\{\Phi_{xx}\}+S_2\{\Phi^2\}=6S_2\{x^3t\}-6S_2\{xt^3\}+S_2\{x^6t^6\} \tag{16}$$

By using the double Sumudu transform's differential property, we have

$$[\frac{1}{v^2}\Phi(u,v)-\frac{1}{v^2}\Phi(u,0)-\frac{1}{v}\frac{\partial\Phi(u,0)}{\partial t}]-S_2\{\Phi_{xx}\}+S_2\{\Phi^2\}=36u^3v-36uv^3+518400u^6v^6 \quad (17)$$

Using a single Sumudu transform to the initial condition and using the result in above equation, we get

$$\frac{1}{v^2}\Phi(u,v)=36u^3v-36uv^3+518400u^6v^6+S_2\{\Phi_{xx}-\Phi^2\}$$

$$\Phi(u,v)=36u^3v_3-36uv^5+518400u^6v^8+v^2S_2\{\Phi_{xx}-\Phi^2\}$$

Using the inverse double Sumudu transform on the equation above, we obtain

$$\Phi(x,t)=x^3t^3-\frac{3}{10}xt^5+\frac{x^6t^8}{56}+S_2^{-1}\left[v^2S_2\left\{\Phi_{xx}-\Phi^2\right\}\right] \quad (18)$$

Now, here we introduce the Adomian decomposition method, which gives the infinite series solution $\Phi(x,t)$ of the form $\Phi(x,t)=\sum_{\infty}^{n=0}\Phi_n(x,t)$, such that

$$\sum_{\infty}^{n=0}\Phi_n(x,t)=x^3t^3-\frac{3}{10}xt^5+\frac{x^6t^8}{56}+S_2^{-1}\left[v^2S_2\left\{(\Phi_n)_{xx}-\sum_{n=0}^{\infty}A_n(\Phi)\right\}\right] \quad (19)$$

The Adomian polynomials for the nonlinear term $N\Phi(x,t)=\Phi^2$ are of the form

$$A_0(\Phi)=\Phi_0^2$$

$$A_1(\Phi)=2\Phi_0\Phi_1$$

$$A_2(\Phi)=2\Phi_0\Phi_2+\Phi_1^2$$

$$\vdots$$

From the equation (20) we have the recursive relations as

$\Phi_0(x,t)=x^3t^3-\frac{3}{10}xt^5+\frac{x^6t^8}{56}$

$\Phi_{k+1}(x,t)=-S_2^{-1}\left[v^2S_2\left\{(\Phi_k)_{xx}-\sum_{k=0}^{\infty}A_k(\Phi)\right\}\right]$

From these recursive relation we have the series solution as

$\Phi_1(x,t)=-S_2^{-1}\left[v^2S_2\left\{(\Phi_0)_{xx}-\sum_{k=0}^{\infty}A_0(\Phi)\right\}\right]$

$\Phi_1(x,t)=\frac{3}{10}xt^5-\frac{2}{4050}x^4t^{10}+\frac{3}{4400}x^2t^{112}-\frac{1}{4368}x^9t^{13}+\frac{1}{19600}x^7t^{15}-\frac{1}{959616}x^{12}t^{18}$

and so on ...

Hence the series solution of given equation (16) is given by

$\Phi(x,t) = \sum_{\infty}^{n=0} \Phi_n(x,t) = x^3t^3$

Example(3) Examine the nonlinear Klein-Gordon equation that follows

$$\Phi_{tt} - \Phi_{xx} + \Phi^2 = x^2t^2 \tag{20}$$

subject to the initial condition

$\Phi(x,0) = 0, \Phi_t(x,0) = x$

Solution:- Double Sumudu transform applied to both sides of the equation (21)

$$S_2\{\Phi_{tt}\} - S_2\{\Phi_{xx}\} + S_2\{\Phi^2\} = S_2\{x^2t^2\} \tag{21}$$

By using the double Sumudu transform's differential property, we have

$$[\frac{1}{v^2}\Phi(u,v) - \frac{1}{v^2}\Phi(u,0) - \frac{1}{v}\frac{\partial\Phi(u,0)}{\partial t}] - S_2\{\Phi_{xx}\} + S_2\{\Phi^2\} = 4u^2v^2 \tag{22}$$

Using a single Sumudu transform to the initial condition and using the result in above equation, we get

$$\frac{1}{v^2}\Phi(u,v) = \frac{u}{v} + 4u^2v^2 + S_2\{\Phi_{xx} - \Phi^2\}$$

$$\Phi(u,v) = uv + 4u^2v^4 + v^2S_2\{\Phi_{xx} - \Phi^2\}$$

Using the inverse double Sumudu transform on the equation above, we obtain

$$\Phi(x,t) = xt + \frac{1}{12}x^2t^4 + S_2^{-1}[v^2S_2\{\Phi_{xx} - \Phi^2\}] \tag{23}$$

Now, here we introduce the Adomian decomposition method, which gives the infinite series solution $\Phi(x,t)$ of the form $\Phi(x,t) = \sum_{\infty}^{n=0} \Phi_n(x,t)$, such that

$$\sum_{\infty}^{n=0} \Phi_n(x,t) = xt + \frac{1}{12}x^2t^4 + S_2^{-1}[v^2S_2\{(\Phi_n)_{xx} - \sum_{n=0}^{\infty} A_n(\Phi)\}] \tag{24}$$

The Adomian polynomials for the nonlinear term $N\Phi(x,t) = \Phi^2$ are of the form

$$A_0(\Phi) = \Phi_0^2$$
$$A_1(\Phi) = 2\Phi_0\Phi_1$$
$$A_2(\Phi) = 2\Phi_0\Phi_2 + \Phi_1^2$$
$$\vdots$$

From the equation (25) we have the recursive relations as

$\Phi_0(x,t) = xt + \frac{1}{12}x^2t^4$

$\Phi_{k+1}(x,t) = -S_2^{-1}\left[v^2 S_2\left\{(\Phi_k)_{xx} - \sum_{k=0}^{\infty} A_k(\Phi)\right\}\right]$

From these recursive relation we have the series solution as

$\Phi_1(x,t) = -S_2^{-1}\left[v^2 S_2\left\{(\Phi_0)_{xx} - \sum_{k=0}^{\infty} A_0(\Phi)\right\}\right]$

$\Phi_1(x,t) = \frac{1}{180}t^6 - \frac{1}{12}x^2t^4 - \frac{1}{252}x^3t^7 - \frac{1}{12960}x^4t^{10}$

and so on ...

Hence the series solution of given equation (21) is given by

$\Phi(x,t) = \sum_{\infty}^{n=0} \Phi_n(x,t) = xt$

CONCLUSION

This paper successfully solves the nonlinear Klein-Gordon equations using the combined double Sumudu transform and Adomian decomposition method.The illustrative examples shows that the effectiveness and power fullness of the combined method.The purpose of combination of Adomian decomposition method with double Sumudu transform is attained by reducing the computational work in solving nonlinear Klein -Gordon equations.

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